Error Propagation Analysis of V-BLAST With Channel-Estimation Errors

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***Abstract*—In this letter, expressions are given for the symbol-error rate (SER) of the Vertical Bell Laboratories Layered Space–Time (V-BLAST) system, taking into account error propagation due to channel-estimation errors. In addition to error propagation, suboptimal substream ordering due to imperfect channel estimates is accounted for. First, the conditional SER is determined using the distribution of the signal-to-interference-plus-noise ratio of each substream, conditioned on the channel estimate. Then, the average SER as a function of the channel estimation error-to-signal ratio (ESR) is upper bounded by averaging over the distribution of the channel estimates. The upper bound on the SER is tighter than previous bounds in the literature. Comparisons with exact simulations demonstrate the accuracy of the SER expressions for a large range of ESRs.**

***Index Terms*—Decision-feedback equalization, multiple-input multiple-output (MIMO) systems, space–time coding, successive interference cancellation (SIC).**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) wireless systems have received significant interest recently due to the large capacities possible. In particular, for independent and identically distributed (i.i.d.) Rayleigh scattering, the link capacity increases as $\min(M_T, M_R)$, where M_T and M_R are the number of transmit and receive antennas, respectively [1]. The Vertical Bell Laboratories Layered Space–Time (V-BLAST) architecture is a method to achieve a significant fraction of the theoretical capacity with a reasonable implementation complexity [2]. A high-rate transmit data stream is demultiplexed into M_T lower-rate independent substreams that are transmitted in parallel through the M_T transmit antennas. At the receiver, the $M_R \geq M_T$ received signals are processed using ordered successive interference cancellation (SIC), in which past decisions on decoded substreams are used to cancel interference caused to the remaining substreams. The order of detection of the substreams is chosen to maximize the signal-to-noise ratio (SNR) at each stage. This ordering is equivalent to the global maximization of the minimum substream SNR [2].

If previous decisions are correct, the diversity order of the ith stage of SIC is $(M_R - M_T + i), i = 1, \ldots, M_T$. In practice, however, error propagation limits the performance of V-BLAST. Furthermore, errors in channel estimates increase the effect of error propagation. Thus, it is of interest to analyze the performance of V-BLAST taking into account channel-estimation errors and error propagation.

Paper approved by I. Lee, the Editor for Switching Systems and Network Performance of the IEEE Communications Society. Manuscript received March 13, 2004; revised July 25, 2004.

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Digital Object Identifier 10.1109/TCOMM.2004.840670

The performance of a layered space–time system, such as V-BLAST, with perfect channel estimation has been analyzed in [3] using the QR decomposition of the channel matrix. The QR decomposition has also been used in [4] to derive a suboptimal but simpler detection algorithm for the V-BLAST system. Imperfect channel estimates affect the performance, not only with crosstalk in the interference-cancellation step, but also in the detection ordering. For a fixed detection order and assuming correct previous decisions, an outage probability of V-BLAST was determined in [5] as a function of the amount of training for channel estimation. An outage is declared if the crosstalk power due to channel-estimation errors is greater than a certain fraction of the thermal noise power. Based on this definition of outage, it was shown in [5] that half of the transmission time should be used for training to maximize the throughput. In [6], the effect of training and channel estimation errors on the channel capacity is studied.

In this letter, we provide an error-propagation analysis of V-BLAST with channel-estimation errors. In Section II, a model of the zero-forcing (ZF) V-BLAST system is presented using the QR decomposition of the channel matrix. In Section III, we derive an upper bound for the symbol-error rate (SER), taking into account error propagation and suboptimal ordering of substreams due to imperfect channel estimates. We then obtain the distribution of the signal-to-interference-plus-noise ratio (SINR) of each substream conditioned on the channel estimate. The error-propagation analysis and the SINR distribution are used to obtain an upper bound on the average SER with channel-estimation errors. The upper bound is tighter than previous bounds in the literature. In Section IV, the SER expressions are evaluated for a range of channel estimation error-to-signal ratios (ESRs) and are compared with exact link simulations. Conclusions are given in Section V.

II. System Model

We consider a narrowband MIMO spatial multiplexing system with M_T transmit and M_R receive antennas. Let $\mathbf{x} = [x_1 \cdots x_{M_T}]^T$ denote the vector of transmit symbols drawn from a constellation normalized such that $E[|x_i|^2] = E_s/M_T$, $i = 1, \ldots, M_T$. Here, the superscript ^{T} denotes transpose, $E[\cdot]$ denotes expectation, and E_s is the total transmit energy. Now, let $\mathbf{H}$ be the $M_R \times M_T$ channel matrix and $\mathbf{y} = [y_1 \cdots y_{M_R}]^T$ denote the received vector. The channel output can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{w}$ is an $M_R \times 1$ additive white Gaussian noise (AWGN) vector with covariance $N_0\mathbf{I}_{M_R}$, and $\mathbf{I}_{M_R}$ is the $M_R \times M_R$ identity matrix. In order to focus on the effect of channel-estimation errors, we assume in this letter that $\mathbf{H}$ is a matrix of i.i.d.

zero-mean complex Gaussian random variables with unit variance [2]. Nevertheless, the analysis presented here is also applicable to channel matrices with antenna correlation.

The V-BLAST receiver [2] performs ordered SIC to estimate $\mathbf{x}$ given the received vector $\mathbf{y}$ and estimates of $\mathbf{H}$. Although we consider ZF V-BLAST receiver processing here, the analysis can be extended to minimum mean-squared error (MMSE) V-BLAST processing in a straightforward manner. The ZF V-BLAST receiver can be analyzed using the QR decomposition of the channel matrix. Theoretically, the optimum detection order (i.e., the order that maximizes the substream SNR at each detection stage) can be determined using $O(M_T^2/2)$ QR decompositions of permutations of the channel matrix [7]. Let $\tilde{\mathbf{H}} = \mathbf{HP}$ be the channel matrix whose columns are reordered according to the optimal $M_T \times M_T$ permutation matrix $\mathbf{P}$. Note that the optimal $\mathbf{P}$ is a function of the channel realization $\mathbf{H}$. Let the QR decomposition of $\tilde{\mathbf{H}}$ be $\tilde{\mathbf{H}} = \mathbf{QR}$, where $\mathbf{Q}$ is an $M_R \times M_T$ matrix with orthonormal columns ($\mathbf{Q}^*\mathbf{Q} = \mathbf{I}_{M_T}$), and $\mathbf{R} = (r_{i,j})$ is an $M_T \times M_T$ upper triangular matrix with real positive diagonal elements. The superscript * denotes conjugate transpose.

Given the permutation matrix $\mathbf{P}$ and assuming perfect channel estimates, the transmitted symbols are detected at the receiver as follows. Using the fact that $\mathbf{PP}^* = \mathbf{I}_{M_T}$, we form the vector $\mathbf{z}$ as given by

$$\begin{aligned}\mathbf{z} &= \mathbf{Q}^*\mathbf{y} \\ &= \mathbf{Q}^*(\mathbf{QRP}^*\mathbf{x} + \mathbf{w}) \qquad (2)\\ &= \mathbf{R}\tilde{\mathbf{x}} + \mathbf{w}' \qquad (3)\end{aligned}$$

where $\tilde{\mathbf{x}} = \mathbf{P}^*\mathbf{x} = [\tilde{x}_1 \cdots \tilde{x}_{M_T}]^T$, and $\mathbf{w}' = \mathbf{Q}^*\mathbf{w}$ is an AWGN vector with covariance $N_0\mathbf{I}_{M_T}$. We now estimate $\tilde{\mathbf{x}}$ by interference cancellation. Let $\mathbf{\Lambda_R}$ denote the matrix of diagonal elements of $\mathbf{R}$. Interference cancellation is achieved by forming

$$\begin{aligned}\mathbf{v} &= \mathbf{z} - (\mathbf{R} - \mathbf{\Lambda_R})\hat{\tilde{\mathbf{x}}} \qquad (4)\\ &= \mathbf{\Lambda_R}\hat{\tilde{\mathbf{x}}} + \mathbf{R}(\tilde{\mathbf{x}} - \hat{\tilde{\mathbf{x}}}) + \mathbf{w}' \qquad (5)\end{aligned}$$

where $\hat{\tilde{\mathbf{x}}}$ denotes the estimate of $\tilde{\mathbf{x}}$. If the previous decisions are correct, $\hat{\tilde{\mathbf{x}}} = \tilde{\mathbf{x}}$ and

$$\mathbf{v} = \mathbf{\Lambda_R}\tilde{\mathbf{x}} + \mathbf{w}' \qquad (6)$$

i.e., $v_i = r_{i,i}\tilde{x}_i + w'_i, i = 1, \ldots, M_T$, where $\mathbf{v} = [v_1 \cdots v_{M_T}]^T$ and $\mathbf{w}' = [w'_1 \cdots w'_{M_T}]^T$. Thus, $\tilde{x}_i$ is estimated using the decision statistic $v_i/r_{i,i}$.

We now consider a model for the channel-estimation error. The distribution of the channel-estimation error matrix using optimal training sequences is derived in [5]. For the benefit of the reader, we summarize the derivation here. For $L \geq M_T$, consider L $M_T \times 1$ training vectors $\mathbf{s}_l, l = 1, \ldots, L$, and form the $M_T \times L$ training matrix $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_L]$. The training symbols are normalized such that $\mathrm{tr}(\mathbf{SS}^*) = E_sL$. The corresponding $M_R \times L$ matrices of received vectors and noise vectors are, respectively, $\mathbf{Y} = [\mathbf{y}_1 \cdots \mathbf{y}_L]$ and $\mathbf{W} = [\mathbf{w}_1 \cdots \mathbf{w}_L]$. The MIMO system equation during training is then

$$\mathbf{Y} = \mathbf{HS} + \mathbf{W}. \qquad (7)$$

The maximum-likelihood (ML) estimate of $\mathbf{H}$ is given by

$$\begin{aligned}\hat{\mathbf{H}} &= \mathbf{YS}^*(\mathbf{SS}^*)^{-1} \\ &= \mathbf{H} + \mathbf{WS}^*(\mathbf{SS}^*)^{-1}. \qquad (8)\end{aligned}$$

Let $\mathbf{\Delta H} = \mathbf{WS}^*(\mathbf{SS}^*)^{-1}$ and $\boldsymbol{\varepsilon}_i^*$ denote the ith row of $\mathbf{\Delta H}$. From the fact that the elements of $\mathbf{W}$ are i.i.d. Gaussian variables with variance N_0, we have

$$E\left[\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_j^*\right] = N_0(\mathbf{SS}^*)^{-1}\delta_{i,j} \qquad (9)$$

where $\delta_{i,j}$ is the Kronecker delta function. To minimize $\mathrm{tr}(E[\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_i^*])$, it can be shown that the matrix $\mathbf{S}$ is of the form $\mathbf{S} = \sqrt{E_sL/M_T}\mathbf{U}$, where the $M_T \times L$ matrix $\mathbf{U}$ satisfies $\mathbf{UU}^* = \mathbf{I}_{M_T}$. For such a training matrix, we have $E[\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_j^*] = (N_0M_T)/(E_sL)\mathbf{I}_{M_T}\delta_{i,j}$. Thus, the channel-estimation error matrix $\mathbf{\Delta H}$ can be modeled as a matrix of i.i.d. complex Gaussian variables. To evaluate V-BLAST performance with various amounts of training, in Section III, we compute the SER for different values of the variance σ_ϵ^2 of the elements of $\mathbf{\Delta H}$.

III. Error Propagation in V-BLAST With Channel-Estimation Errors

We now derive expressions for the SER of V-BLAST with channel-estimation errors. The effect of error propagation is taken into account. Following Section II, we let $\hat{\tilde{\mathbf{H}}}$ denote the estimated channel matrix with columns reordered using the estimated permutation matrix $\hat{\mathbf{P}}$, i.e., $\hat{\tilde{\mathbf{H}}} = \hat{\mathbf{H}}\hat{\mathbf{P}}$. Note that $\hat{\mathbf{P}}$ is a function of $\hat{\mathbf{H}}$. We also express $\hat{\tilde{\mathbf{H}}}$ in terms of its QR decomposition $\hat{\tilde{\mathbf{H}}} = \hat{\mathbf{Q}}\hat{\mathbf{R}}$. Now, premultiplying the received vector $\mathbf{y}$ by $\hat{\mathbf{Q}}^*$, we have

$$\begin{aligned}\hat{\mathbf{z}} &= \hat{\mathbf{Q}}^*\mathbf{y} \qquad (10)\\ &= \hat{\mathbf{Q}}^*\mathbf{H}\hat{\mathbf{P}}\tilde{\mathbf{x}}' + \mathbf{w}'' \qquad (11)\end{aligned}$$

where $\tilde{\mathbf{x}}' = \hat{\mathbf{P}}^*\mathbf{x} = [\tilde{x}'_1 \cdots \tilde{x}'_{M_T}]^T$ is the ordered transmit vector using the estimated permutation matrix $\hat{\mathbf{P}}$, and $\mathbf{w}'' = [w''_1 \cdots w''_{M_T}]^T$ is an AWGN vector with covariance $N_0\mathbf{I}_{M_T}$. In the following, we define $\hat{\tilde{\mathbf{x}}}' = [\hat{\tilde{x}}'_1 \cdots \hat{\tilde{x}}'_{M_T}]^T$ to be the estimate of $\tilde{\mathbf{x}}'$.

The substreams are detected in the order $\tilde{x}'_{M_T}, \tilde{x}'_{M_T-1}, \ldots, \tilde{x}'_1$. Consider the substream $\tilde{x}'_i$ and define the substream error $e_i = \tilde{x}'_i - \hat{\tilde{x}}'_i$. Let $P_{e_i|\hat{\mathbf{H}}}$ be the probability that $e_i \neq 0$, given the estimated channel matrix $\hat{\mathbf{H}}$. The average SER is

$$P_e = \frac{1}{M_T}\sum_{i=1}^{M_T}\int P_{e_i|\hat{\mathbf{H}}}f(\hat{\mathbf{H}})d\hat{\mathbf{H}} \qquad (12)$$

where $f(\hat{\mathbf{H}})$ is the probability density function (pdf) of $\hat{\mathbf{H}}$. Now, define the conditional error event $\mathcal{E}_i = \{e_i \neq 0|\hat{\mathbf{H}}\}$. Denoting $\mathcal{E}_i^c$ to be the complement of $\mathcal{E}_i$, we have

$$\begin{aligned} P_{e_i|\hat{\mathbf{H}}} = & \Pr(\mathcal{E}_i|\mathcal{E}_{M_T})\Pr(\mathcal{E}_{M_T}) \\ & + \Pr\left(\mathcal{E}_i|\mathcal{E}_{M_T-1}\cap\mathcal{E}_{M_T}^c\right)\Pr\left(\mathcal{E}_{M_T-1}\cap\mathcal{E}_{M_T}^c\right) + \cdots \\ & + \Pr\left(\mathcal{E}_i|\mathcal{E}_{i+1}\cap\left[\bigcap_{l=i+2}^{M_T}\mathcal{E}_l^c\right]\right)\Pr\left(\mathcal{E}_{i+1}\cap\left[\bigcap_{l=i+2}^{M_T}\mathcal{E}_l^c\right]\right) \\ & + \Pr\left(\mathcal{E}_i|\bigcap_{l=i+1}^{M_T}\mathcal{E}_l^c\right)\Pr\left(\bigcap_{l=i+1}^{M_T}\mathcal{E}_l^c\right) \quad (13) \\ \leq & \Pr(\mathcal{E}_{M_T}) + \Pr\left(\mathcal{E}_{M_T-1}\cap\mathcal{E}_{M_T}^c\right) + \cdots \\ & + \Pr\left(\mathcal{E}_i\cap\left[\bigcap_{l=i+1}^{M_T}\mathcal{E}_l^c\right]\right) \quad (14) \\ = & \sum_{k=i}^{M_T}\Pr\left(\mathcal{E}_k|\bigcap_{l=k+1}^{M_T}\mathcal{E}_l^c\right)\Pr\left(\bigcap_{l=k+1}^{M_T}\mathcal{E}_l^c\right). \quad (15) \end{aligned}$$

The upper bound (14) is obtained by assuming that given a decision error in an earlier step of the SIC procedure, the probability of a subsequent decision error is unity. The accuracy of this assumption increases with deteriorating channel-estimate quality. One also observes that if an earlier error results in a uniform distribution over the constellation for a subsequent symbol decision, the conditional probability of a subsequent error is given by $(1-1/M)$ for M-ary quadrature amplitude modulation (M-QAM). We note that compared with the alternative upper bound $P_{e_i|\hat{\mathbf{H}}} \leq \sum_{k=i}^{M_T}\Pr(\mathcal{E}_k|\bigcap_{l=k+1}^{M_T}\mathcal{E}_l^c)$ that can be obtained from [8], (15) is tighter. The difference in the bounds is most noticeable at high SER (e.g., SER$> 10^{-2}$) since $\Pr(\bigcap_{l=k+1}^{M_T}\mathcal{E}_l^c)$ is close to unity for low SER. Now, let $\mathcal{P}_k = \Pr(\mathcal{E}_k|\bigcap_{l=k+1}^{M_T}\mathcal{E}_l^c)$. From repeated application of the chain rule for conditional probability, (15) implies

$$P_{e_i|\hat{\mathbf{H}}} \leq \sum_{k=i}^{M_T}\mathcal{P}_k\prod_{l=k+1}^{M_T}(1-\mathcal{P}_l) \quad (16)$$

and

$$\begin{aligned} \frac{1}{M_T}\sum_{i=1}^{M_T}P_{e_i|\hat{\mathbf{H}}} & \leq \frac{1}{M_T}\sum_{i=1}^{M_T}\sum_{k=i}^{M_T}\mathcal{P}_k\prod_{l=k+1}^{M_T}(1-\mathcal{P}_l) \quad (17) \\ & = \frac{1}{M_T}\sum_{k=1}^{M_T}\sum_{i=1}^{k}\mathcal{P}_k\prod_{l=k+1}^{M_T}(1-\mathcal{P}_l) \quad (18) \\ & = \frac{1}{M_T}\sum_{k=1}^{M_T}k\mathcal{P}_k\prod_{l=k+1}^{M_T}(1-\mathcal{P}_l). \quad (19) \end{aligned}$$

Finally, from (12) and (19), an upper bound for the SER of V-BLAST with channel-estimation errors and error propagation is

$$P_e \leq \frac{1}{M_T}\sum_{k=1}^{M_T}k\int\mathcal{P}_k\left[\prod_{l=k+1}^{M_T}(1-\mathcal{P}_l)\right]f(\hat{\mathbf{H}})d\hat{\mathbf{H}}. \quad (20)$$

We now proceed to determine $\mathcal{P}_i$, $i = 1, \ldots, M_T$. Our approach is to determine the distribution of the SINR (given $\hat{\mathbf{H}}$) from the distribution of $\mathbf{\Delta H}$, compute the probability of error given the SINR, and finally, average over the SINR distribution to obtain $\mathcal{P}_i$. Letting $\mathbf{\Lambda}_{\hat{\mathbf{R}}}$ be the matrix of diagonal elements of $\hat{\mathbf{R}}$, we write the interference-cancellation step as

$$\begin{aligned} \hat{\mathbf{v}} & = \hat{\mathbf{z}} - (\hat{\mathbf{R}} - \mathbf{\Lambda}_{\hat{\mathbf{R}}})\hat{\tilde{\mathbf{x}}}' \quad (21) \\ & = \mathbf{\Lambda}_{\hat{\mathbf{R}}}\hat{\tilde{\mathbf{x}}}' + \hat{\mathbf{Q}}^*\mathbf{H}\hat{\mathbf{P}}\tilde{\mathbf{x}}' - \hat{\mathbf{R}}\hat{\tilde{\mathbf{x}}}' + \mathbf{w}'' \quad (22) \\ & = \mathbf{\Lambda}_{\hat{\mathbf{R}}}\hat{\tilde{\mathbf{x}}}' + \hat{\mathbf{Q}}^*(\hat{\mathbf{H}} - \mathbf{\Delta H})\hat{\mathbf{P}}\tilde{\mathbf{x}}' - \hat{\mathbf{R}}\hat{\tilde{\mathbf{x}}}' + \mathbf{w}'' \quad (23) \\ & = \mathbf{\Lambda}_{\hat{\mathbf{R}}}\hat{\tilde{\mathbf{x}}}' + \hat{\mathbf{Q}}^*(\hat{\mathbf{Q}}\hat{\mathbf{R}} - \mathbf{\Delta H}\hat{\mathbf{P}})\tilde{\mathbf{x}}' - \hat{\mathbf{R}}\hat{\tilde{\mathbf{x}}}' + \mathbf{w}'' \quad (24) \\ & = \mathbf{\Lambda}_{\hat{\mathbf{R}}}\hat{\tilde{\mathbf{x}}}' + \hat{\mathbf{R}}(\tilde{\mathbf{x}}' - \hat{\tilde{\mathbf{x}}}') - \hat{\mathbf{Q}}^*\mathbf{\Delta H}\hat{\mathbf{P}}\tilde{\mathbf{x}}' + \mathbf{w}''. \quad (25) \end{aligned}$$

Since the probability $\mathcal{P}_i$ is conditioned on correct previous decisions, setting $\hat{\tilde{\mathbf{x}}}' = \tilde{\mathbf{x}}'$ in (25), we have

$$\begin{aligned} \hat{\mathbf{v}} & = \mathbf{\Lambda}_{\hat{\mathbf{R}}}\tilde{\mathbf{x}}' - \hat{\mathbf{Q}}^*\mathbf{\Delta H}\hat{\mathbf{P}}\tilde{\mathbf{x}}' + \mathbf{w}'' \quad (26) \\ & = \mathbf{\Lambda}_{\hat{\mathbf{R}}}\tilde{\mathbf{x}}' - \hat{\mathbf{Q}}^*\mathbf{\Delta}\tilde{\mathbf{H}}\tilde{\mathbf{x}}' + \mathbf{w}'' \quad (27) \end{aligned}$$

where $\mathbf{\Delta}\tilde{\mathbf{H}} = \mathbf{\Delta H}\hat{\mathbf{P}}$. Thus, the ith element of $\hat{\mathbf{v}} = [\hat{v}_1 \ \cdots \ \hat{v}_{M_T}]^T$ is

$$\hat{v}_i = \hat{r}_{i,i}\tilde{x}_i' - \sum_{j=1}^{M_T}(\hat{\mathbf{Q}}^*\mathbf{\Delta}\tilde{\mathbf{H}})_{i,j}\tilde{x}_j' + w_i'' \quad (28)$$

where $\hat{r}_{i,i}$ is the ith diagonal element of $\hat{\mathbf{R}}$. The receiver treats the crosstalk from the desired substream due to the unknown channel-estimation error as interference, and estimates $\tilde{x}_i'$ using $\hat{v}_i/\hat{r}_{i,i}$.

We see that the channel-estimation errors result in two effects for V-BLAST receivers: suboptimal ordering of substreams and crosstalk. The suboptimal ordering $\hat{\mathbf{P}}$ is a function of $\hat{\mathbf{H}}$, and is accounted for in (20) by averaging over $f(\hat{\mathbf{H}})$. The crosstalk term in (28) is analogous to intersymbol interference in single-carrier equalization [9]. Following a common practice in equalization analysis, we determine the receiver SINR for V-BLAST with channel-estimation errors, and compute the SER at a given SINR using the nearest-neighbor union bound [9]. Letting γ_i denote the SINR for the ith substream, we have

$$\gamma_i = \frac{\frac{|\hat{r}_{i,i}|^2E_s}{M_T}}{\frac{E_s}{M_T}\sum_{j=1}^{M_T}\left|(\hat{\mathbf{Q}}^*\mathbf{\Delta}\tilde{\mathbf{H}})_{i,j}\right|^2 + N_0}. \quad (29)$$

In order to obtain the distribution of γ_i given $\hat{\mathbf{H}}$, we now consider the distribution of the crosstalk power in (29). As in [5], we neglect the second-order effect of the estimation errors associated with $\hat{\mathbf{Q}}^*\mathbf{\Delta}\tilde{\mathbf{H}}$, and make the first-order approximation $\hat{\mathbf{Q}}^*\mathbf{\Delta}\tilde{\mathbf{H}} \approx \mathbf{Q}^*\mathbf{\Delta}\tilde{\mathbf{H}}$. Now, since $\mathbf{Q}$ is independent of $\mathbf{\Delta}\tilde{\mathbf{H}}$ and $\mathbf{Q}^*\mathbf{Q} = \mathbf{I}_{M_T}$, $\mathbf{Q}^*\mathbf{\Delta}\tilde{\mathbf{H}}$ is a matrix of i.i.d. zero-mean complex Gaussian elements with variance σ_ϵ^2. Thus

$$\frac{E_s}{M_T}\sum_{j=1}^{M_T}\left|(\hat{\mathbf{Q}}^*\mathbf{\Delta}\tilde{\mathbf{H}})_{i,j}\right|^2 \approx \frac{E_s\sigma_\epsilon^2}{2M_T}G_i \quad (30)$$

where G_i is a chi-squared random variable with $2M_T$ degrees of freedom and pdf given by

$$f_{G_i}(g) = \frac{g^{M_T-1}e^{-\frac{g}{2}}}{\Gamma(M_T)2^{M_T}}, \quad g \geq 0 \tag{31}$$

where $\Gamma(M_T) = (M_T-1)!$ is the gamma function. Note that the average crosstalk-to-signal power is proportional to the number of substreams M_T (since $E[G_i] = 2M_T$).

We now use (31) to compute the pdf of γ_i (given $\hat{\mathbf{H}}$, or from (29), given $\hat{r}_{i,i}$, $i = 1, \ldots, M_T$). The cumulative distribution function (cdf) of γ_i is given by

$$\begin{aligned} F_{\gamma_i}(a) &= \Pr(\gamma_i \leq a) \\ &= \Pr\left(G_i \geq \frac{2|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\left[\frac{1}{a} - \frac{M_T N_0}{E_s|\hat{r}_{i,i}|^2}\right]\right) \\ &= 1 - F_{G_i}\left(\frac{2|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\left[\frac{1}{a} - \frac{1}{\gamma_{0,i}}\right]\right) \end{aligned} \tag{32}$$

where $F_{G_i}(\cdot)$ is the cdf of G_i and $\gamma_{0,i} = E_s|\hat{r}_{i,i}|^2/(M_T N_0)$. Thus, the pdf of γ_i given $\hat{r}_{i,i}$ is

$$\begin{aligned} f_{\gamma_i}(a) &= \frac{d}{da}F_{\gamma_i}(a) \\ &= \frac{2|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2 a^2} f_{G_i}\left(\frac{2|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\left[\frac{1}{a} - \frac{1}{\gamma_{0,i}}\right]\right), \ 0 \leq a \leq \gamma_{0,i} \\ &= \left(\frac{|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\right)^{M_T} \frac{1}{\Gamma(M_T)a^2}\left(\frac{1}{a} - \frac{1}{\gamma_{0,i}}\right)^{M_T-1} \\ &\quad \cdot \exp\left[-\frac{|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\left(\frac{1}{a} - \frac{1}{\gamma_{0,i}}\right)\right], \ 0 \leq a \leq \gamma_{0,i}. \end{aligned} \tag{33}$$

The pdf $f_{\gamma_i}(\cdot)$ is now used to obtain the conditional probability $\mathcal{P}_i$. In the following analysis, we restrict attention to an uncoded quadrature phase-shift keying (QPSK) substream constellation. This analysis can be easily extended to other constellations. Using the nearest-neighbor union bound [9], we have

$$\begin{aligned} \mathcal{P}_i &\approx \int_0^{\gamma_{0,i}} 2Q\left(\sqrt{a}\right) f_{\gamma_i}(a)da \\ &= \frac{2}{\Gamma(M_T)}\left(\frac{|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\right)^{M_T} \int_0^{\gamma_{0,i}} \frac{Q\left(\sqrt{a}\right)}{a^2}\left(\frac{1}{a} - \frac{1}{\gamma_{0,i}}\right)^{M_T-1} \\ &\quad \cdot \exp\left[-\frac{|\hat{r}_{i,i}|^2}{\sigma_\epsilon^2}\left(\frac{1}{a} - \frac{1}{\gamma_{0,i}}\right)\right] da \end{aligned} \tag{34}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$, and the approximation arises from approximating the interference plus noise as Gaussian. As is evident from the simulation results in Section IV, such an approximation often provides a good degree of accuracy and, hence, is often used [9]. After a change of variable, we have

$$\begin{aligned} \mathcal{P}_i \approx \frac{2}{\Gamma(M_T)}\left(\frac{M_T N_0}{E_s\sigma_\epsilon^2}\right)^{M_T} \int_0^1 \frac{Q\left(\sqrt{\gamma_{0,i}u}\right)}{u^2}\left(\frac{1}{u} - 1\right)^{M_T-1} \\ \cdot \exp\left[-\frac{M_T N_0}{E_s\sigma_\epsilon^2}\left(\frac{1}{u} - 1\right)\right] du. \end{aligned} \tag{35}$$

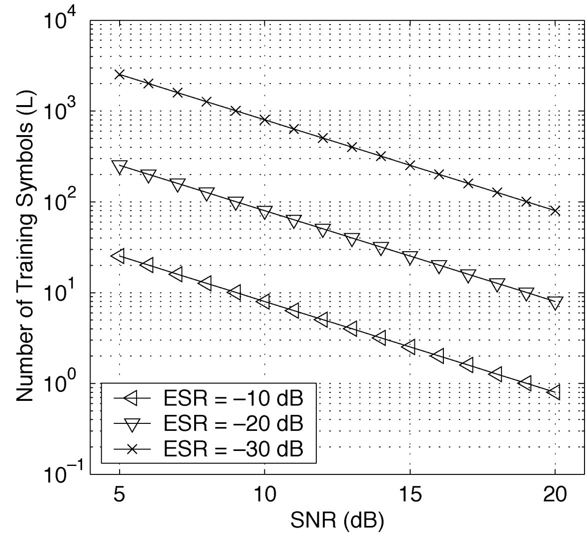

Fig. 1. Number of training symbols versus SNR for $M_T = 8$ and various ESRs.

TABLE I
NUMBER OF TRAINING SYMBOLS FOR $M_T = 8$ AND SNR = 15 dB

ESR (dB)	Number of Training Symbols (L)
−10	3
−20	26
−30	253

This expression for $\mathcal{P}_i$ is used in (20) to obtain an upper bound on the SER of V-BLAST with channel-estimation errors. In the case of perfect channel estimates (i.e., $\sigma_\epsilon^2 = 0$), we have

$$\mathcal{P}_i \approx 2Q\left(\sqrt{\gamma_{0,i}}\right). \tag{36}$$

IV. Simulation Results

In this section, we compare the SER obtained using (20), (35), and (36) with exact link simulations of the V-BLAST receiver. The integral in (20) is evaluated by Monte Carlo integration over 1000 channels. The estimation ESR is the ratio of the energy of the elements of $\boldsymbol{\Delta}\mathbf{H}$ to the energy of the elements of $\mathbf{H}$, and is given by

$$\begin{aligned} \text{ESR} &= \frac{E\left[|\boldsymbol{\Delta}\mathbf{H}_{i,j}|^2\right]}{E\left[|\mathbf{H}_{i,j}|^2\right]} && (37) \\ &= \sigma_\epsilon^2 && (38) \end{aligned}$$

since $\mathbf{H}$ is a matrix of i.i.d. complex Gaussian variables with unit variance. To evaluate the sensitivity of V-BLAST to channel-estimation errors, we evaluate the SER performance for a range of ESRs. The average SNR per receive antenna is given by SNR $= E_s/N_0$.

For a given ESR, the required number of training symbols can be computed using (9) with orthogonal training sequences. Fig. 1 is a plot of the required number of training symbols (L) versus SNR for $M_T = 8$ and various ESRs. Table I lists the number of training symbols for SNR = 15 dB.

Fig. 2 is a plot of the uncoded SER versus SNR for the V-BLAST system with QPSK transmission, $(M_T, M_R) = (8, 12)$, and: 1) perfect channel estimates; 2) ESR $= -10$ dB;

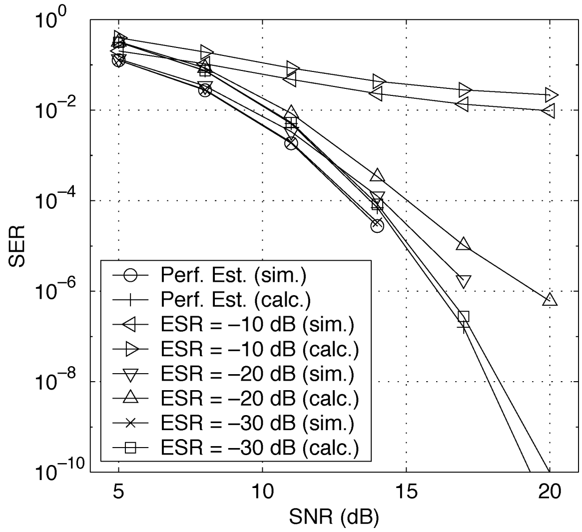

Fig. 2. Calculation and exact simulation for $(M_T, M_R) = (8, 12)$ of SER versus SNR for QPSK with channel-estimation errors.

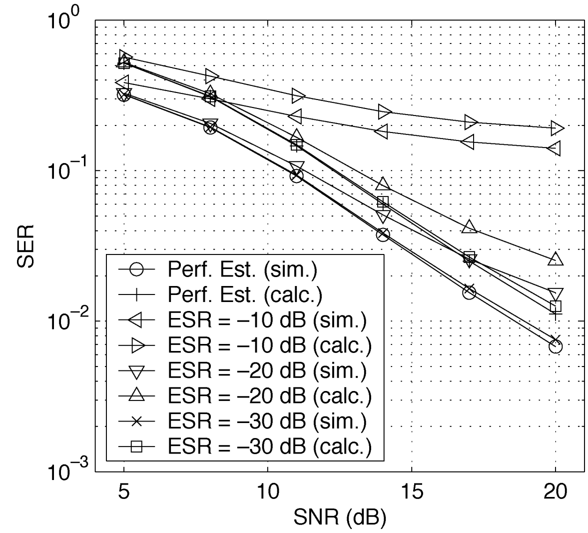

Fig. 4. Calculation and exact simulation for $(M_T, M_R) = (4, 4)$ of SER versus SNR for QPSK with channel-estimation errors.

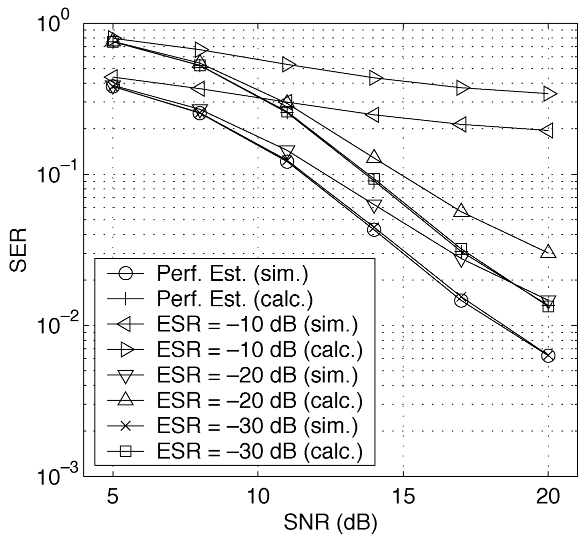

Fig. 3. Calculation and exact simulation for $(M_T, M_R) = (8, 8)$ of SER versus SNR for QPSK with channel-estimation errors.

3) ESR $= -20$ dB; and 4) ESR $= -30$ dB. From the figure, we see that at SER $= 10^{-4}$, the calculated upper bounds are within 1 dB of the exact simulations for ESR ≤ -20 dB. For ESR $= -10$ dB, the calculated SER upper bound is around twice the simulated SER. We note that the performance with ESR $= -30$ dB is nearly identical to the performance with perfect channel estimates.

Figs. 3 and 4 are plots of the SER performance for $(M_T, M_R) = (8, 8)$ and $(M_T, M_R) = (4, 4)$ with QPSK transmission. The SNR differences between the calculated upper bounds and the simulations are larger than in Fig. 2 because of the lower diversity order for $M_T = M_R$. Nevertheless, the calculated SER upper bound is around 1.4–2 times the simulated SER, and the performance with ESR $= -30$ dB is similar to the performance with perfect channel estimates. Finally, additional simulations showed that V-BLAST processing with channel-estimation errors and the detection order based on perfect channel estimates produced no significant change in performance.

V. Conclusion

In this letter, upper bounds are derived for the uncoded SER of V-BLAST, taking into account error propagation in the presence of channel-estimation errors. The QR decomposition of the channel matrix is used to derive an expression for the SINR of each substream conditioned on the channel estimate. The conditional pdf of the SINR is then derived to obtain an expression for the upper bound of the SER with error propagation. The SER upper bound, which is tighter than previous bounds in the literature, is found to be around 1.4–2 times the SER obtained from exact simulations. The performance with ESR $= -30$ dB is nearly identical to the performance with perfect channel estimates. Furthermore, there is no significant performance degradation due to the suboptimal substream-detection order caused by the channel-estimation errors.

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