Receiver-Initiated Collision Avoidance in Multi-Hop Ad Hoc Networks

Renato M. de Moraes
Department of Electrical Engineering
University of California at Santa Cruz
Santa Cruz, CA 95064, USA

J.J. Garcia-Luna-Aceves
Department of Computer Engineering
University of California at Santa Cruz
Santa Cruz, CA 95064, USA

***Abstract*— Previous analysis of collision avoidance in ad hoc networks with hidden terminals has focused on sender-initiated schemes, like the IEEE 802.11 distributed coordination function. This paper presents analytical modeling of receiver-initiated collision avoidance schemes in multi-hop ad hoc networks. Our analysis shows that receiver-initiated schemes can offer substantial improvements over sender-initiated techniques, provided that nodes have a good estimate of the traffic offered by their neighbors.**

***Index Terms*—Ad hoc networks, analytical modeling, collision avoidance, network protocols, wireless networks**

I. INTRODUCTION

The hidden-terminal problem in wireless networks was first addressed by Tobagi and Kleinrock [1], who showed that protocols based on carrier sensing perform as poorly as ALOHA [2] when hidden terminals exist in the network. As a result, collision avoidance schemes have been proposed that try to avoid collisions of data packets at the receivers without having to employ more than one receiver or transmitter per node. In these schemes, small control packets are used to establish a handshake between sender and receiver, such that nodes around the two nodes are made to back off or wait, until the receiver is able to obtain the data packet from the sender successfully, and in such cases as the 802.11 standard [3], the sender receives a positive acknowledgment from the receiver.

The collision avoidance handshake can be sender-initiated or receiver-initiated. In the sender-initiated case, a sender that needs to transmit data to a receiver first sends a request-to-send (RTS) packet to the receiver, who responds with a clear-to-send (CTS) if it receives the RTS correctly. A sender transmits a data packet only after receiving a CTS successfully, and the receiver sends a positive acknowledgment to the sender. ALOHA [2] or CSMA [1] can be used by the senders to transmit RTS's, and nodes around senders and receivers back off for long enough periods to allow the three- or four-way handshake to complete. In a receiver-initiated scheme, the receiver polls a sender with a ready-to-receive (RTR) packet, and the sender sends a data packet or an indication that no data packet is forthcoming.

Most of the analytical models presented to analyze sender- and receiver-initiated collision avoidance protocols consider only single-hop networks [4], [5], [6]. Recently, however, Wang and Garcia-Luna-Aceves [7] presented an analytical model for sender-initiated collision avoidance in multi-hop networks based on the preliminary results from [8].

This paper summarizes the analytical model presented in [7] and [8], and extends it to study the performance of the Receiver Initiated Multiple Access (RIMA) scheme with simple polling and dual polling introduced in [4], which we denote by RIMA-SP and RIMA-DP, respectively. In both cases, time series representation of the channel around the nodes are presented, and then, the throughput for RIMA-SP and RIMA-DP are derived.

We consider two traffic cases: *heavy* and *equiprobable*. In the heavy traffic case, a polled neighbor always has traffic to send to the polling node. In the equiprobable case, a polled neighbor has traffic to send to the polling node with a probability that is uniformly distributed over all its neighbors. Numerical results are presented from the models obtained for RIMA-SP and RIMA-DP. Short and long data packet length are analyzed for the heavy and equiprobable traffic cases. The results indicate that the receiver-initiated schemes perform better than sender-initiated schemes for both traffic models, but obviously much better performance is attained when a polled node always has traffic for the polling node.

The remainder of the paper is organized as follows. Section II introduces the basic analytical model and its parameters. Section III presents the RIMA-SP study with the appropriate model extension. Section IV presents the

This work was supported in part by the Baskin Chair of Computer Engineering at UCSC.

Renato M. de Moraes was supported by CAPES/Brazil.

RIMA-DP analysis. Section V shows the numerical results obtained and compares them to the sender-initiated scheme. Section VI concludes the paper with implementation considerations for making a receiver-initiated scheme feasible in ad hoc networks.

II. Approximated Model Analysis

We adopt the same channel model description used by Wang and Garcia-Luna-Aceves [7] and Wu and Varshney [8]. The channel is modeled as a circular region containing some nodes which can transmit to each other. It is assumed that these nodes can have *weak interactions* with nodes outside the region and this means that communication decisions (transmit, defer, or back off) are rarely affected by those outer nodes. These assumptions result in a very simple model behavior for a node x, such that we can apply a three-state Markov chain describing the possible states for that node. Figure 1 shows the Markov chain where x is the polling node. The *wait* state corresponds to the case in which the node x is not ready to poll, or is idle. The *succeed* state corresponds to the case when the node can complete a successful handshake with other nodes. The *fail* state describes the case in which a collision occurs.

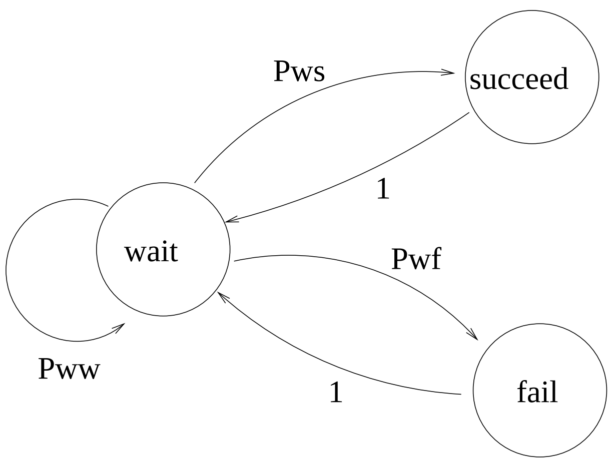

Fig. 1. Markov chain model for node x.

In the network model we consider, nodes are bi-dimensionally Poisson distributed with density λ, so that the probability of finding i nodes in an area A is given by:

$$p(i, A) = \frac{(\lambda A)^i}{i!} e^{-\lambda A}. \quad (1)$$

If we denote by N the average number of nodes within a circular region of radius R, then $N = \lambda \pi R^2$.

To simplify our analysis, we assume that nodes operate in a time-slotted mode, because the maximum propagation delay τ is considered to be much smaller than packet transmission time. Thus, each time slot has duration τ seconds. Also the duration of RTR, CTS, data, and acknowledgment (ACK) packets are normalized to the slot time duration and are assumed to be a multiple of this time slot. They are respectively denoted by l_{rtr}, l_{cts}, l_{data}, and l_{ack}.

The time spent in *wait* is τ seconds, a slot duration. The time spent in *fail* and *succeed* depends on the receiver-initiated scheme and they will be described in subsequent sections. The transition probability from *succeed* to *wait* is one as we assume that no node is allowed to transmit continuously. The transition probability from *fail* to *wait* is also one, because a node must become idle after detecting a collision.

Now, suppose that a receiver node x is in *wait* state at the beginning of a slot. At the beginning of the next slot, x initiates a successful handshake to y with probability P_{ws}, where y is the node polled by x, assumed to be at a distance r from x. Any node at most distance R from node x can communicate with node x. Figure 2 shows the regions involved in this handshake where now we need to consider the hidden node area, $B(r)$, affecting the x transmission to y. Takagi and Kleinrock [9] presented this interfering area, and Wu and Varshney [8] were the first to use this result to calculate the performance analysis introducing a Markov chain model for this type of communication channel. In reference [9] the region $B(r)$ has been shown to be:

$$B(r) = R^2 \left[\pi - 2q \left(\frac{r}{2R} \right) \right], \quad (2)$$

where $q(t) = arccos(t) - t\sqrt{1 - t^2}$.

Hence, the transition probability $P_{ws}(r)$ that node x can successfully communicate with node y is a function of the distance r between them, and also depends on how the hidden nodes located in $B(r)$ will behave during x transmission. The next sections will consider two cases of receiver-initiated polling schemes in order to calculate $P_{ws}(r)$ and the corresponding throughputs.

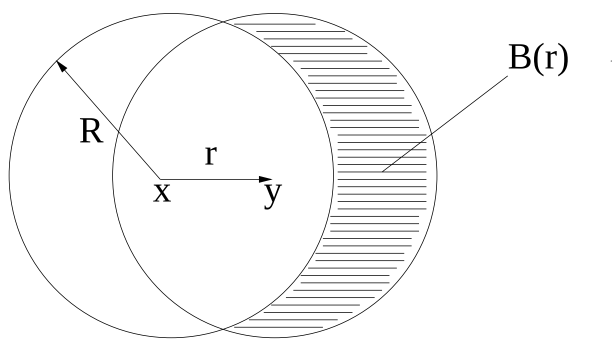

Fig. 2. Illustration of communication regions involved. x is the receiver initiating handshake. B(r) is the hidden region to x node.

We also assume that the receiver is ready to poll a node with probability p and not ready with probability $(1-p)$, so being p a protocol design parameter. However, it is important to note that, even when the node is ready to poll, it may be that the channel is being already in use by other nodes. Thus, we want to consider the likelihood that the receiver can actually use the channel to poll another node, and this probability we call p'. In [8] it was shown that p and p' are related by

$$p' = p \cdot Pr\{\text{Channel is sensed idle in a slot}\}. \quad (3)$$

So p' is indeed the protocol design parameter used to obtain the throughput performance.

III. Receiver-Initiated with Simple Polling (RIMA-SP)

The analysis considered here is analogous to that in [4] but includes the effect of hidden nodes as shown in Figure 2. Furthermore, we also consider that the receiver sends an acknowledgment over the same channel after successfully receiving the data packet from sender. In this case the receiver, x, polls the sender, y, by sending an RTR packet, and if y has data to transmit it does so (see Figure 3). After that, x sends the ACK to y and then returns to idle state. Figure 3 shows the channel time series for node x and its Vulnerable Period (VP) from nodes in the hidden region, i.e., if an interfering node attempts to transmit during VP, then its packet and the x packet will collide at node y so preventing node x to take control of the channel.

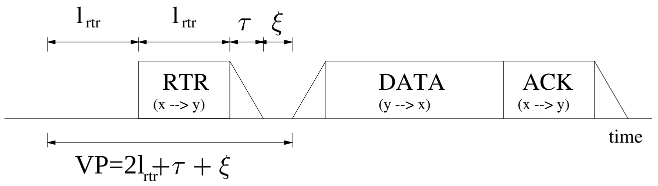

Fig. 3. Time series representation for channel around x in RIMA-SP successful handshake case. Vulnerable Period (VP) is $2l_{rtr} + \tau + \xi$.

Figure 3 illustrates that the vulnerable period of node x, regarding terminals in region $B(r)$, begins l_{rtr} seconds before RTR is sent to y and ends at ξ seconds after node y receives RTR from x. Therefore, $VP = 2l_{rtr} + \tau + \xi$. Otherwise, packets sent from $B(r)$ collide with the RTR packet at node y.

Hence, ξ is an additional collision-avoidance waiting period at the polled node prior to answering an RTR. From Theorem 1 in [4], RIMA-SP provides correct collision avoidance in presence of hidden terminals, provided that $\xi = \tau$.

In addition, as part of the RIMA-SP and RIMA-DP schemes, node x senses the channel after sending its RTR packet. If it detects carrier immediately after transmitting its RTR, node x assumes that a collision or a successful data transfer to a hidden node is taking place. Accordingly, it sends a No-Transmission-Request (NTR) packet to y to stop y from sending data that would only collide at x [4].

Now, from Figure 1 and Figure 3, the time spent in each state of the Markov chain is given by:

$$T_w = \tau,$$

$$T_s = l_{rtr} + \tau + \xi + l_{data} + \tau + l_{ack} + \tau,$$

$$T_f = l_{rtr} + \tau + \xi + \tau,$$

where T_f reflects the fact that the receiver sending the poll will realize that either a collision happens or y has no data to x if after a time of $(l_{rtr} + \tau + \xi + \tau)$ seconds there is no answer from the polled sender y.

We now compute the probability P_{ws}. From Figures 1, 2, and 3, a transition from *wait* state to *succeed* state will depend on the following simultaneous events: $\{x$ *polls* y *in a slot*$\}$, $\{y$ *does not transmit in a slot*$\}$, $\{$*none of the terminals within* R *of* x *transmits in the same slot*$\}$, $\{$*none of the terminals in* $B(r)$ *transmits for (*$2l_{rtr} + \tau + \xi$*) slots, given that the distance between* x *and* y *is* $r\}$, $\{y$ *has data to send to* $x\}$, where they are all independent from each other. In terms of probabilities we have:

P_1 = Pr$\{x$ *polls* y *in a slot*$\}$,
P_2 = Pr$\{y$ *does not transmit in a slot*$\}$,
P_3 = Pr$\{$*none of the terminals within* R *of* x *transmits in the same slot*$\}$,
$P_4(r)$ = Pr$\{$*none of the terminals in* $B(r)$ *transmits for* $(2l_{rtr} + \tau + \xi)$ *slots / r*$\}$,
P_5 = Pr$\{y$ *has data to send to* $x\}$,

so that,

$$P_{ws}(r) = P_1 \cdot P_2 \cdot P_3 \cdot P_4(r) \cdot P_5\ . \quad (4)$$

From Eq. (3) we see that $P_1 = p'$ and $P_2 = (1 - p')$. Remembering that we are considering Poisson bi-dimensional distribution of nodes, then using equation Eq. (1) we have

$$\begin{aligned} P_3 &= \sum_{i=0}^{\infty}(1-p')^i \frac{(\lambda\pi R^2)^i}{i!} e^{-\lambda\pi R^2} \\ &= \sum_{i=0}^{\infty}(1-p')^i \frac{N^i}{i!} e^{-N} \\ &= e^{-p'N} \end{aligned}$$

where πR^2 is the circular area of radius R centered at x.

To obtain P_4 we need to find the probability that none of the terminals within $B(r)$ transmits in a time slot, call it $p_4(r)$, so

$$\begin{aligned} p_4(r) &= \sum_{i=0}^{\infty}(1-p')^i \frac{(\lambda B(r))^i}{i!} e^{-\lambda B(r)} \\ &= e^{-p'\lambda B(r)}. \end{aligned}$$

Thus, $P_4(r)$ must account for the entire vulnerable period time. Accordingly we obtain

$$\begin{aligned} P_4(r) &= (p_4(r))^{2l_{rtr}+\tau+\xi} \\ &= e^{-p'\lambda B(r)(2l_{rtr}+\tau+\xi)}. \end{aligned}$$

The probability P_5 is indeed a network design parameter. The probability that y has data to send when polled by x is assumed here in two contexts. The first one is to consider that each node around x has data to transmit to it with equal probability. Thus, if there are N nodes within the circular region of radius R centered at x then $P_5 = 1/N$. We call it *equiprobable traffic assumption*. This assumption preserves the validity of prior analytical results [4], [5], [10].

The other condition over P_5 is to assume that a polled node has always data to send. Then $P_5 = 1$ and we call it *heavy traffic assumption*.

Finally, to calculate P_{ws}, we assume in our model that terminal x may choose any neighboring terminal as its polled node with equal probability. It results that the average number of nodes within a region of radius r is proportional to r^2, and the probability density function of the distance r between node x and y is given by

$$f(r) = 2r, \text{ for } 0 < r < 1,$$

where r was normalized to the radius R.

Hence, P_{ws} is obtained by

$$\begin{aligned} P_{ws} &= \int_0^1 2rP_{ws}(r)dr \\ &= 2p'(1-p')e^{-p'N}\int_0^1 re^{-p'\lambda B(r)(2l_{rtr}+\tau+\xi)}dr. \end{aligned}$$

Now, from Figure 1, we want to obtain P_{ww} the probability that the receiver node x remains in *wait* state in a slot. It is given by the product of the probability that x does not poll any node and the probability that none of the nodes within R of x transmits in the same slot. Thus,

$$P_{ww} = (1-P_1)\cdot P_3 = (1-p')e^{-p'N}.$$

Denote by π_s, π_w and π_f the steady-state probabilities of states *succeed*, *wait* and *fail*, respectively, for the Markov chain in Figure 1. Thus, we have

$$\begin{aligned} \pi_w P_{ww} + \pi_s + \pi_f &= \pi_w \\ \pi_w P_{ww} + 1 - \pi_w &= \pi_w \\ \pi_w &= \frac{1}{2-P_{ww}} \\ \pi_w &= \frac{1}{2-(1-p')e^{p'N}}. \end{aligned}$$

Solving for π_s we obtain

$$\pi_s = \pi_w P_{ws} = \frac{P_{ws}}{2-(1-p')e^{-p'N}}.$$

The stationary probability for *fail* state can simply be obtained by using the normalizing probability condition, so

$$\pi_f = 1 - \pi_s - \pi_w.$$

Now we can obtain the throughput of the receiver-initiated protocol. Again we emphasize we study two situations: *equiprobable traffic assumption* (Th_{equip}) and *heavy traffic assumption* (Th_{heavy}). From Figure 1, the throughput is obtained by the rate of time the channel is used to transmit successful data to the total time spent in all states of the Markov chain. So, the throughput is given by (all the terms shown below were calculated above):

$$Th_{equip} = \frac{\frac{1}{N}\pi_s \cdot l_{data}}{\pi_w T_w + \frac{1}{N}\pi_s T_s + \pi_f T_f} \tag{5}$$

and

$$Th_{heavy} = \frac{\pi_s \cdot l_{data}}{\pi_w T_w + \pi_s T_s + \pi_f T_f}. \tag{6}$$

Note that the NTR control packet is not considered in the throughput calculation as the capacity analysis assumes only the successful data packets handshakes.

IV. Receiver-Initiated with Dual Polling (RIMA-DP)

The analysis considered in this section is analogous to the one presented for RIMA-DP in [4], with the inclusion of hidden nodes as shown in Figure 2, and the use of an acknowledgment sent by the polled node to the polling node over the same channel, as shown in Figure 4.

From Theorem 2 in [4], RIMA-DP provides correct collision avoidance in presence of hidden terminals, provided that $\xi > 7\tau$ (in our results we take $\xi = 8\tau$). In this case the receiver, x, polls the sender, y, by sending an RTR

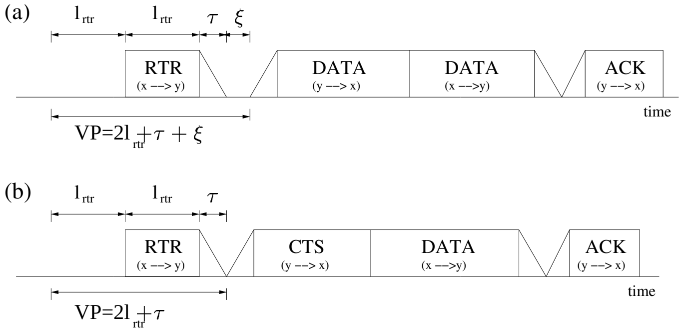

Fig. 4. Time series representation for channel around x in RIMA-DP successful handshake cases. (a) Both x and y have data to send, so Vulnerable Period (VP) is $2l_{rtr}+\tau+\xi$. (b) Only x has data to send, so Vulnerable Period (VP) is $2l_{rtr}+\tau$.

packet, and if y has data to transmit it does so (we call it *heavy traffic assumption*), see Figure 4. Immediately after that, the x also sends data to y (and we assume that this data packet also serves like an acknowledgment to the previous y-data packet received). After correctly receiving the data packet from x, then y sends the acknowledgment for x-data packet, and then x returns to idle state (see Figure 4(a)). Another situation arises when the polled node y does not have data to send in which case after receiving the RTR it immediately sends CTS to x informing that it is ready to receive (see Figure 4(b)).

From Figure 4, the vulnerable period for receiver node x regarding terminals in region $B(r)$ begins l_{rtr} seconds before RTR is sent to y and ends at ξ seconds after node y receives RTR from x in case of *heavy traffic assumption*. Otherwise the vulnerable period ends just after node y receives the RTR as y will immediately send CTS. If any node in $B(r)$ send packets during vulnerable period then collision will occur at y.

From Figures 1 and 4, the time spent in each state of the Markov chain is expressed by:

$$T_w = \tau,$$

$$T_{sh} = l_{rtr}+\tau+\xi+2(l_{data}+\tau)+l_{ack}+\tau,$$

$$T_{se} = \frac{1}{N}T_{sh}+\left(1-\frac{1}{N}\right)(l_{rtr}+\tau+l_{cts}+\tau+l_{data}+ \\ +\tau+l_{ack}+\tau),$$

$$T_f = l_{rtr}+\tau+l_{cts}+\tau,$$

where T_{sh} is the time spent in *succeed* state if *heavy traffic assumption* is considered and T_{se} is also for time in *succeed* state but when general cases are considered (as *heavy traffic assumption* is a particular case). The value for T_f comes from the fact that the x node needs to wait $l_{rtr}+\tau+l_{cts}+\tau$ to realize that a collision occurs at y.

Under the *heavy traffic assumption* then the transition probability P_{ws} is the same expression as Eq. (4), (i.e. $P_{ws}(r) = P_1P_2P_3P_4(r)P_5$) where $P_5 = 1$.

The approach to obtain P_1, P_2, and P_3 is the same to that used in the RIMA-SP case. Thus we have $P_1 = p'$, $P_2 = (1-p')$, $P_3 = e^{-p'N}$. The other probabilities depend on whether we consider the *heavy traffic assumption* or not. If we do consider, then we obtain the same expressions as those for RIMA-SP but the only change will occur for the throughput which should be given by

$$Th_{heavy} = \frac{2\cdot l_{data}\cdot \pi_s}{\pi_w T_w + \pi_s T_{sh} + \pi_f T_f}, \tag{7}$$

where the factor 2 comes from the fact that we are transmitting 2 data packets per handshake (see Figure 4(a)).

If we do not consider *heavy traffic assumption* then

$$P_{ws}(r) = P_1\cdot P_2\cdot P_3\cdot[P_4(r)\cdot P_5 + P_6(r)\cdot P_7], \tag{8}$$

where

P_1 = Pr$\{$*x polls y in a slot*$\}$,
P_2 = Pr$\{$*y does not transmit in a slot*$\}$,
P_3 = Pr$\{$*none of the terminals within R of x transmits in the same slot*$\}$,
$P_4(r)$ = Pr$\{$*none of the terminals in $B(r)$ transmits for $(2l_{rtr}+\tau+\xi)$ slots / r*$\}$,
P_5 = Pr$\{$*y has data to send to x*$\}$,
$P_6(r)$ = Pr$\{$*none of the terminals in B(r) transmits for $(2l_{rtr}+\tau)$ slots / r*$\}$,
P_7 = Pr$\{$*y does not have data to send to x*$\}$.

Using a similar procedure to that presented for the RIMA-SP case, we obtain

$$P_4(r) = e^{-p'\lambda B(r)(2l_{rtr}+\tau+\xi)},$$

$$P_6(r) = e^{-p'\lambda B(r)(2l_{rtr}+\tau)}.$$

P_5 and P_7 are network design parameters. Again we can assume for these parameters an equiprobable likelihood so that at the time node y is polled it has data to send with probability $P_5 = \frac{1}{N}$ and not with $P_7 = 1-\frac{1}{N}$. Using the same development used in RIMA-SP we find that

$$P_{ws} = \int_0^1 2rP_{ws}(r)dr \\ = 2p'(1-p')e^{-p'N}\cdot \\ \int_0^1 re^{-p'\lambda B(r)[\frac{1}{N}(2l_{rtr}+\tau+\xi)+(1-\frac{1}{N})(2l_{rtr}+\tau)]}dr.$$

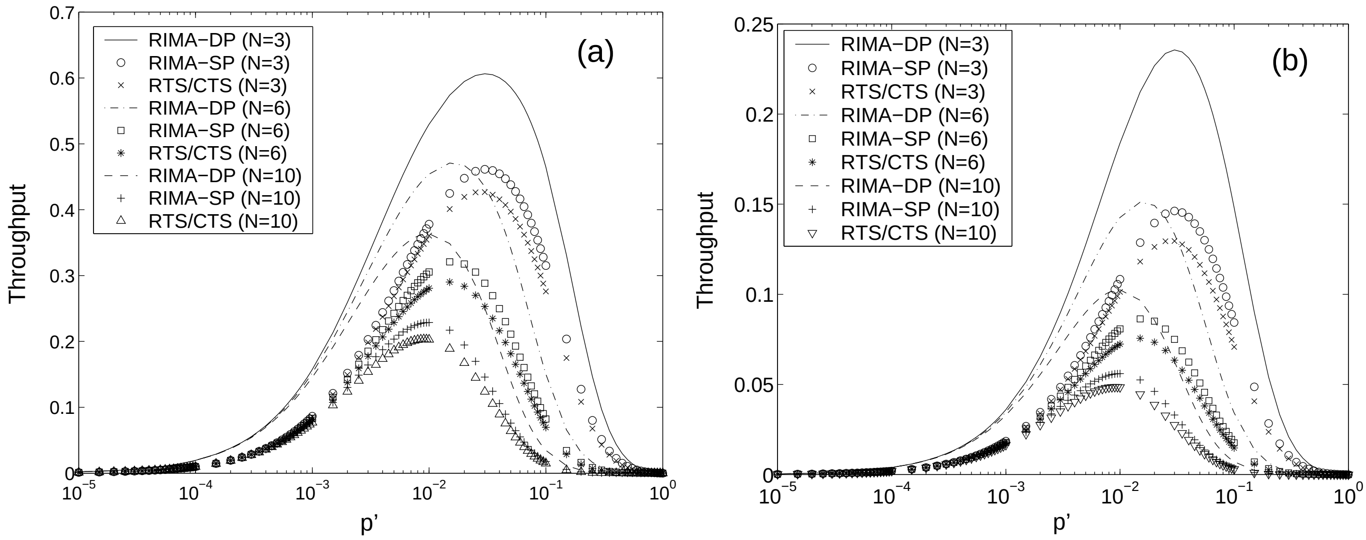

Fig. 5. Throughput vs p' in *heavy traffic assumption*. (a) long data packet: $l_{data} = 100\tau$. (b) short data packet: $l_{data} = 20\tau$.

The throughput in this case must account for the equiprobable condition. Thus,

$$Th_{equip} = \frac{\frac{1}{N} \cdot l_{data} \cdot \pi_s}{\pi_w T_w + \pi_s T_{se} + \pi_f T_f}. \tag{9}$$

V. Numerical Results

We present numerical results of the throughout attained with RIMA-SP and RIMA-DP, as well as the ideal sender-initiated collision avoidance (RTS/CTS) scheme in which no data packets collide with other packets [5], for comparison purpose. For the latter, we use the results obtained by Wang and Garcia-Luna-Aceves [7]. The numerical results were obtained using $l_{rtr} = l_{cts} = l_{ack} = 5\tau$. We vary the data packet length l_{data} and consider two cases: long data packet ($l_{data} = 100\tau$), and short data packet ($l_{data} = 20\tau$).

Figure 5 shows the numerical results obtained for the *heavy traffic assumption* (Eq. (6) and Eq. (7)). We see that the throughput obtained for either RIMA-DP and RIMA-SP is substantially better than the throughput attained for the sender-initiated case. It is also clear that performance degrades quickly as N increases.

Figure 6 shows numerical results for *equiprobable traffic assumption* (Eq. (5) and Eq. (9)). In this case, only RIMA-DP outperforms the sender-initiated scheme. RIMA-SP performs poorly, because the polled node need not have data to send.

VI. Conclusions

We have presented the first analytical model of the performance of receiver-initiated collision avoidance schemes in multihop ad hoc networks. Two receiver initiated schemes were compared against the ideal sender-initiated scheme in which no data packet collides with other packets [5]. The receiver-initiated schemes analyzed were the Receiver Initiated Multiple Access with Simple Polling (RIMA-SP) and Receiver Initiated Multiple Access with Dual Polling (RIMA-DP) schemes.

The numerical results show that RIMA-DP outperforms the sender-initiated scheme when either a heavy traffic assumption or an equiprobable traffic assumption is considered, and that RIMA-SP outperforms the sender-initiated scheme only under the heavy traffic assumption, and performs poorly under the equiprobable traffic assumption.

Our analytical results indicate that, to make a receiver-oriented scheme feasible, polling nodes must have a good estimate of the likelihood of receiving data from the polled nodes. This can be attained by nodes transmitting beacons periodically describing the state of their transmission queues. A beacon is transmitted following the same collision avoidance rules observed for the transmission of RTR packets, and specifies a list of destinations for which the beaconing node has traffic. Furthermore, the state of a node's transmission queue can also be included in the control packets and acknowledgments sent as part of the receiver-initiated handshakes in order to reduce the number of beacons sent independently of handshakes. The performance analysis of this modification to receiver-initiated schemes is the subject of another publication.

References

[1] F. A. Tobagi and L. Kleinrock. "Packet Switching in Radio Channels: Part II - the Hidden Terminal Problem in Carrier Sense Multiple-access Modes and the Busy-tone Solution". *IEEE Trans. on Comm.*, 23(12), pp. 1417-1433, 1975.

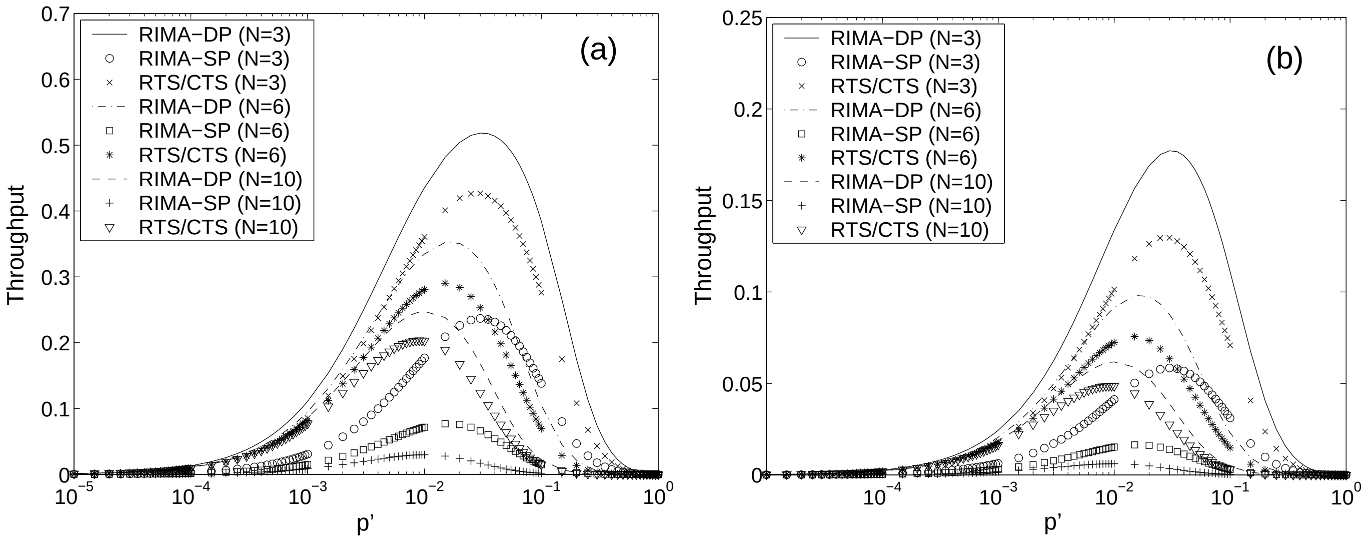

Fig. 6. Throughput vs p' in *equiprobable traffic assumption*. (a) long data packet: $l_{data} = 100\tau$. (b) short data packet: $l_{data} = 20\tau$.

[2] N. Abramson, "The Aloha System - Another Alternative for Computer Communications". In *Proc. of AFIP*, vol.36, pp. 295-298, 1970.

[3] *IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, Nov. 1997. P802.11

[4] J.J. Garcia-Luna-Aceves and A. Tzamaloukas. "Reversing the Collision-Avoidance Handshake in Wireless Networks". *Proc. ACM Mobicom 99*, Seattle, Washington, August 15–20, 1999.

[5] C. L. Fullmer and J.J. Garcia-Luna-Aceves. "Solutions to Hidden Terminals Problems in Wireless Networks". In *Proc. of ACM SIGCOMM'97*, 1997.

[6] G. Bianchi. "Performance Analysis of the IEEE 802.11 Distributed Coordination Function". *IEEE Journal on Selected Areas in Communications*, 18(3), pp. 535-547, Mar 2000.

[7] Y. Wang and J.J. Garcia-Luna-Aceves. "Performance of Collision Avoidance in Single-Channel Ad Hoc Networks". *Proc. of IEEE Tenth International Conference on Network Protocols (ICNP)*, Paris, France, November 12-15, 2002.

[8] L. Wu and P. Varshney. "Performance Analysis of CSMA and BTMA Protocols in Multi-hop Networks (I). Single Channel Case". *Information Science, Elsevier Sciences INC.*, 120, pp. 159-177, 1999.

[9] H. Takagi and L. Kleinrock. "Optimal Transmission Range for Randomly Distributed Packet Radio Terminals". *IEEE Trans. on Comm.*, 32(3), pp. 246-257, 1984.

[10] F. Talucci and M. Gerla, "MACA-BI (MACA by invitation). A Wireless MAC Protocol for High Speed Ad hoc Networking," *Proc. IEEE ICUPC*, 1997.