Inverted Pendulum Operational Manual

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3 Introduction

The Inverted Pendulum system shown in Figure 1 below consists of several parts. These are the counterweight, pendulum, pendulum weight, pendulum encoder, horizontal link, motor and platform.

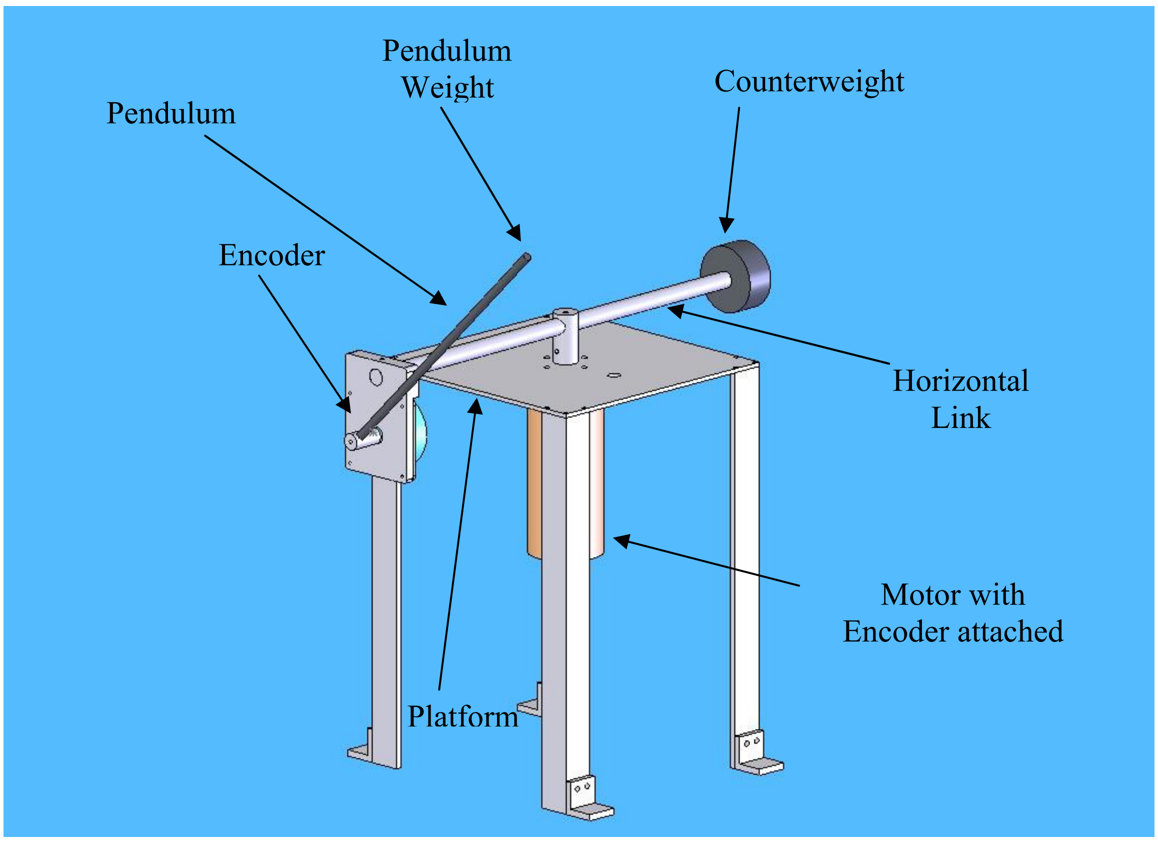

Figure 1: Inverted Pendulum System

3.1 Equations of Motion

The following governing equations for the inverted pendulum system were derived using the program autolev.

$$\tau_1 = -\tfrac{1}{3} L_b \left[\left(m_b L_b + 3 m_d L_b\right) \left(\sin(\theta_1)\cos(\theta_1)\,\dot{\theta}_2^2 - \ddot{\theta}_1 \right) + \left(1.5 m_b + 3 m_d\right) \left(g \sin(\theta_1) - \left(L_a - L_c\right)\cos(\theta_1)\,\ddot{\theta}_2 \right) \right]$$

$$\tau_2 = \tfrac{1}{2} L_b \cos(\theta_1)\left[\tfrac{4}{3} L_b (m_b + 3m_d)\sin(\theta_1)\dot{\theta}_2\,\dot{\theta}_1 + (L_a - L_c)(m_b + 2m_d)\ddot{\theta}_2\right] +$$

$$\tfrac{1}{12}\left[m_a L_a{}^2 + 12 m_c L_c{}^2 + 3m_a (L_a - 2L_c)^2 + 12 m_b (L_a - L_c)^2 + 12 m_d (L_a - L_c)^2 + 12 m_e (L_a - L_c)^2\right]\ddot{\theta}_2$$

$$- \tfrac{1}{3} L_b \sin(\theta_1)\left[1.5(L_a - L_c)(m_b + 2m_d)\dot{\theta}_1{}^2 - L_b (m_b + 3m_d)\sin(\theta_1)\ddot{\theta}_2\right]$$

where m_e is the mass of the encoder and its attachment (.1972 Kg) m_d is the weight of the blob at the end of the pendulum (.0528 Kg) m_c is the mass of the counterweight (0.3313 Kg) m_a is the mass of the horizontal link (.1390 Kg) m_b is the mass of the pendulum link (.0629 Kg) L_a is the length of the horizontal link (.4064m) L_b is the length of the pendulum link (.254m), L_c is the distance between the counterweight and the motor (.184 m) b_1 is the viscous friction term in the pendulum and b_2 is the viscous friction term in the motor. The angles θ_1 and θ_2 are defined as shown in the figure below

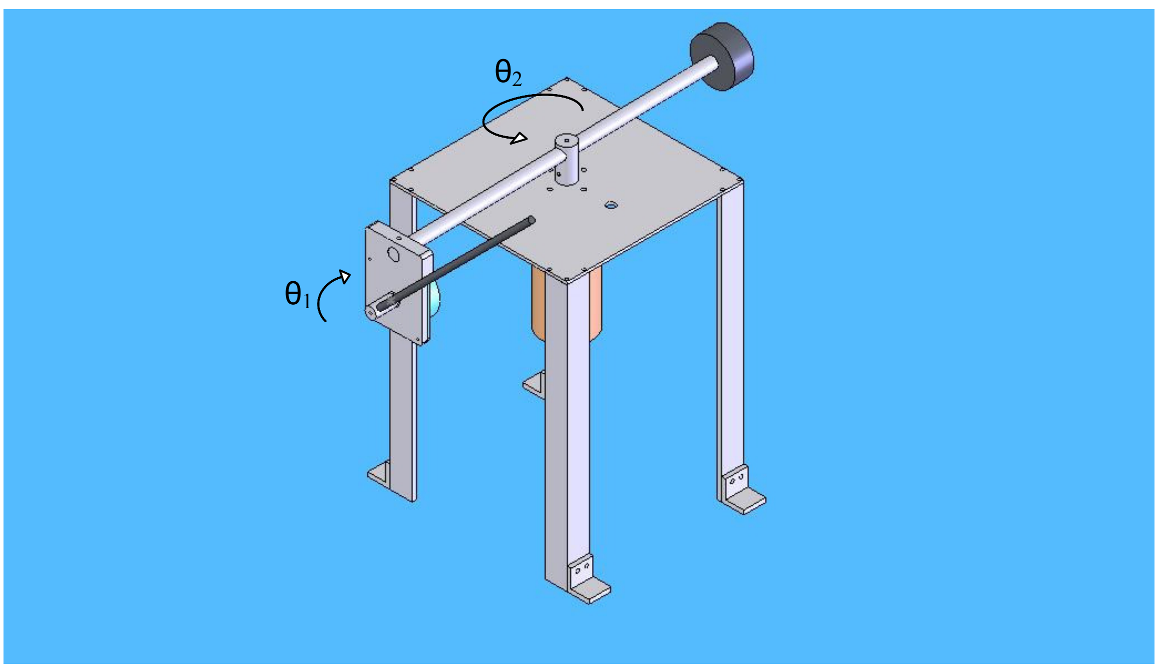

Figure 2: Angle Definitions

Putting the equations in the form,

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

Results in the equations

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} m_b L_b^2 + m_d L_B^2 & L_b (L_a - L_c) \cos(\theta_1) \left(\frac{1}{2} m_b + m_d \right) \\ L_b (L_a - L_c) \cos(\theta_1) \left(\frac{1}{2} m_b + m_d \right) & L_b^2 \sin^2 \theta_1 \left(\frac{1}{3} m_b + m_d \right) + (L_a - L_c)^2 (m_d + m_b + m_e) + m_c L_c^2 + \frac{1}{12} m_a L_a^2 + \frac{1}{4} m_a (L_a - 2L_c)^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$
$$+ \begin{bmatrix} -L_b g \sin(\theta_1)\left(m_d + \frac{1}{2} m_b\right) - \left(\frac{1}{3} m_b + m_d \right) \left(L_b^2 \sin(\theta_1) \cos(\theta_1) \dot{\theta}_2^2 \right) + b_1 \dot{\theta}_1 \\ L_b^2 \left(\frac{2}{3} m_b + 2m_d \right) \sin(\theta_1) \cos(\theta_1) \dot{\theta}_2 \dot{\theta}_1 - L_b (L_a - L_c) \left(\frac{1}{2} m_b + m_d \right) \sin(\theta_1) \dot{\theta}_2^2 + b_2 \dot{\theta}_2 \end{bmatrix}$$

With the equations in this form $\ddot{\theta}_1$ and $\ddot{\theta}_2$ can be found by solving the equation shown below.

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \alpha^{-1} (\tau - \beta)$$

3.2 Inverted Pendulum Control System

The equations of motion for the inverted pendulum system are highly nonlinear. Consequently a unique method of controlling the system had to be derived since most control theory is based on a linear system assumption. The control system designed used two types of control, energy control and state space feedback control. The energy control was used when the pendulum was in the non-linear range while the feedback control was used when the pendulum neared a equilibrium point thus approaching linearity.

3.2.1 Linearization and Linear Control

To apply a linear control law to the inverted pendulum system the equations of motion were linearised about the desired equilibrium point by using the Jacobean. After the linearization of the system, feedback gains were calculated by using the linear-quadratic regulator on the A and B matrices of the system.

3.2.2 Non Linear / swing up Control

The Non-Linear swing up control works by supplying the system with enough energy to go from the stable equilibrium state (down) to the unstable equilibrium state (up). At each time interval the energy of the system (potential energy of pendulum + kinetic energies of pendulum and horizontal link) is compared to the desired energy in the system (in our case 0). Based on the difference in energy and the position of the pendulum a torque is applied to the link. The relationship between the torque (τ) and Energy (E) is given below.

$$\tau = kEsign\left(\dot{\theta}\cos(\theta)\right)$$

Where k is some gain. The equation shows that the amount of torque that is put in is proportional to the energy in the system. k determines the reaction time, however if k is too large, then the system will most likely overshoot. The sign term deals with the timing and direction issues of the pendulum to give it more net energy.

3.2.3 Motor System

The governing equations of the inverted pendulum system have torque terms while the input to the motor is a voltage. Consequently some method of converting voltages to torques and vice versa was needed in order to control the system. By analyzing the equations of motion for the motor it was discovered that the Voltage input of the motor was related to the Torque output at steady state by the following equation.

$$\tau = \frac{K_T V}{R}$$

Where Kt is the Torque Constant (0.0923 Nm/A) and R is the Terminal Resistance of the motor (1.6Ω).

4 Inverted Pendulum Control Hardware

The control hardware for the Inverted Pendulum system consists of many components.

4.1 Harris Board 2.0

The majority of the control hardware for the inverted pendulum system is located on the Harris Board 2.0. A detailed description of the Board as well as a schematic of the board can be found in Appendix G.

4.2 PIC 18F452 Microcontroller

The PIC microcontroller is the heart of the control system. It first acquires the position and velocity for the pendulum and horizontal link from the encoder counter. These values are then multiplied by a gain vector to determine the magnitude of the voltage that should be applied to the motor. The microcontroller then outputs the voltage using PWM (Pulse Width Modulation) to the Motor Drive Circuitry.

4.3 Xilinx Spartan 3 FPGA

The FPGA was used to implement 2 quadrature encoder counters so as to measure the positions and velocities of both the pendulum and link. The velocities of the pendulum and the link were estimated by differentiating the position data that is taking the difference between the two most recent position values and dividing the result by the sampling time of the control system.

4.4 Freescale H-Bridge MC33886

The microcontroller could not be used to drive the Motor due the large torques and consequently large currents required to stabilize the inverted pendulum. An H-Bridge was used to create a motor drive circuit. The H-Bridge was able to deliver currents of up to 5A.

4.5 AD 7233 D/A Converters

D/A converters were used to convert the digital encoder counter data to analog voltages in order to real-time control in MATLAB.

4.6 4N35-M Opto-Isolators

Opto-Isolators were used to isolate the H-Bridge from the microcontroller to prevent large current surges from resetting the microcontroller or FPGA. These current surges arise from the DC motor and frequently occur when the motor changes direction.

4.7 US Digital H3-2048-HS Ball Bearing Optical Shaft Encoder

To measure the pendulum angle a US digital encoder with 2048 counts per revolution was used.

4.8 National Instruments DAQ-6036E PCMCIA Card

The DAQ-6036E card is used to acquire the pendulum and link position data and also to output the control voltage to the motor.

4.9 LOKO DPS-5050 Power Supply

A LOKO DPS-5050 was used to power the H-Bridge. The power supply has a maximum voltage rating of 50 V and a maximum current rating of 5A. In the control system this power supply was run at approximately 6V.

4.10 HP 6236B 5V 3A Power Supply

A HP 6236B power supply was used to power the microcontroller, FPGA, pendulum encoder and motor encoders. The power supply has a maximum voltage rating of 40V and a maximum current rating of 3A. In the control system this power supply was run at approximately 5V.

4.11 Reliance Brush Type DC Servo Motor

A 55oz-in continuous torque Reliance Brush DC motor was used to drive the horizontal link of the inverted pendulum system. An encoder with a resolution of 1000 counts per revolution was attached to the motor in order to sense the position of the horizontal link.

5 MATLAB Control

The first task in performing Real-Time control in MATLAB is wiring the two bread boards properly. The encoders need to be wired to the bread board with the two D/A converters on it as shown in Figure 3 below. The wiring diagram for this board can be found in Appendix A: Figure 10

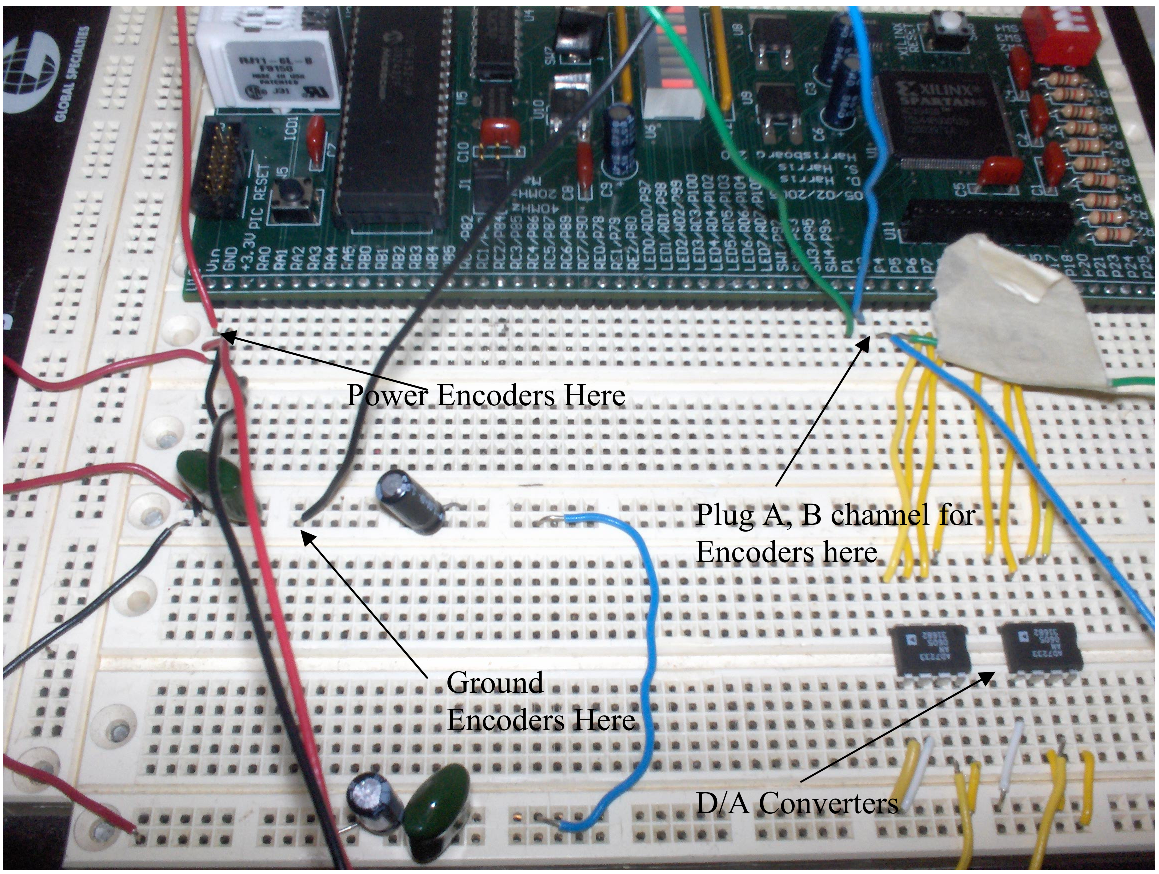

Figure 3: Wiring D/A Bread Board

The user should now connect the outputs of the D/A converters to the National Instruments Box, using BNC cables as shown in Figure 4 below. θ_1 should be connected to Analog channel 1 and θ_2 should be connected to analog channel 0.

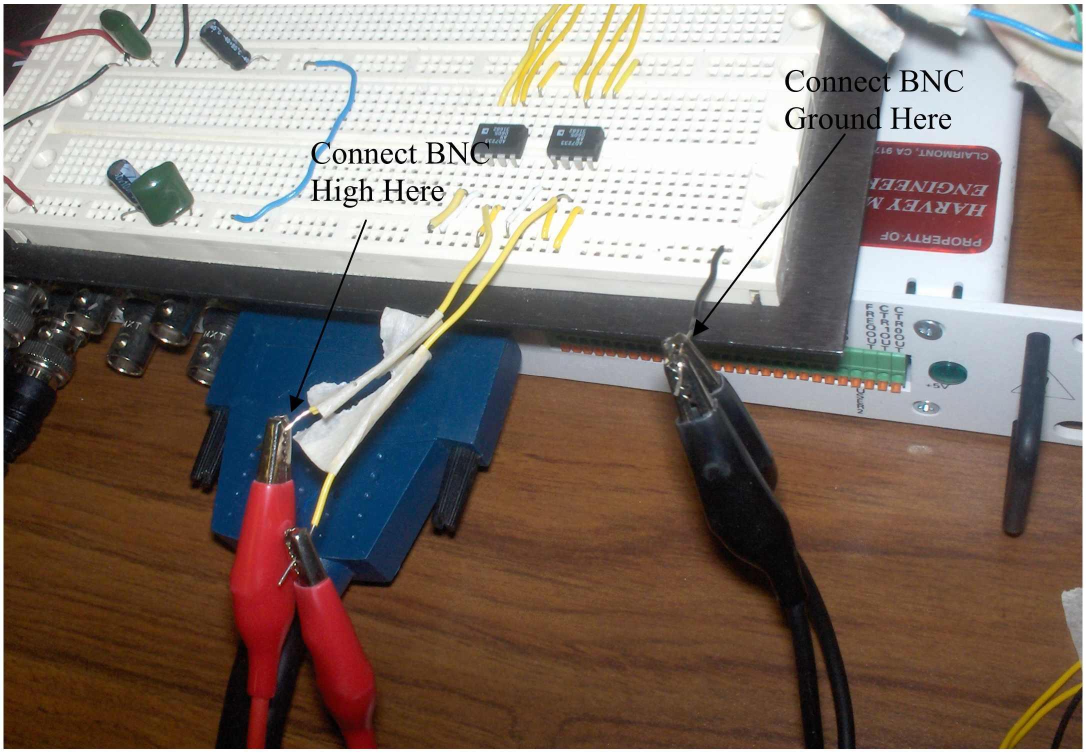

Figure 4: Wiring D/A outputs to NI box

After the D/A board has been wired properly the user should power the bread board using two power supplies. The user should plug both COM outputs from the power supply into the ground input on the board. The user should then plug -15V into V1, +15V into V2 and 5V into V3 as shown in the figure below.

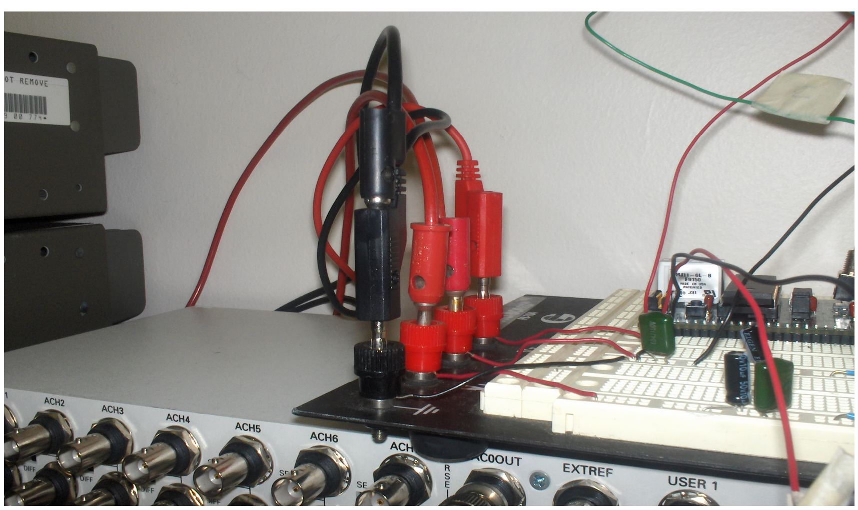

Figure 5: Powering D/A Bread Board

After the D/A bread board has been properly wired the user should no wire the Motor Drive bread board. The wiring diagram for this board can be found in Appendix A: Figure 11. Figure 6 shows where analog outputs from the National Instruments box should be plugged into on the board.

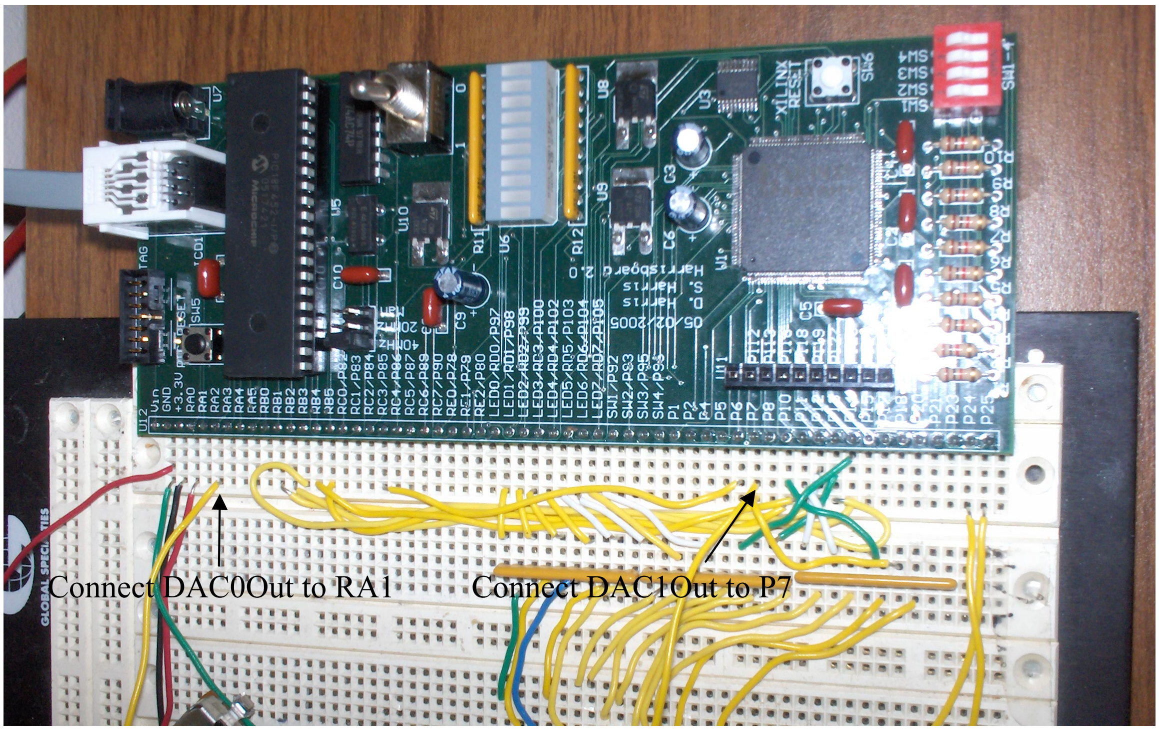

Figure 6: Motor Drive Circuit

Figure 7 shows what H-Bridge pins the leads from the motor should be plugged into on the board.

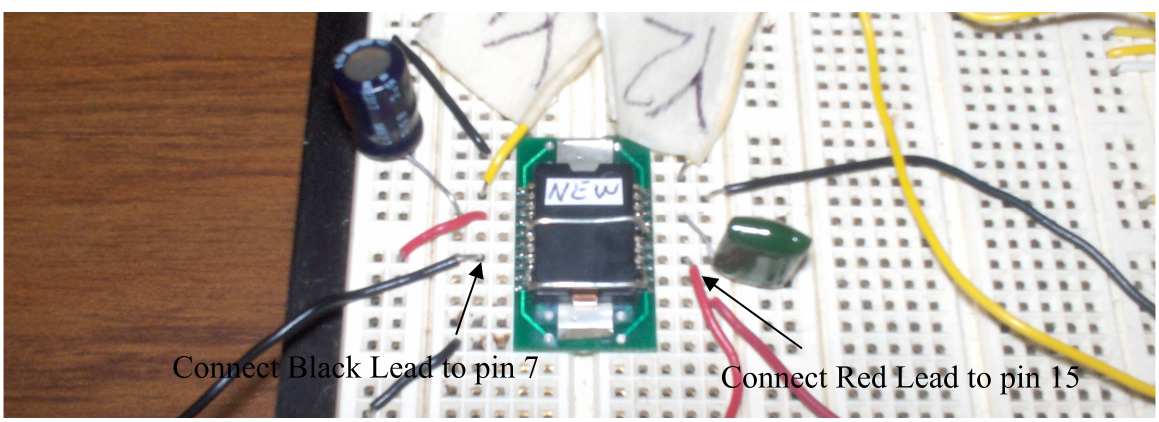

Figure 7: Connecting Motor to H-Bridge

The next step in setting up the Motor Drive bread board is to power it. The board should be powered by two separate power supplies. The first power supply the Loko Power DPS-5050 should be set to 6V and the positive terminal connected to V3 on the bread board and the negative terminal connected to V1. The second power supply a HP 6236 B should be set to 5V with the positive terminal going to V2 and the COM terminal going to V1.

After the board have been wired it may be necessary to program the FPGA's and Microprocessor on the respective breadboards. The FPGA on the D/A board should be programmed with Xilinx project found in C:\Eout while the FPGA on the Motor Drive board should be programmed with the project found in C:\PendFirmFinal. The PIC on the Motor Drive board should be programmed with the pwm.mcp project found in C:\Pendulum\PICFiles. Information on how to program the PIC and FPGA can be found in the PIC and FPGA manual. The user should also set Switch 1 on the Motor Drive board to the open position as shown in the Figure 8 below.

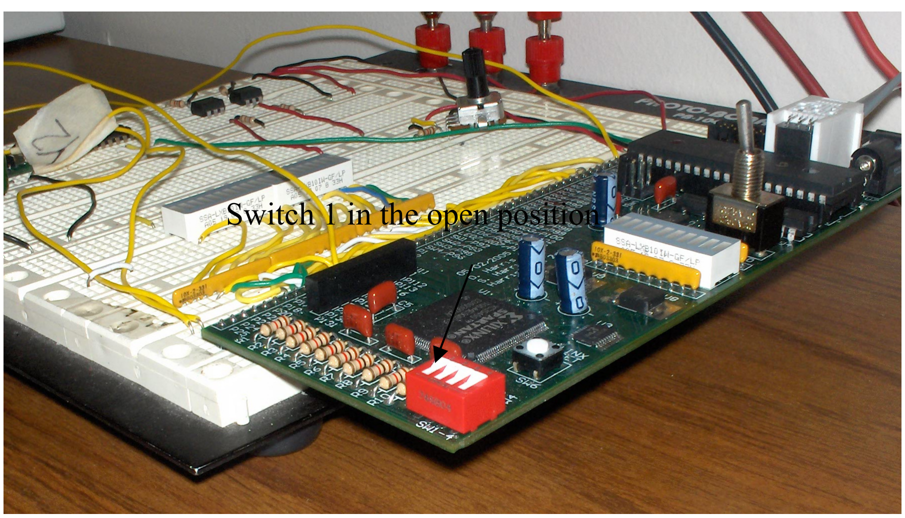

Figure 8: Opened Switch 1

The next step in running MATAB Real-Time control of the Inverted Pendulum System is to open MATLAB and set the current directory to C:\Pendulum\Pendulum. Open up the Inverted Pendulum GUI and run a simulation of the StateSpace controller. More information on operating the GUI can be found in the MATLAB manual. Next open the StateSpaceRT SIMULINK model. Next click on Tools-> Real Time Workshop ->Build model. Then click on the connect to target icon in the model window and then click on play button as shown in the figure below.

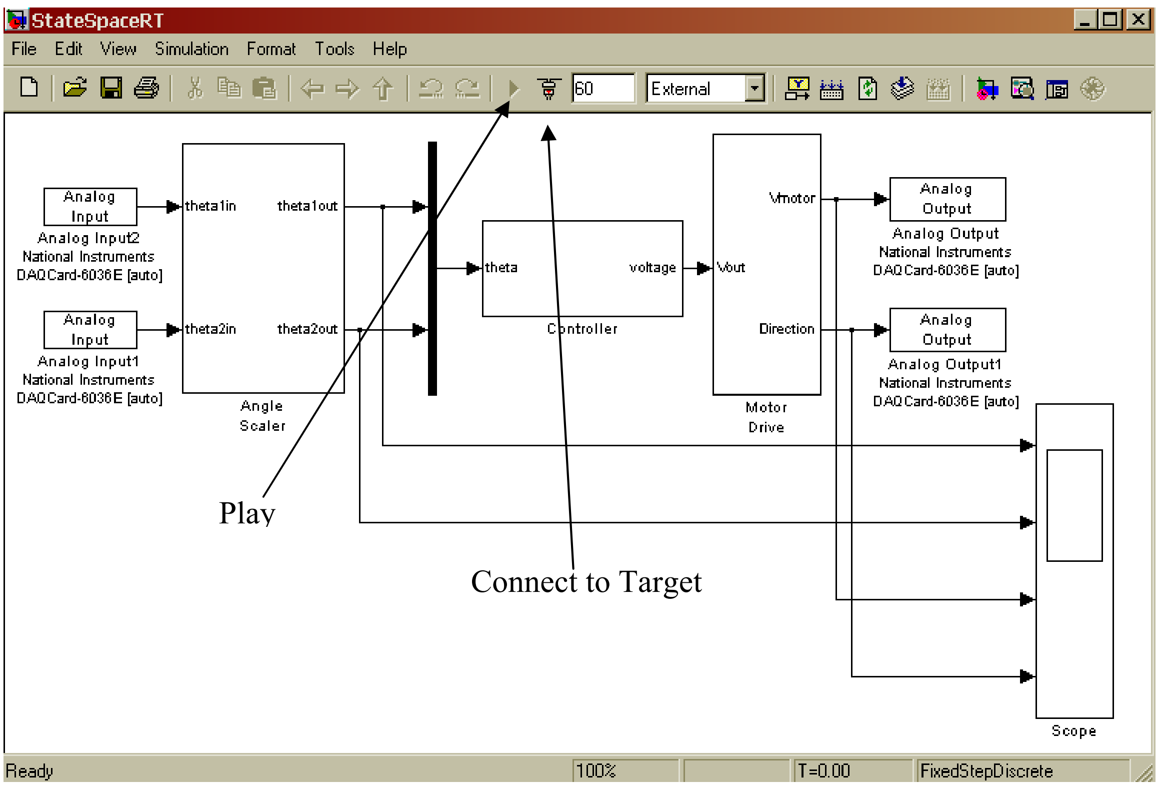

Figure 9: StateSpaceRT Model

6 Microprocessor Control

The first task in performing microprocessor control on the inverted pendulum system is to wire the motor drive breadboard properly. The circuit diagram for wiring the motor drive bread board can be found in Appendix A: Figure 12. Connect the encoders to the motor drive breadboard in a similar fashion as shown in Figure 3. Connect the motor leads to the H-Bridge as shown in Figure 7. A wire should be placed between pin RC3 (PIC) and P7 (FPGA) as shown in the circuit diagram. Also switch 1 should be placed in the closed position. After the wiring has been accomplished the breadboards should be powered in a similar fashion as mentioned in the MATLAB Control section. V2 should be connected to 5V, V3 should be connected to 6V (Loko Power Supply) and V1 should be a common ground for the power supplies.

After the board have been wired it may be necessary to program the FPGA's and Microprocessor on breadboards. The FPGA on the breadboard should be programmed with the project found in C:\PendFirmFinal. The PIC on the breadboard should be programmed with the project found in C:\Pendulum\I Research Files. Information on how to program the PIC and FPGA can be found in the PIC and FPGA manual.

7 Appendix A: Electrical Circuit Diagrams

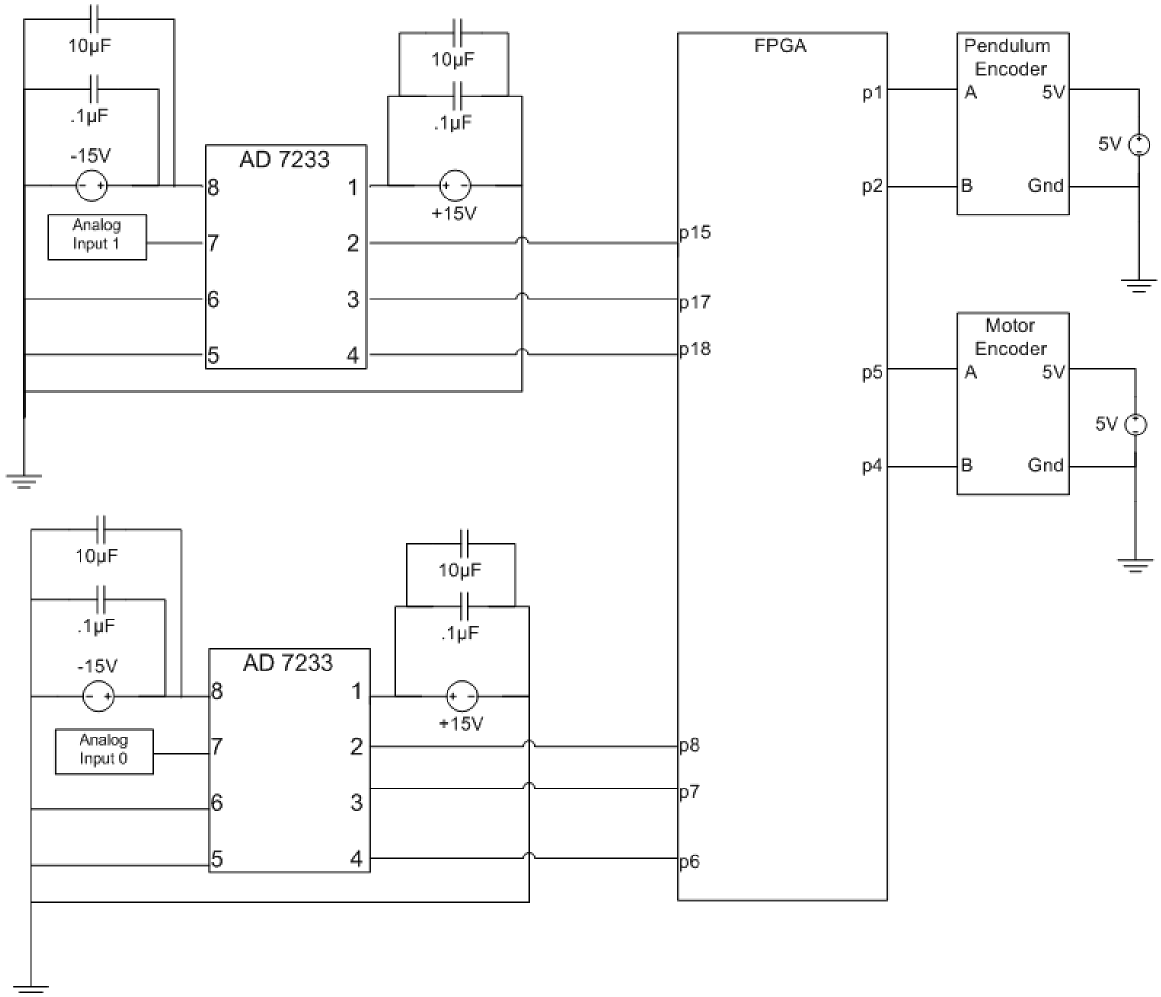

Figure 10: D/A Breadboard Circuit Diagram

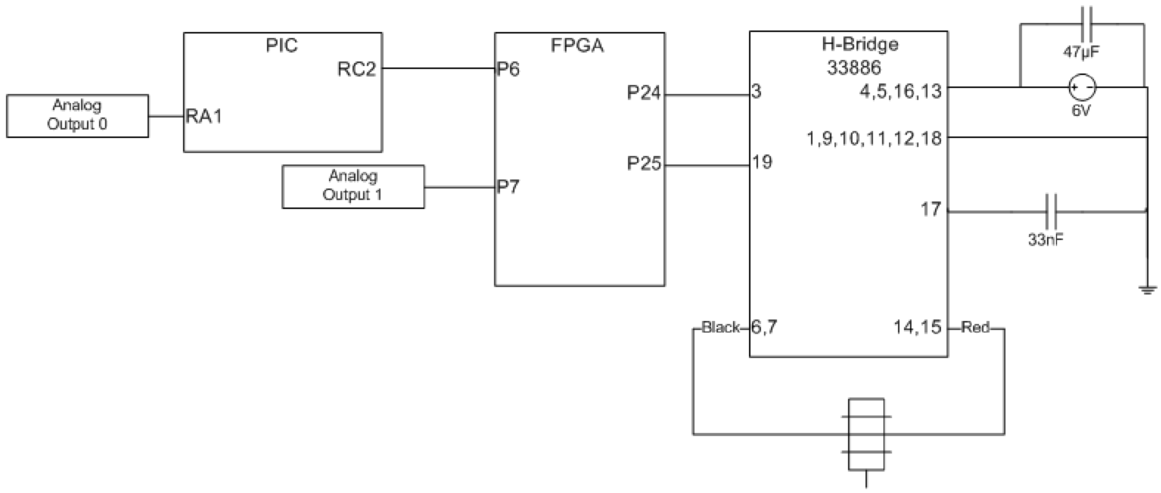

Figure 11: Motor Drive Circuit (MATLAB Control)

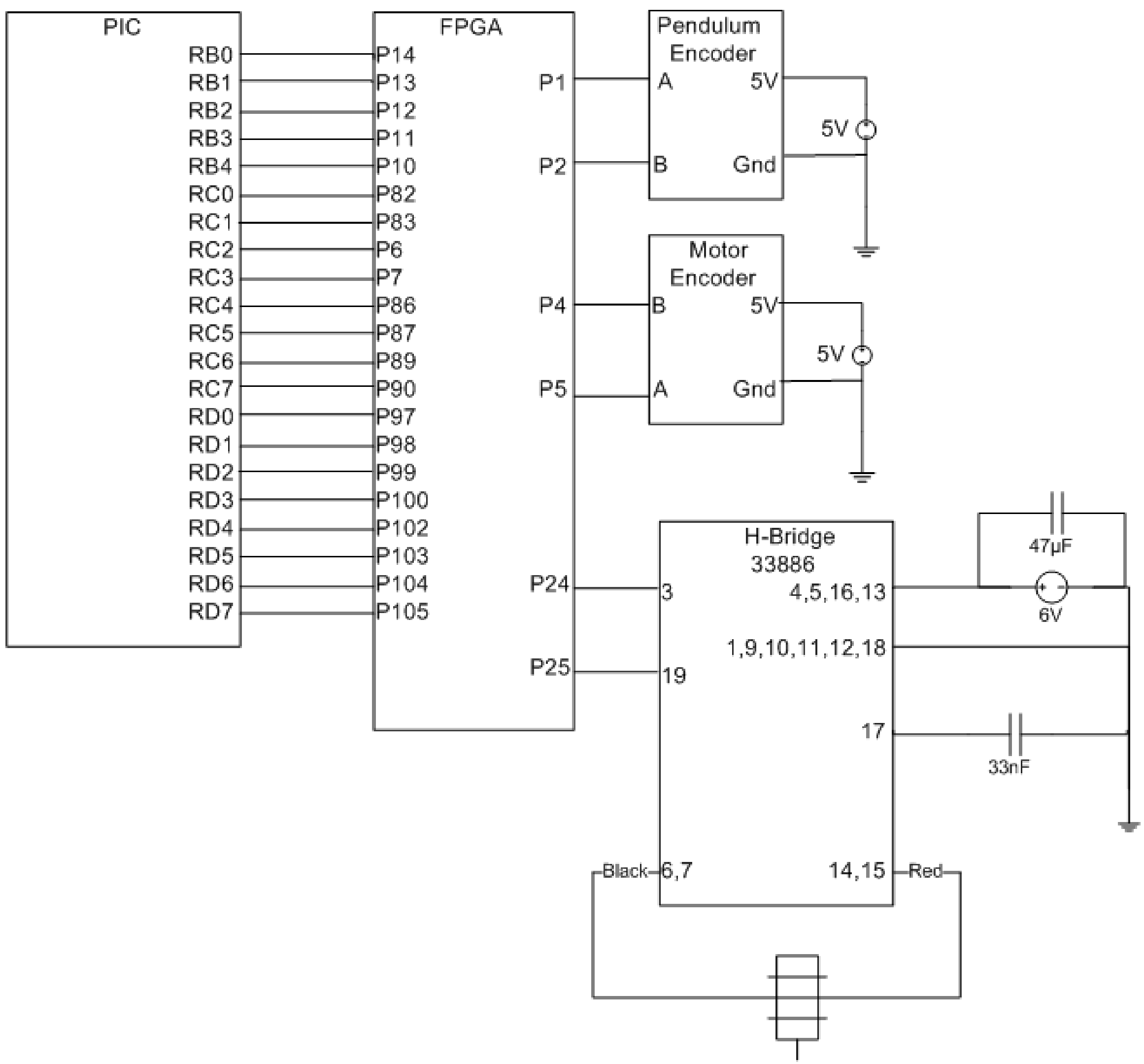

Figure 12: Motor Drive Circuit (Microprocessor Control)

8 Appendix B: Inverted Pendulum Mechanical Drawings

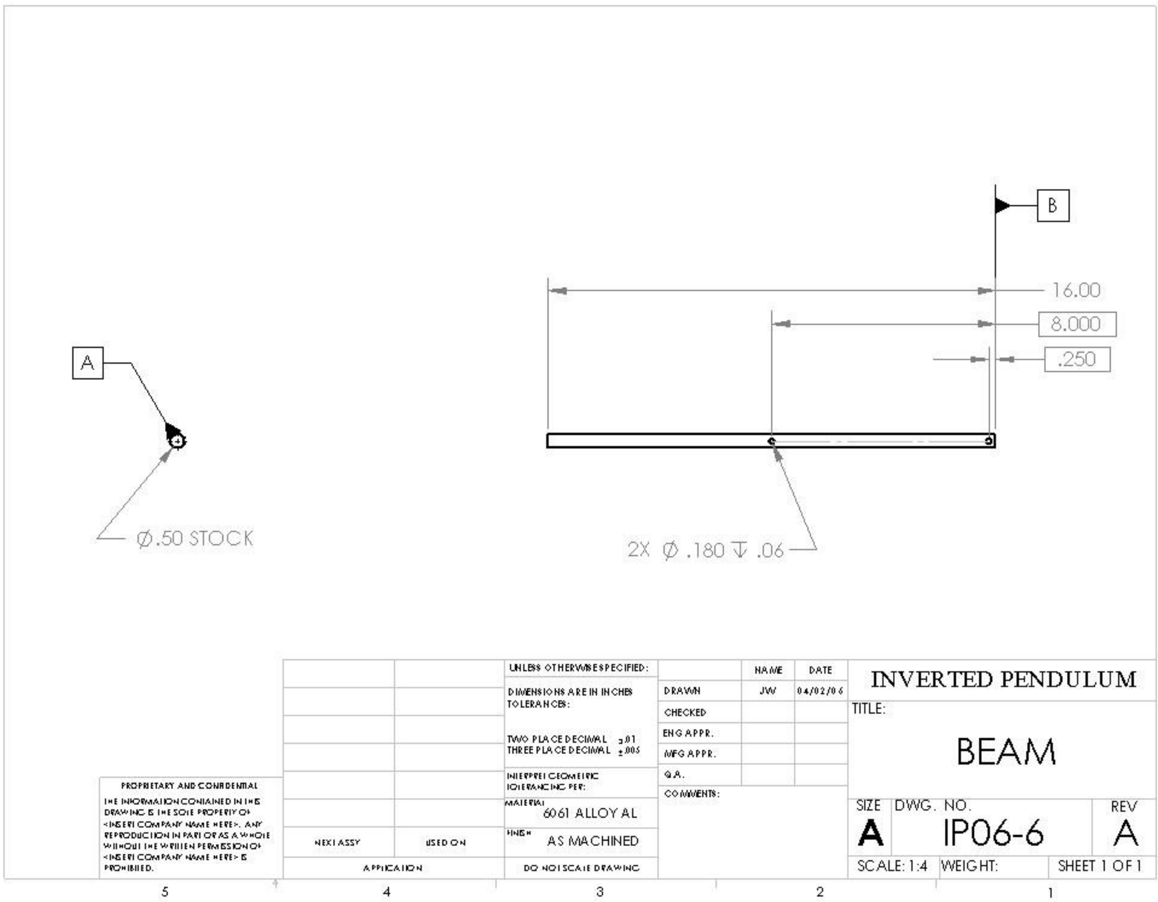

Figure 13: Beam Engineering Drawing

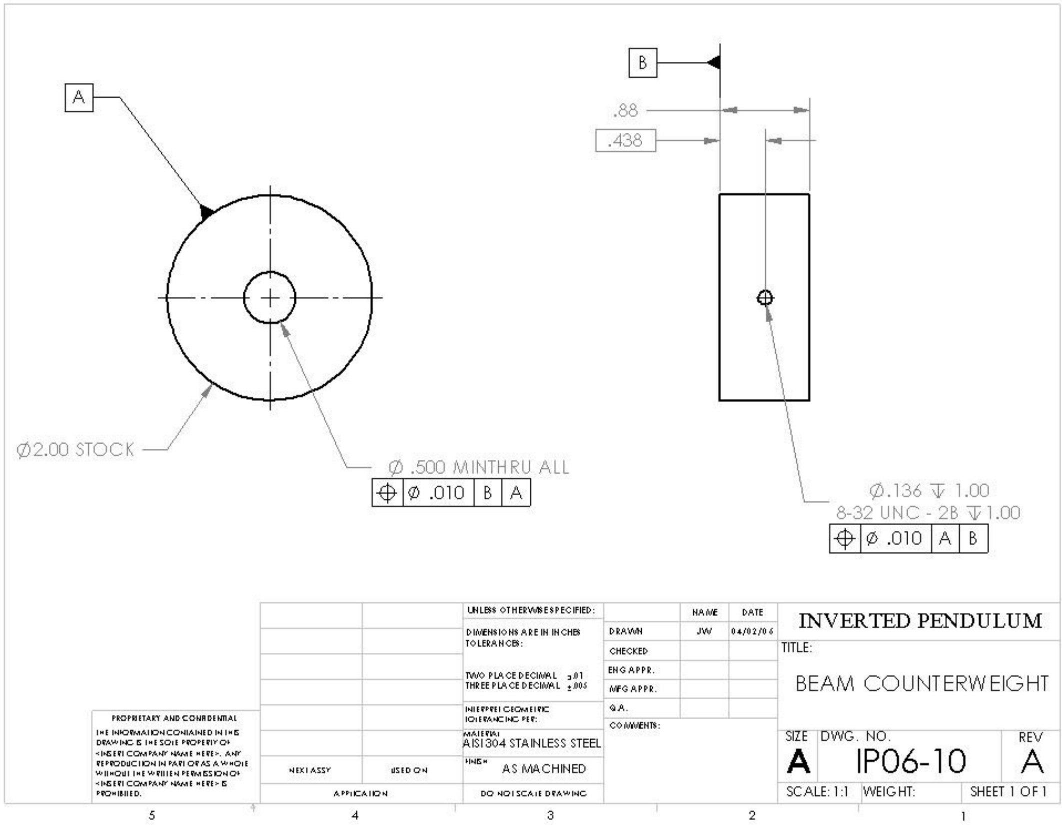

Figure 14: Counterweight Engineering Drawing

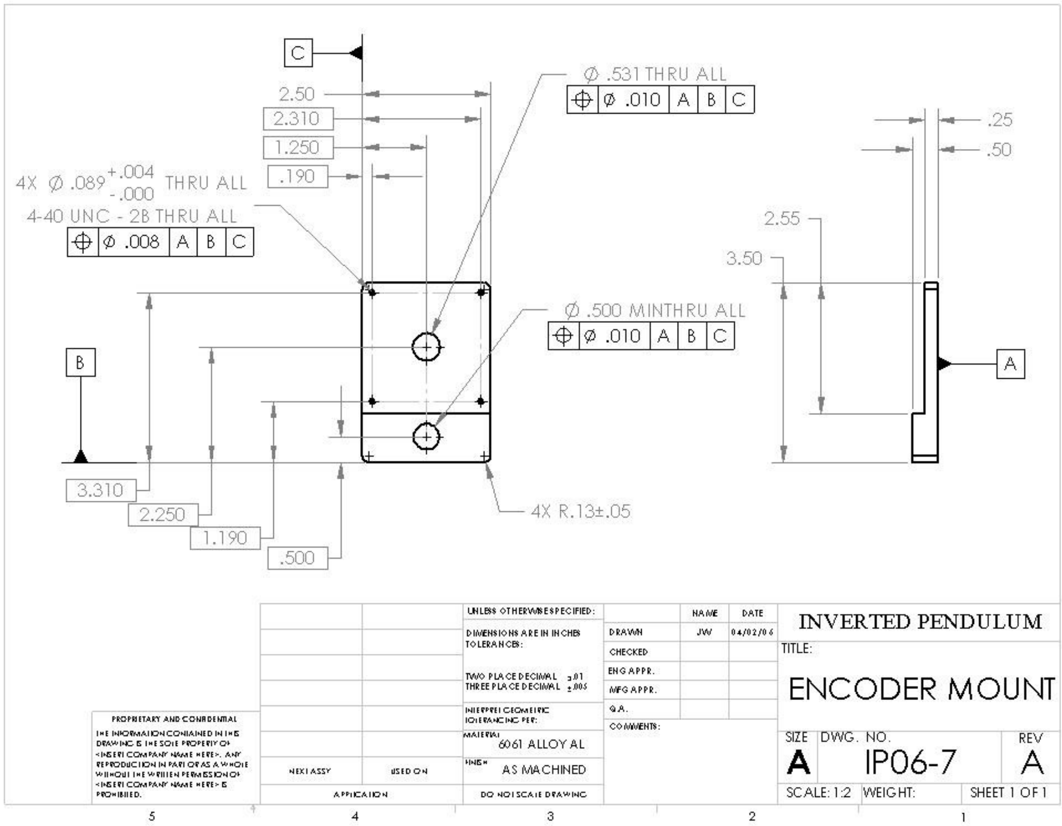

Figure 15: Encoder Mount Engineering Drawing

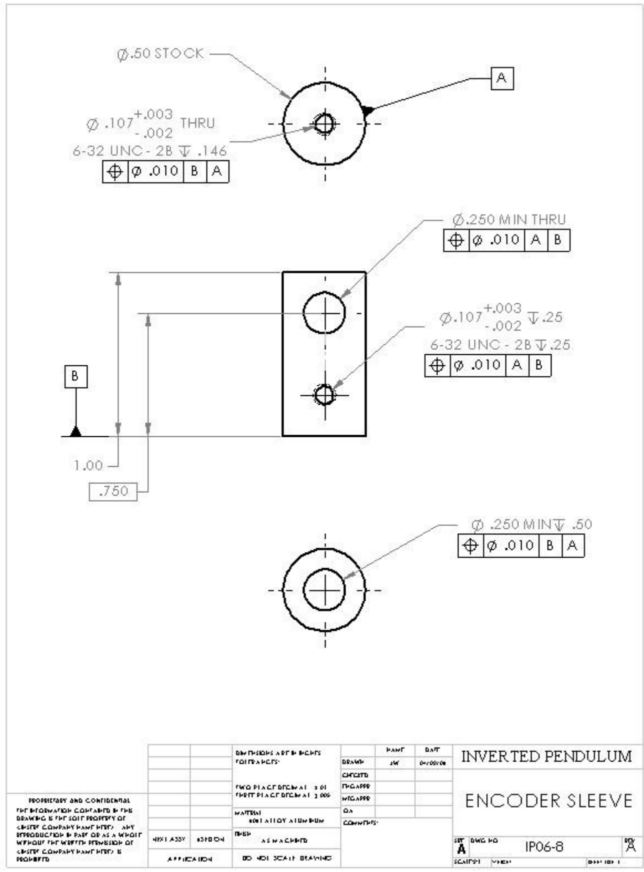

Figure 16: Encoder Sleeve Engineering Drawing

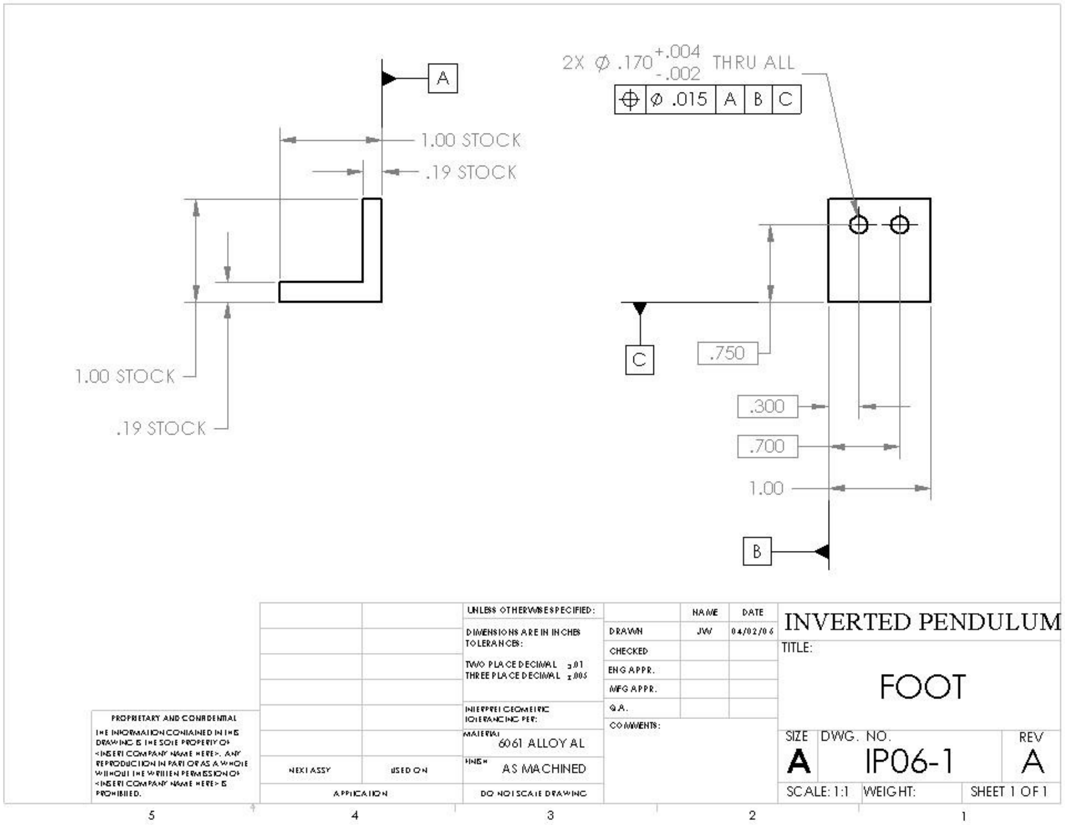

Figure 17: Foot Engineering Drawing

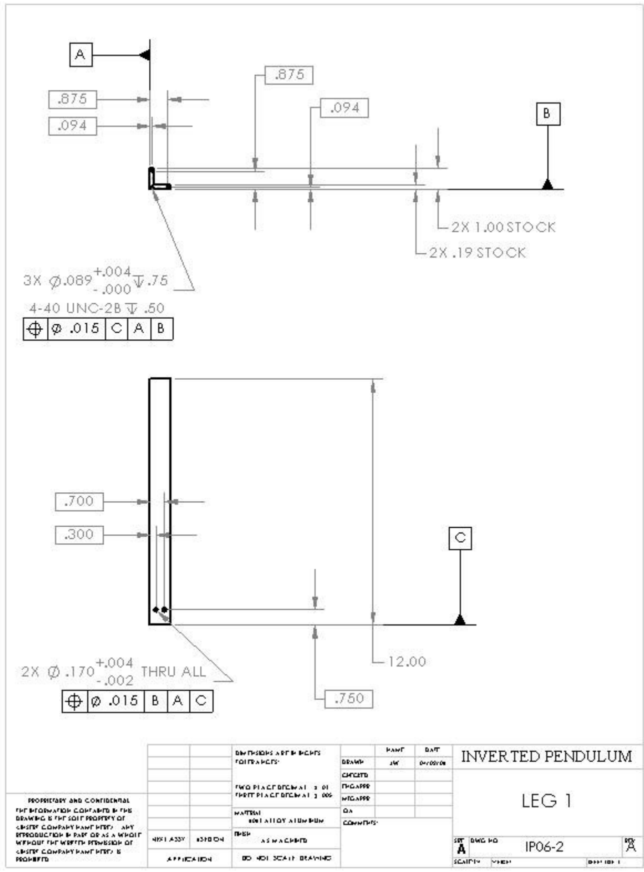

Figure 18: Leg(1) Engineering Drawing

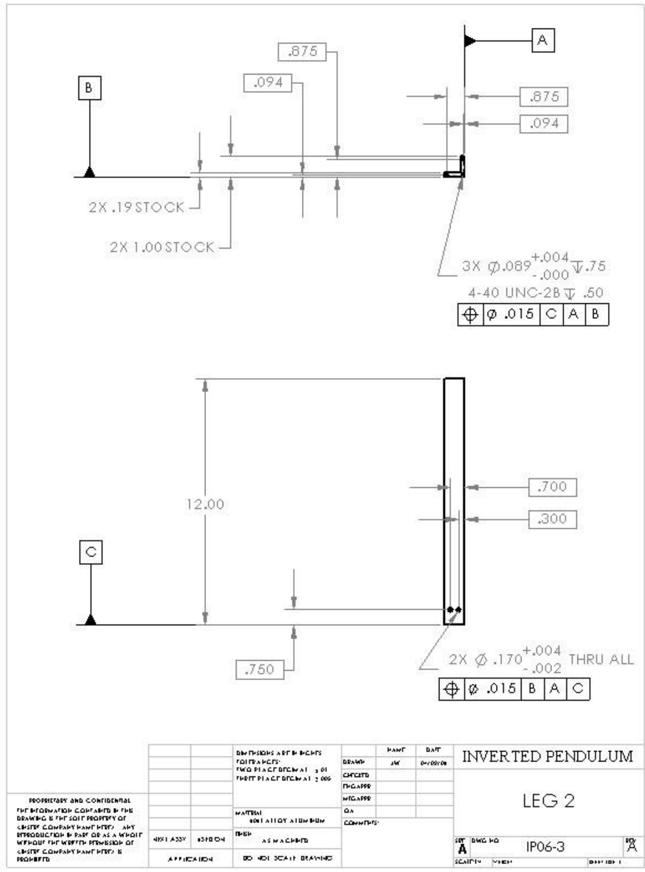

Figure 19: Leg(2) Engineering Drawing

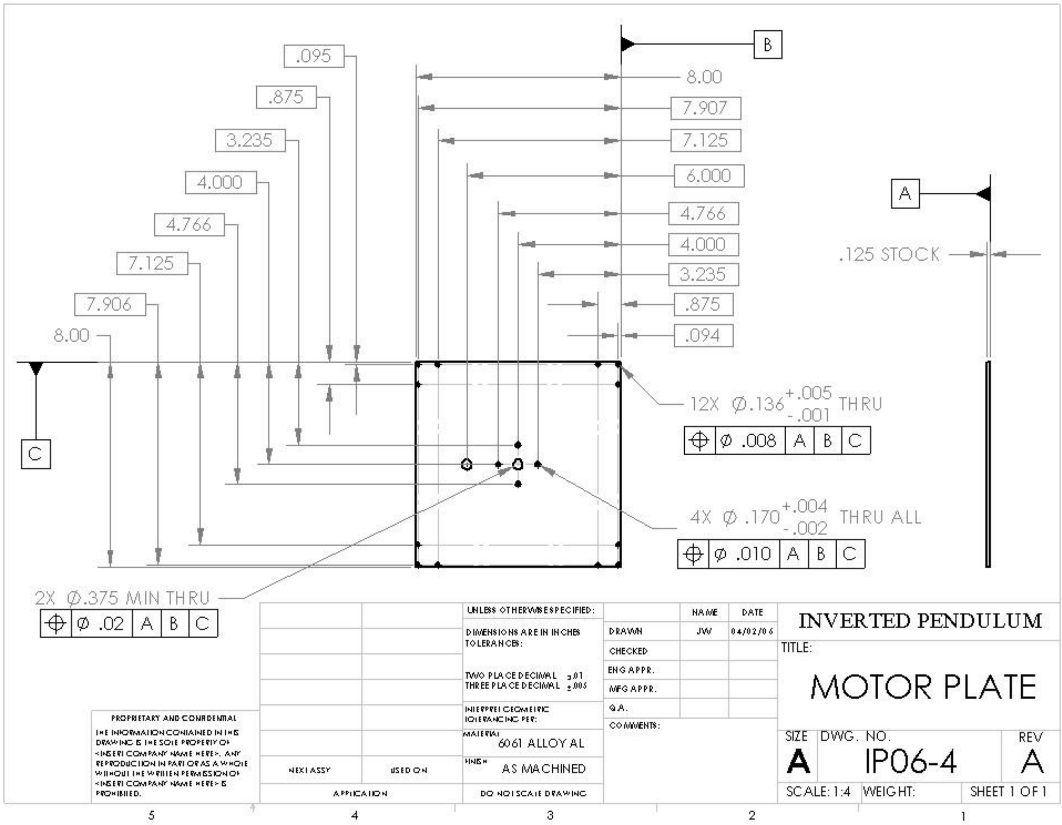

Figure 20: Motor Plate Engineering Drawing

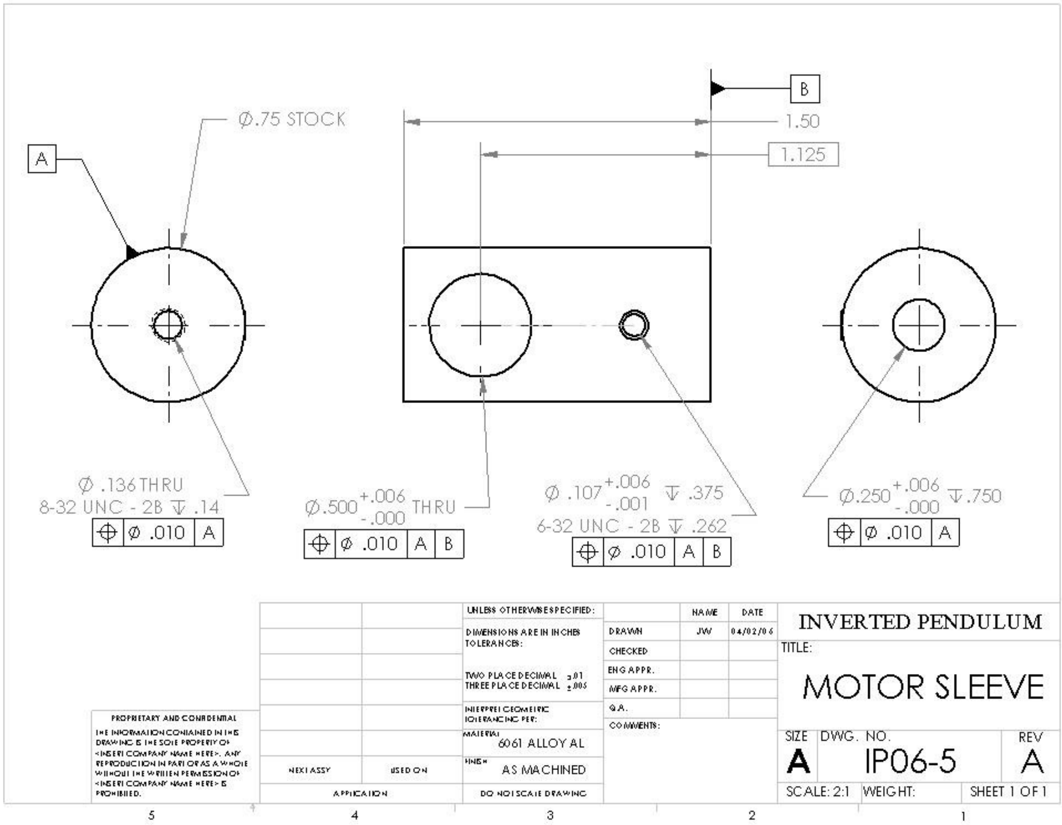

Figure 21: Motor Sleeve Engineering Drawing

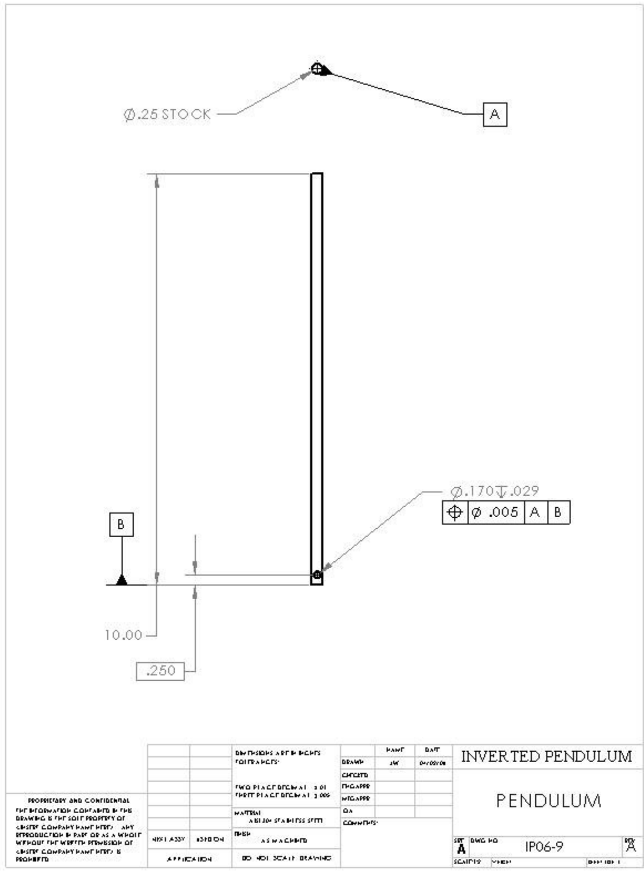

Figure 22: Pendulum Engineering Drawing