Individual Outage Rate Regions for Fading Multiple Access Channels

Ravi Narasimhan
Department of Electrical Engineering, University of California, Santa Cruz, CA 95064, USA
Email: ravi@soe.ucsc.edu

Abstract— **In this paper, outage regions are computed for the quasi-static fading multiple access channel (MAC) with constraints on the individual user outage probabilities and no channel state information at the transmitters (CSIT). An outage rate region is defined as the set of rate vectors for which the user outage probability constraints are satisfied *simultaneously*. Outage probability is typically defined in terms of a *common* outage event, defined as the event that a target user rate vector lies outside the achievable region of the MAC, conditioned on the fading state. In contrast, rate regions without CSIT are computed in this paper based on *individual* outage events and individual probability constraints that are satisfied simultaneously. An individual outage event for a given user occurs if the user's message is not correctly decoded, irrespective of the decoding success of the messages for other users. Individual outage rate regions are useful for the MAC with heterogeneous user channels and quality-of-service (QoS) requirements. Explicit outage rate regions and total throughputs are computed for the two-user fading MAC. It is shown that rate regions and throughputs are significantly larger using individual outage probabilities than using the common outage probability.**

I. Introduction

In many wireless systems, such as indoor wireless local area networks (WLANs), the fading states of the users are constant over a coding block or packet. For such a quasi-static fading multiple access channel (MAC), traffic delay constraints often do not permit averaging over several channel states to select user transmission rates. Thus, rate regions based on target *outage* probabilities rather than *ergodic* rate regions are required to meet quality-of-service (QoS) constraints in quasi-static fading.

The rate regions of interest depend on the outage probability and the signal-to-noise ratios (SNRs) computed from transmit power, path loss and shadowing. In [1], a zero-outage capacity region, defined as the rate region that can be maintained under all fading channel conditions using channel state information at the transmitters (CSIT) for optimal power control. Common and individual outage capacity regions are defined in [2] based on CSIT, power control and a finite probability of not transmitting due to poor channel conditions.

In previous works, CSIT is assumed to be available at the user terminals. However, the channel may change sufficiently rapidly such that accurate CSIT is difficult to obtain. Without CSIT, each user transmits at a fixed rate, and the *common* outage probability is defined as the probability that a target user rate vector lies outside the achievable region of the MAC, conditioned on the fading state. This notion of common outage probability agrees with the definitions given in [2], [3] using a joint maximum likelihood (ML) decoder at the access point. For heterogeneous user channels and QoS requirements, it is necessary to evaluate *individual* user outage probabilities as opposed to the common system outage probability.

Individual outage probabilities without CSIT are discussed in [4] for a MAC with multiple transmit antennas. In [4], a successive group decoder (SGD) is proposed where at each decoding stage, a *subset* of users is jointly decoded. The interference from the decoded users at each stage is subtracted before proceeding to the next stage. The optimal successive group decoder (OSGD) consists of the optimal partitioning of users into groups and the optimal group decoding order to minimize the common and individual outage probabilities. For OSGD, although theorems related to outage probabilities and error exponents are given in [4], analytical results are difficult to compute in closed form; hence, high SNR asymptotes and simulations are presented in [4] for outage capacity and outage probability. Furthermore, [5] discusses a joint decoder and a successive decoder for a two-user symmetric MAC with *equal* user data rates and transmit powers. If the joint decoder does not successfully decode both user messages, either of the two successive decoders is used to determine if any user can be decoded successfully regarding the other user as interference. In [6], the MAC throughput is computed using multiuser detection and assuming *equal* data rates for all users. Order statistics are used in [7] to determine an optimal decoding order to maximize system goodputs, computed based on decoding success at each successive cancellation stage.

In the above works, explicit outage rate regions without CSIT are not given. Outage rate regions are useful to optimize cross-layer queuing algorithms, such as those described in [8], to quasi-static fading channels. Motivated by these considerations, individual outage rate regions obtained from individual outage probabilities are computed in this paper for the quasi-static fading MAC without CSIT. An individual outage probability is the probability that a given user's message cannot be correctly decoded by successive interference cancellation, irrespective of the decoding success of the messages for other users. In other words, a particular user may not experience an outage even though the rate vector lies outside the achievable region of the MAC. In contrast to SGD, the interference is cancelled *individually* rather than in groups of users. Hence, the complexity due to partitioning users into groups and determining the group decoding order is avoided.

Intuitively, the individual outage probability of a user can be computed as follows. For a given target rate vector, the access point determines the subset $\mathcal{G}$ of users whose messages can be decoded successfully. For a desired user in the subset $\mathcal{G}$, the access point removes the interference from all other users in $\mathcal{G}$ by successive cancellation. The interference from the remaining users not in $\mathcal{G}$ is treated as additional noise. With this scheme, the individual outage probability of User i is the sum over all $\mathcal{G}$ that do not contain User i of the probability that a rate vector is associated with $\mathcal{G}$. Common and individual outage probabilities are defined in this paper for the general K-user MAC. In addition, explicit common and individual outage rate regions are determined for a two-user MAC.

The remainder of the paper is organized as follows. In Section II, a K-user MAC model and outage probability definitions are discussed. Explicit computations of the outage probabilities for $K = 2$ are presented in Section III. Common and individual outage rate regions are characterized for the two-user case in Section IV. In addition, the data rates are optimized to maximize the total throughput at each SNR with and without user outage probability constraints. Section V contains numerical results that compare two-user rate regions using the individual and common outage probabilities for different outage probability constraints. Conclusions are given in Section VI.

II. System Model and Outage Probability Definitions

A. System Model

Consider a quasi-static MAC with K users. As in [9], each user has a single transmit antenna, and the access point has a single receive antenna. The channel between the i-th user and the access point is denoted by h_i, where $\mathbf{h} = \begin{bmatrix} h_1 & \cdots & h_K \end{bmatrix}^T$ is a zero-mean Gaussian random vector with identity covariance. This channel model represents quasi-static, independent and identically distributed (i.i.d.) Rayleigh fading. The average received SNR at the access point is denoted by ρ, and the fraction of the average received SNR contributed by the i-th user is $\beta_i \rho$, where $\sum_{i=1}^{K} \beta_i = 1$. The β_i represent the differences in transmit power, path loss, and shadowing among the users.

The received signal y at the access point can be written as

$$y = \sqrt{\rho} \sum_{i=1}^{K} \sqrt{\beta_i} h_i x_i + n \tag{1}$$

where x_i is the signal from the i-th user and n is an additive white Gaussian noise (AWGN) with $E[|n|^2] = 1$. Note that $E[x_i x_j^*] = \delta_{i,j}$, where * denotes complex conjugation and $\delta_{i,j}$ is the Kronecker delta function. In order to minimize the outage probability for this MAC, the inputs x_i are chosen to be i.i.d. Gaussian [1], [2]. Let $\mathbf{R} = \begin{bmatrix} R_1 \cdots R_K \end{bmatrix}^T \in \mathbb{R}_+^K$ denote the target rate vector, where $\mathbb{R}_+^K$ denotes the positive orthant of $\mathbb{R}^K$. The achievable rate region conditioned on $\mathbf{h}$ is given by

$$\mathcal{R}_{\text{MAC}} = \Big\{ \mathbf{R} \in \mathbb{R}_+^K : \sum_{i \in \mathcal{S}} R_i \le I(\mathbf{x}_\mathcal{S}; y | \mathbf{x}_{\mathcal{S}^c}, \mathbf{h}), \quad \forall\, \mathcal{S} \subseteq \{1, \ldots, K\} \Big\} \tag{2}$$

where $\mathcal{S}^c$ is the complement of $\mathcal{S}$, $\mathbf{x}_\mathcal{S}$ and $\mathbf{x}_{\mathcal{S}^c}$ are vectors of transmitted symbols from the users in $\mathcal{S}$ and $\mathcal{S}^c$, respectively, and I denotes mutual information. Note that for the MAC modeled by (1),

$$I(\mathbf{x}_\mathcal{S}; y | \mathbf{x}_{\mathcal{S}^c}, \mathbf{h}) = \log_2 \Big(1 + \rho \sum_{i \in \mathcal{S}} \beta_i X_i \Big) \tag{3}$$

where $X_i = |h_i|^2$. The X_i are i.i.d. exponential variables with unit mean.

B. Outage Probability Definitions

In order to compute common and individual outage probabilities, let $\mathcal{G} \subseteq \{1, \ldots, K\}$ denote the subset of users whose messages can be decoded successfully by successive interference cancellation. Also, let $\mathcal{B} = \mathcal{G}^c \subseteq \{1, \ldots, K\}$ denote the remaining subset of users whose messages are decoded erroneously. Conditions on the rate vector $\mathbf{R}$ are now derived for this scenario.

As discussed in the Introduction, users in $\mathcal{B}$ act as interferers to users in $\mathcal{G}$. Since $\mathbf{x}$ is a Gaussian vector, the rates $\{R_i\}_{i \in \mathcal{G}}$ must satisfy the Gaussian MAC achievability constraints (conditioned on the channel $\mathbf{h}$) with additional AWGN from users in $\mathcal{B}$, as given by

$$\mathcal{R}_{1,\mathcal{G}} = \Big\{ \mathbf{R} \in \mathbb{R}_+^K : \sum_{i \in \mathcal{S}} R_i \le I(\mathbf{x}_\mathcal{S}; y | \mathbf{x}_{\mathcal{S}_c}, \mathbf{h}), \forall \mathcal{S} \subseteq \mathcal{G} \Big\} \tag{4}$$

where $\mathcal{S}_c = \mathcal{S}^c \cap \mathcal{G}$ and

$$I(\mathbf{x}_\mathcal{S}; y | \mathbf{x}_{\mathcal{S}_c}, \mathbf{h}) = \log_2 \Big(1 + \frac{\rho \sum_{i \in \mathcal{S}} \beta_i X_i}{1 + \rho \sum_{i \in \mathcal{B}} \beta_i X_i} \Big). \tag{5}$$

These constraints on the rates $\{R_i\}_{i \in \mathcal{G}}$ ensure successful decoding for users in $\mathcal{G}$. After successful decoding, the signals from users in $\mathcal{G}$ are subtracted from y. Thus, the remaining signals are from users in $\mathcal{B}$ only. Since *none* of the messages from users in $\mathcal{B}$ are decoded correctly, the rates $\{R_i\}_{i \in \mathcal{B}}$ must satisfy the following inequalities:

$$\mathcal{R}_{2,\mathcal{G}} = \Big\{ \mathbf{R} \in \mathbb{R}_+^K : \sum_{i \in \mathcal{T}} R_i > I(\mathbf{x}_\mathcal{T}; y | \mathbf{x}_\mathcal{G}, \mathbf{h}), \forall \mathcal{T} \subseteq \mathcal{B} \Big\} \tag{6}$$

where

$$I(\mathbf{x}_\mathcal{T}; y | \mathbf{x}_\mathcal{G}, \mathbf{h}) = \log_2 \Big(1 + \frac{\rho \sum_{i \in \mathcal{T}} \beta_i X_i}{1 + \rho \sum_{i \in \mathcal{T}_c} \beta_i X_i} \Big) \tag{7}$$

and $\mathcal{T}_c = \mathcal{T}^c \cap \mathcal{B}$. From the above discussion, the region in $\mathbb{R}_+^K$ that corresponds to correct decoding of messages from

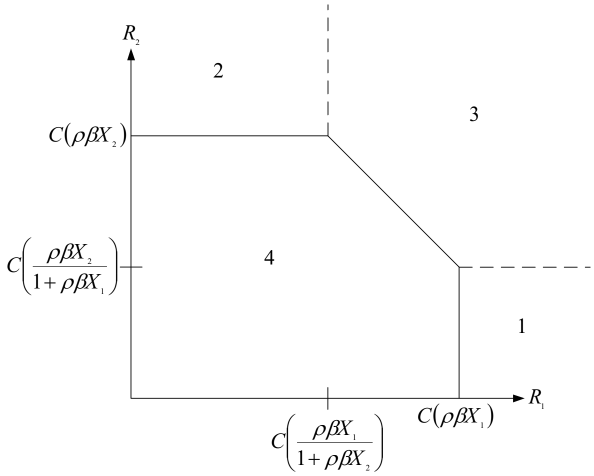

Fig. 1. Achievable region (labeled by "4") conditioned on channel state for two-user MAC. Note $\mathcal{C}(a) = \log_2(1+a)$.

users in $\mathcal{G}$ and incorrect decoding of messages from users in $\mathcal{B}$ is the intersection of the regions in (4) and (6)

$$\mathcal{R}_{\mathcal{G}} = \mathcal{R}_{1,\mathcal{G}} \cap \mathcal{R}_{2,\mathcal{G}}. \tag{8}$$

Note that by comparing a given rate vector $\mathbf{R}$ with the 2^K disjoint regions $\mathcal{R}_{\mathcal{G}}$, the access point can determine the appropriate subsets $\mathcal{G}$ and $\mathcal{B}$. Once the subset $\mathcal{G}$ is known, the access point performs decoding as discussed above, *i.e.*, successive interference cancellation of users in $\mathcal{G}$ with users in $\mathcal{B}$ treated as Gaussian interference.

Finally, the individual outage probability of the i-th user is determined by summing the probabilities of $\mathcal{R}_{\mathcal{G}}$ over the 2^{K-1} subsets $\mathcal{G}$ that do not contain index i:

$$P_{\text{out},i} = \sum_{\mathcal{G}\subseteq\{1,\ldots,i-1,i+1,\ldots,K\}} \Pr(\mathcal{R}_{\mathcal{G}}). \tag{9}$$

Furthermore, the common outage probability of the MAC is given by

$$P_{\text{out,common}} = 1 - \Pr(\mathcal{R}_{\mathcal{G}}),\ \mathcal{G} = \{1,\ldots,K\}. \tag{10}$$

III. Outage Probabilities for Two-User Case

Although computing the probabilities (9) and (10) is feasible for $K = 2, 3$, the analysis becomes difficult as the number of users increases. Hence, for the remainder of the paper, the two-user MAC is considered in detail. For simplicity, let $\beta_1 = \beta_2 = \beta = 1/2$, which corresponds to equal user transmit powers, path losses and shadowing. The achievable region for the two-user MAC conditioned on $\mathbf{h}$ is labeled by "4" in Fig. 1. In the figure, $\mathcal{C}(a) = \log_2(1+a)$. Note that the (R_1, R_2)-plane in Fig. 1 is divided into $2^2 = 4$ regions. Region 1 corresponds to a decoding error of the message from User 1; however, the message from User 2 can be successfully decoded in Region 1 since the rate of User 2 is less than the maximum rate when User 1 is regarded as additional AWGN, *i.e.*, $R_2 \leq \log_2[1 + \rho\beta X_2/(1+\rho\beta X_1)]$. Similarly, Region 2 corresponds to a decoding error of User 2's message and successful decoding of User 1's message. Finally, Region 3 corresponds to decoding errors of the messages from both users.

From Section II-B, the individual outage probabilities of Users 1 and 2 are given by, respectively,

$$P_{\text{out},1} = P_1 + P_3 \tag{11}$$
$$P_{\text{out},2} = P_2 + P_3 \tag{12}$$

where P_i denotes the probability of Region i in Fig. 1. Note that the common outage probability of the MAC is given by

$$\begin{aligned} P_{\text{out,common}} &= 1 - P_4 \\ &= P_1 + P_2 + P_3. \end{aligned} \tag{13}$$

The common outage probability is typically used to evaluate MAC performance in quasi-static channels without CSIT. However, as will be seen in Section IV, the use of individual outage probabilities yields significantly larger rate regions for heterogeneous user QoS requirements. In order to determine these outage rate regions, the individual and common outage probabilities are computed in this section for the two-user MAC.

The outage probabilities are computed by first determining the probabilities of Regions 1, 2 and 3 in Fig. 1. The probability of Region 1 is calculated as follows:

$$\begin{aligned} P_1 &= \Pr\left[R_1 > \log_2(1+\rho\beta X_1), R_2 \leq \log_2\left(1 + \frac{\rho\beta X_2}{1+\rho\beta X_1}\right)\right] \\ &= \Pr\left[X_1 < \frac{2^{R_1}-1}{\rho\beta}, X_2 \geq (2^{R_2}-1)X_1 + \frac{2^{R_2}-1}{\rho\beta}\right]. \end{aligned} \tag{14}$$

Since X_1 and X_2 are i.i.d. exponential variables with unit mean,

$$\begin{aligned} P_1 &= \int_{X_1=0}^{\frac{2^{R_1}-1}{\rho\beta}} \int_{X_2=(2^{R_2}-1)X_1+\frac{2^{R_2}-1}{\rho\beta}}^{\infty} e^{-(X_1+X_2)}\, dX_1\, dX_2 \\ &= \frac{1}{2^{R_2}}\left[\exp\left(-\frac{2^{R_2}-1}{\rho\beta}\right) - \exp\left(-\frac{2^{R_1+R_2}-1}{\rho\beta}\right)\right]. \end{aligned} \tag{15}$$

By symmetry,

$$P_2 = \frac{1}{2^{R_1}}\left[\exp\left(-\frac{2^{R_1}-1}{\rho\beta}\right) - \exp\left(-\frac{2^{R_1+R_2}-1}{\rho\beta}\right)\right]. \tag{16}$$

The probability of Region 3 is computed as follows:

$$\begin{aligned} P_3 = \Pr\Bigg[&R_1 > \log_2\left(1 + \frac{\rho\beta X_1}{1+\rho\beta X_2}\right), \\ &R_2 > \log_2\left(1 + \frac{\rho\beta X_2}{1+\rho\beta X_1}\right), \\ &R_1 + R_2 > \log_2(1+\rho\beta[X_1+X_2])\Bigg] \\ = &\int_{X_1=0}^{\frac{2^{R_1}-1}{\rho\beta}} \int_{X_2=0}^{(2^{R_2}-1)X_1+\frac{2^{R_2}-1}{\rho\beta}} e^{-(X_1+X_2)}\, dX_1\, dX_2 \end{aligned}$$

$$+\int_{X_1=\frac{2^{R_1}-1}{\rho\beta}}^{\frac{2^{R_2}(2^{R_1}-1)}{\rho\beta}} \int_{X_2=\frac{X_1}{2^{R_1}-1}-\frac{1}{\rho\beta}}^{\frac{2^{R_1+R_2}-1}{\rho\beta}-X_1} e^{-(X_1+X_2)}\,dX_1\,dX_2$$
$$= 1 - \frac{1}{2^{R_1}}e^{-\frac{2^{R_1}-1}{\rho\beta}} - \frac{1}{2^{R_2}}e^{-\frac{2^{R_2}-1}{\rho\beta}}$$
$$- e^{-\frac{2^{R_1+R_2}-1}{\rho\beta}}\left[1-\frac{1}{2^{R_1}}-\frac{1}{2^{R_2}}+\frac{(2^{R_1}-1)(2^{R_2}-1)}{\rho\beta}\right]. \quad (17)$$

Finally, the individual and common outage probabilities can be computed by substituting (15), (16) and (17) into (11), (12) and (13). The results are

$$P_{\text{out},1} = 1-\frac{1}{2^{R_1}}e^{-\frac{2^{R_1}-1}{\rho\beta}} - e^{-\frac{2^{R_1+R_2}-1}{\rho\beta}}\left[1-\frac{1}{2^{R_1}}+\frac{(2^{R_1}-1)(2^{R_2}-1)}{\rho\beta}\right] \quad (18)$$
$$P_{\text{out},2} = 1-\frac{1}{2^{R_2}}e^{-\frac{2^{R_2}-1}{\rho\beta}} - e^{-\frac{2^{R_1+R_2}-1}{\rho\beta}}\left[1-\frac{1}{2^{R_2}}+\frac{(2^{R_1}-1)(2^{R_2}-1)}{\rho\beta}\right] \quad (19)$$
$$P_{\text{out,common}} = 1 - e^{-\frac{2^{R_1+R_2}-1}{\rho\beta}}\left[1+\frac{(2^{R_1}-1)(2^{R_2}-1)}{\rho\beta}\right]. \quad (20)$$

These outage probabilities are used to characterize outage rate regions in the next section.

IV. Outage Rate Regions and Throughput Maximization

Individual and common outage rate regions for the two-user case are characterized using the individual and common outage probabilities computed in Section III. In addition, user data rates are optimized at each SNR to maximize the total uplink throughput.

A. Outage Rate Regions

Suppose that based on QoS requirements, the target outage probabilities for Users 1 and 2 are $P_{\text{out},1,t}$ and $P_{\text{out},2,t}$, respectively. For SNR ρ, the individual outage rate region can be written as follows:

$$\mathcal{R}_{\text{indiv}}(\rho) = \{(R_1,R_2) : P_{\text{out},1}(R_1,R_2,\rho) \le P_{\text{out},1,t}, P_{\text{out},2}(R_1,R_2,\rho) \le P_{\text{out},2,t}\} \quad (21)$$

where the individual outage probabilities are viewed as functions of (R_1, R_2, ρ).

In accordance with Eqn. (4) of [2], the common outage probability is the probability that the rate vector lies outside the MAC achievable region, given the fading channel gains and power constraints. Hence, with the common outage probability viewed as a function of (R_1, R_2, ρ), the common outage rate region of the MAC is

$$\mathcal{R}_{\text{common}}(\rho) = \{(R_1,R_2) : P_{\text{out,common}}(R_1,R_2,\rho) \le \min(P_{\text{out},1,t}, P_{\text{out},2,t})\}. \quad (22)$$

Note that the common outage probability must be less than the target outage probabilities of all users to ensure that the user with the most stringent QoS requirements (*i.e.*, smallest target outage probability) is satisfied. Without the minimization in (22), QoS constraints, such as delay, would not be met for some users.

The following lemma is useful to characterize the boundaries of the outage rate regions.

Lemma 1: For a fixed SNR ρ, the outage rate regions $\mathcal{R}_{\text{indiv}}(\rho)$ and $\mathcal{R}_{\text{common}}(\rho)$ are convex in (R_1, R_2).

As noted in [1], since the rate regions are convex, the boundary points of the rate regions are solutions to the optimization problem $\max_{(R_1,R_2)}(\mu_1 R_1 + \mu_2 R_2)$ subject to $(R_1, R_2) \in \mathcal{R}$, where $\mu_1 \ge 0$, $\mu_2 \ge 0$, $\mathcal{R} = \mathcal{R}_{\text{indiv}}(\rho)$ for the individual outage rate region and $\mathcal{R} = \mathcal{R}_{\text{common}}(\rho)$ for the common outage rate region. Note that from (18), (19) and (20), the individual and common outage probabilities are not convex functions of (R_1, R_2). However, given a good initial guess, there are efficient algorithms to solve this optimization problem [10]. An initial guess of $R_1^{(0)} = R_2^{(0)} = \frac{1}{2}\log_2(1+\rho)$ has been successful for this optimization problem.

B. Throughput Maximization

An important performance metric for the MAC is the total uplink throughput. In this paper, it is assumed that there is an infinite backlog of packets to be transmitted from each user. The effective throughput is the success probability multiplied by the data rate. Hence, the total uplink throughput using individual outage probabilities for SNR ρ is given by

$$G_{t,\text{indiv}}(R_1,R_2,\rho) = P_4(R_1+R_2) + P_2R_1 + P_1R_2 = [1-P_{\text{out},1}(R_1,R_2,\rho)]R_1 + [1-P_{\text{out},2}(R_1,R_2,\rho)]R_2. \quad (23)$$

The corresponding throughput using the common outage probability is

$$G_{t,\text{common}}(R_1,R_2,\rho) = [1-P_{\text{out,common}}(R_1,R_2,\rho)](R_1+R_2). \quad (24)$$

The throughputs (23) and (24) are maximized over (R_1, R_2) to yield $G_{t,\text{indiv,max}}(\rho)$ and $G_{t,\text{common,max}}(\rho)$, respectively. Since such a maximization ignores user outage probability constraints, user delay performance is not guaranteed. Delay constraints translate into packet error rate (PER) constraints. Furthermore, the PER equals the outage probability if capacity-achieving codes are used for each packet. Hence, in networks with strict user delay constraints, outage probabilities must be less than target values. In these situations, (23) and (24) must be maximized over $(R_1, R_2) \in \mathcal{R}$, where $\mathcal{R}$ is the corresponding outage rate region, to yield $G_{t,\text{indiv,constr.}}(\rho)$ and $G_{t,\text{common,constr.}}(\rho)$, respectively. The four throughputs $G_{t,\text{indiv,max}}(\rho)$, $G_{t,\text{common,max}}(\rho)$, $G_{t,\text{indiv,constr.}}(\rho)$ and $G_{t,\text{common,constr.}}(\rho)$ are compared in Section V. As in Section IV-A, the initial guess of $R_1^{(0)} = R_2^{(0)} = \frac{1}{2}\log_2(1+\rho)$ is used for the maximization.

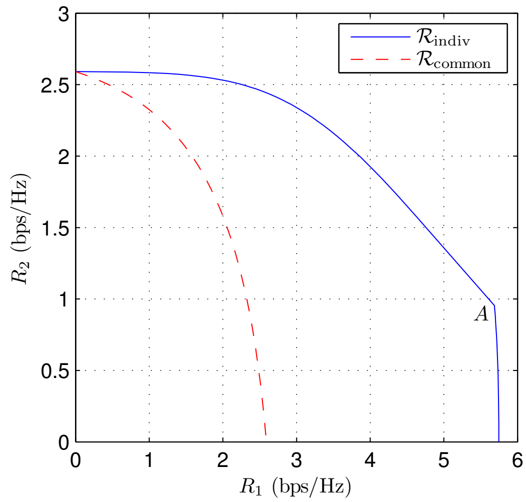

Fig. 2. Individual ($\mathcal{R}_{\text{indiv}}$) and common ($\mathcal{R}_{\text{common}}$) outage rate regions for SNR $= 30$ dB, $P_{\text{out},1,t} = 0.1$ and $P_{\text{out},2,t} = 0.01$.

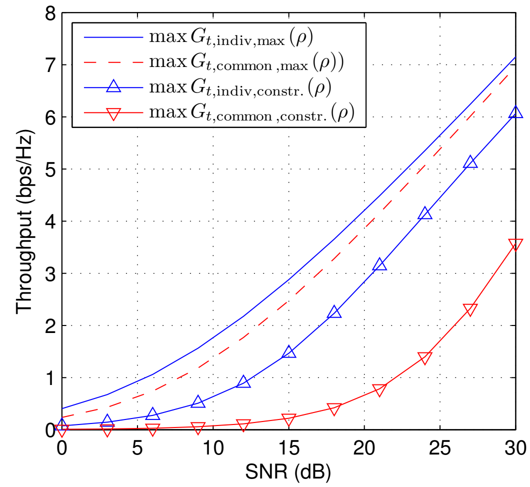

Fig. 3. Maximum uplink throughputs using individual and common outage probabilities with (subscript "constr.") and without (subscript "max") the outage probability constraints of $P_{\text{out},1,t} = 0.1$ and $P_{\text{out},2,t} = 0.01$.

V. Numerical Results

In this section, individual and common outage rate regions and corresponding throughputs are compared for the two-user MAC. For the comparison, the outage probability constraints of Users 1 and 2 are $P_{\text{out},1,t} = 0.1$ and $P_{\text{out},2,t} = 0.01$, respectively. Fig. 2 is a plot of $\mathcal{R}_{\text{indiv}}(\rho)$ and $\mathcal{R}_{\text{common}}(\rho)$ for $\rho = 30$ dB. As evident from the figure, using individual outage probabilities results in a significantly larger rate region than using the common outage probability. Note that for $\mathcal{R}_{\text{common}}(\rho)$, the common outage probability must be less than or equal to the smallest outage probability constraint among all users. Hence, the gap in rate regions is particularly large when the user outage probability constraints are different, as in the case of Fig. 2. For the individual outage rate region $\mathcal{R}_{\text{indiv}}$, the rate point A in Fig. 2 corresponds to $P_{\text{out},1} = P_{\text{out},1,t}$ and $P_{\text{out},2} = P_{\text{out},2,t}$. User 1's outage constraint dominates (*i.e.*, $P_{\text{out},1} = P_{\text{out},1,t}$ and $P_{\text{out},2} < P_{\text{out},2,t}$) for the boundary points below and to the right of A, while User 2's outage constraint dominates (*i.e.*, $P_{\text{out},1} < P_{\text{out},1,t}$ and $P_{\text{out},2} = P_{\text{out},2,t}$) for the boundary points above and to the left of A.

Fig. 3 is a plot of the maximum uplink throughputs using individual and common outage probabilities. Throughputs with and without outage probability constraints ($P_{\text{out},1,t} = 0.1$ and $P_{\text{out},2,t} = 0.01$) are plotted. Without outage probability constraints, the throughputs (23) and (24) are similar. However, with outage probability constraints (due to QoS requirements), use of the individual outage rate region yields much larger throughputs than using the common outage rate region.

VI. Conclusion

Individual outage rate regions from successive decoding in a MAC allow wireless systems to account for disparate user QoS requirements in slow fading environments. Common and individual outage probabilities for the K-user quasi-static fading MAC have been characterized based on successive interference cancellation at the access point. The two-user case has been considered in detail to obtain individual and common outage rate regions. Maximum uplink throughputs have been computed. It is seen that the use of individual rather than common outage probabilities results in significantly larger outage rate regions and throughputs.

Acknowledgment

The author thanks Prof. J. M. Cioffi and K. Seong for helpful comments.

References

[1] S. V. Hanly and D. N. C. Tse, "Multiaccess fading channels–part II: delay-limited capacities," *IEEE Trans. Inform. Theory*, vol. 44, no. 7, pp. 2816–2831, Nov. 1998.

[2] L. Li, N. Jindal and A. Goldsmith, "Outage capacities and optimal power allocation for fading multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, Apr. 2005.

[3] D. N. C. Tse, P. Viswanath and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1859–1874, Sep. 2004.

[4] N. Prasad, G. Yue, M. A. Khojastepour, X. Wang and M. Madihian, "Optimal successive group decoder for the slow-fading multiple access channel," in *Proc. Conf. Inform. Sci. and Syst. (CISS 2006)*, Princeton, NJ, pp. 1025–1030, Mar. 2006.

[5] S. Shamai and A. Steiner, "A broadcast approach for a single-user slowly fading MIMO channel," *Proc. IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2617–2635, Oct. 2003.

[6] D. Tuninetti and G. Caire, "The throughput of some wireless multiaccess systems," *IEEE Trans. Inform. Theory*, vol. 48, no. 10, pp. 2773–2785, Oct. 2002.

[7] C. Y. Sin and V. K. N. Lau, "Exact packet outage expressions of multi-user detection with adaptive successive interference cancellation for wireless systems in slow fading channels," in *Proc. IEEE ICC 2006*, pp. 4994–4999, Jun. 2006.

[8] K. Seong, R. Narasimhan and J. M. Cioffi, "Queue proportional scheduling via geometric programming in fading broadcast channels," *IEEE J. Selected Areas Commun.*, vol. 24, no. 8, pp. 1593–1602, Aug. 2006.

[9] R. Narasimhan, "Outage probabilities and finite-SNR diversity gains in rate-adaptive fading multiple access channels," in *Proc. IEEE ISIT 2006*, Seattle, USA, pp. 1339–1343, July 2006.

[10] P. E. Gill, W. Murray and M. H. Wright, *Practical Optimization*. London, U.K.: Academic, 1981.