

Outage Probabilities and Finite-SNR Diversity Gains in Rate-Adaptive Fading Multiple Access Channels

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Abstract—In this paper, bounds on the outage probability and estimates of the diversity gain at finite signal-to-noise ratios (SNRs) are obtained for quasi-static fading multiple access channels in which user data rates increase with SNR. The data rate of each user is equal to the multiplexing gain multiplied by the capacity of an additive white Gaussian noise (AWGN) channel. Given the user multiplexing gains, the finite-SNR diversity gain is defined in terms of the slope of the common outage probability versus SNR at a particular point. This nonasymptotic framework represents an extension of previous work for point-to-point multiantenna channels to rate-adaptive multiple access channels. In contrast to asymptotic diversity-multiplexing tradeoffs as the SNR tends to infinity, finite-SNR analysis provides useful insight into achievable multiple access performance with different transmit power, path loss, and shadowing among users.

I. INTRODUCTION

The multiple access channel represents a useful model of the uplink in a single-cell wireless system. In this model, all users transmit cochannel signals simultaneously to the access point, which performs optimal decoding. In many wireless systems, the channels from the users to the access point are quasi-static. Hence, each transmission will experience only a single channel realization since coding across realizations would violate traffic delay constraints. Furthermore, the variation of path loss and shadowing among the users and the access point can be large. These variations can be viewed as variations in user transmit power. Thus, it is of interest to study the outage performance of multiple access channels for different user powers. Related papers on the outage capacity of fading multiple access channels include [1], [2].

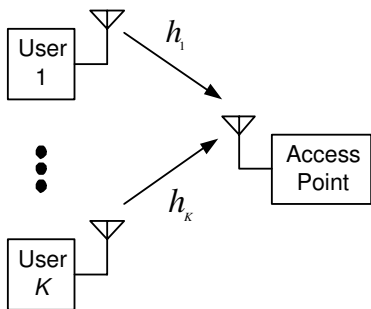
Recently, a tradeoff between diversity gain and multiplexing gain has been computed in [3] for multiple access channels with multiple transmit and receive antennas. This tradeoff characterizes the diversity and multiplexing gains that can be achieved simultaneously in multiple access channels as the system signal-to-noise ratio (SNR) tends to infinity. Differences in user transmit powers do not affect the tradeoff in this asymptotic regime. In practice, however, differences in path loss and shadowing can result in a large variation in user power. Furthermore, due to limited transmit power available at the user terminals, uplink SNRs in systems such as wireless

metropolitan area networks (WMANs) are low to moderate, in many situations around 0 dB [4], [5]. Under these conditions, asymptotic analysis can lead to optimistic results. Hence, it is necessary to analyze the outage and diversity performance in multiple access channels at finite SNR for various multiplexing gains.

In this paper, the performance of rate-adaptive fading multiple access channels is analyzed for the case of single-antenna users and a single-antenna access point. A quasi-static fading model is assumed such that the fading is constant for one coding block (or packet) and varies independently from packet to packet. The focus of this paper is on the common outage probability of the multiple access channel. With capacity-achieving codes applied per packet, the common packet error rate (PER) is equal to the common outage probability, defined as the probability that a rate vector lies outside the achievable region of the multiple access channel.

In this rate-adaptive setting, the desired data rate of each user is varied as a function of SNR and is equal to the capacity of an additive white Gaussian noise (AWGN) channel multiplied by a finite-SNR multiplexing gain. The finite-SNR diversity gain of the multiple access channel is given by the negative slope of the common outage probability versus SNR on a log-log scale. Similar definitions of finite-SNR multiplexing and diversity gains were proposed in [6], [7] for point-to-point multiantenna channels. The resulting finite-SNR diversity-multiplexing tradeoff for multiple access channels captures the performance impact of disparate user transmit powers.

The remainder of the paper is organized as follows. Section II introduces the system model and definitions of finite-SNR multiplexing and diversity gains. Since exact calculation of the common outage probability in a general rate-adaptive multiple access channel is not tractable, upper and lower bounds are derived in Section III. These bounds are used to estimate the diversity gain as a function of the user multiplexing gains and SNR. Section IV presents numerical results for outage and diversity performance, including comparisons to high-SNR asymptotic results. Conclusions are given in Section V.

Fig. 1. Multiple access channel with K users.

II. SYSTEM MODEL

As shown in Fig. 1, a quasi-static fading multiple access channel with K users is considered. Each user has a single transmit antenna, and the access point has a single receive antenna. The channel between the i -th user and the access point is denoted by h_i . The vector of channel gains $\mathbf{h} = [h_1 \cdots h_K]^T$ is assumed to be a zero-mean Gaussian random vector with identity covariance, corresponding to independent and identically distributed (i.i.d.) Rayleigh fading. The average received SNR at the access point is denoted by ρ , and the fraction of the average received SNR contributed by the i -th user is $\beta_i \rho$, where $\sum_{i=1}^K \beta_i = 1$. Hence, the received signal y at the access point can be written as

$$y = \sqrt{\rho} \sum_{i=1}^K h_i \sqrt{\beta_i} x_i + n \quad (1)$$

where x_i is the signal from the i -th user and n is the AWGN with $E[|n|^2] = 1$. The factors $\{\beta_i\}_{i=1}^K$ account for differences in transmit power, path loss, and shadowing among the users. Note that $E[x_i x_j^*] = \delta_{i,j}$, where $*$ denotes complex conjugation and $\delta_{i,j}$ is the Kronecker delta function.

Let R_i denote the target spectral efficiency of the i -th user. The common outage probability of the multiple access channel is defined as the probability that the target rate vector $\mathbf{R} = [R_1 \cdots R_K]^T$ lies outside the achievable rate region (conditioned on \mathbf{h}). Mathematically, consider as in [3] a subset $\mathcal{S} \subseteq \{1, \dots, K\}$ and the outage event

$$\mathcal{O}_{\mathcal{S}} = \left\{ \mathbf{h} \in \mathbb{C}^K : I(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \mathbf{h}) < \sum_{i \in \mathcal{S}} R_i \right\} \quad (2)$$

where \mathcal{S}^c is the complement of \mathcal{S} , $\mathbf{x}_{\mathcal{S}}$ and $\mathbf{x}_{\mathcal{S}^c}$ are vectors of transmitted symbols from the users in \mathcal{S} and \mathcal{S}^c , respectively, \mathbb{C}^K is the K -dimensional complex vector space, and I denotes mutual information. The common outage probability is then

$$P_{\text{out}} = P\left(\bigcup_{\mathcal{S} \subseteq \{1, \dots, K\}} \mathcal{O}_{\mathcal{S}}\right). \quad (3)$$

The inputs x_i are chosen to be i.i.d. Gaussian. Hence, the mutual information in (2) can be shown to be equal to

$$I(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \mathbf{h}) = \log_2 \left(1 + \rho \sum_{i \in \mathcal{S}} \beta_i |h_i|^2 \right). \quad (4)$$

The notions of multiplexing and diversity gains are traditionally defined in the limit as the SNR tends to infinity. For the high-SNR diversity-multiplexing tradeoff computed in [3], the target spectral efficiencies are $R_i \sim r_{i,\text{asymptotic}} \log_2 \rho$, where $r_{i,\text{asymptotic}}$ is the asymptotic multiplexing gain, $i = 1, \dots, K$. For these target rates, the common diversity gain is $d_{\text{asymptotic}} = -\lim_{\rho \rightarrow \infty} (\log_2 P_{\text{out}} / \log_2 \rho)$. In the asymptotic regime, the diversity-multiplexing tradeoff for a single-antenna multiple access channel can be computed using [3] and is given by:

$$d_{\text{asymptotic}}(r_{1,\text{asymptotic}}, \dots, r_{K,\text{asymptotic}}) = \min_{\mathcal{S} \subseteq \{1, \dots, K\}} \left[|\mathcal{S}| \left(1 - \sum_{i \in \mathcal{S}} r_{i,\text{asymptotic}} \right)^+ \right] \quad (5)$$

where $(t)^+ = \max(0, t)$ and $|\mathcal{S}|$ is the number of elements in \mathcal{S} . It can be seen from (5) that for the same set of $(r_{1,\text{asymptotic}}, \dots, r_{K,\text{asymptotic}})$, any set of nonzero β_i results in the same asymptotic diversity-multiplexing tradeoff. Hence, it is necessary to consider a finite-SNR (nonasymptotic) formulation to account for different user powers.

In this paper, the finite-SNR framework presented in [6], [7] for point-to-point multiantenna channels is extended to multiple access channels as described in the following. The nonasymptotic multiplexing gain for the i -th user is defined by the ratio of R_i to the capacity of an AWGN channel at SNR ρ :

$$r_i = \frac{R_i}{\log_2(1 + \rho)}, \quad i = 1, \dots, K. \quad (6)$$

The multiplexing gain r_i indicates the sensitivity to SNR of the i -th user's rate adaptation strategy. From (3) and (6), the common outage probability of the multiple access channel is a function of the user multiplexing gains and SNR, i.e., $P_{\text{out}} = P_{\text{out}}(r_1, \dots, r_K, \rho)$. The common diversity gain as a function of SNR is defined by

$$d(r_1, \dots, r_K, \rho) = -\frac{\partial \ln P_{\text{out}}}{\partial \ln \rho} = -\frac{\rho}{P_{\text{out}}} \cdot \frac{\partial P_{\text{out}}}{\partial \rho}. \quad (7)$$

As illustrated in Fig. 2, the finite-SNR diversity gain is the slope of the outage probability curve at a particular SNR for a given set of multiplexing gains. Hence, this diversity gain can be used to estimate the change in SNR necessary to change the outage probability by a specified amount.

The finite-SNR framework discussed above is used in the next section to analyze the outage and diversity performance of rate-adaptive multiple access channels.

III. OUTAGE AND DIVERSITY COMPUTATIONS

A. Outage Probability Bounds

The common outage probability given in (3) involves a union of $(2^K - 1)$ subsets \mathcal{S} . Because these subsets are neither mutually independent nor disjoint, exact computation of this probability is not tractable except for a few special cases.

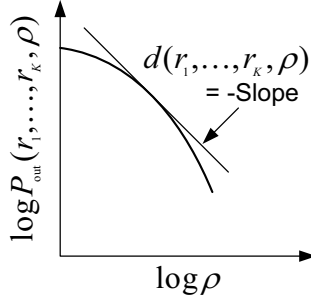


Fig. 2. Illustration of finite-SNR diversity gain.

Thus, in order to gain insight into finite-SNR outage and diversity performance, upper and lower bounds on the outage probability are computed. A simple upper bound is given by the union bound:

$$P_{\text{out}} \leq \sum_{\mathcal{S} \subseteq \{1, \dots, K\}} P(\mathcal{O}_{\mathcal{S}}). \quad (8)$$

A lower bound is computed as follows:

$$P_{\text{out}} \geq \max_{\mathcal{S} \subseteq \{1, \dots, K\}} P(\mathcal{O}_{\mathcal{S}}) = P(\mathcal{O}_{\mathcal{S}_m}) \quad (9)$$

where $\mathcal{S}_m = \operatorname{argmax}_{\mathcal{S} \subseteq \{1, \dots, K\}} P(\mathcal{O}_{\mathcal{S}})$.

In the rate-adaptive context, let $r_{\mathcal{S}} = \sum_{i \in \mathcal{S}} r_i$ denote the sum of the multiplexing gains for users in \mathcal{S} . From (2), (4), and (6), the probability of the outage event $\mathcal{O}_{\mathcal{S}}$ is given by

$$P(\mathcal{O}_{\mathcal{S}}; r_1, \dots, r_K, \rho) = P\left(\log_2(1 + \rho \sum_{i \in \mathcal{S}} \beta_i |h_i|^2) < r_{\mathcal{S}} \log_2(1 + \rho)\right). \quad (10)$$

Note that $G_{\mathcal{S}} = \sum_{i \in \mathcal{S}} \beta_i |h_i|^2$ is a sum of independent gamma random variables with parameters $(1, \beta_i)$. Hence, the cumulative distribution function (cdf) of $G_{\mathcal{S}}$ can be written as a single gamma series [8], [9]:

$$F_{G_{\mathcal{S}}}(t) = \prod_{i \in \mathcal{S}} \left(\frac{\beta_{\mathcal{S}, \min}}{\beta_i}\right) \sum_{k=0}^{\infty} \frac{\delta_{k, \mathcal{S}} \gamma(|\mathcal{S}| + k, t/\beta_{\mathcal{S}, \min})}{\Gamma(|\mathcal{S}| + k)} \quad (11)$$

where $\beta_{\mathcal{S}, \min} = \min_{i \in \mathcal{S}} \beta_i$, $\gamma(m, x) = \int_0^x t^{m-1} e^{-t} dt$, and

$$\begin{aligned} \delta_{0, \mathcal{S}} &= 1 \\ \delta_{k+1, \mathcal{S}} &= \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j \in \mathcal{S}} \left(1 - \frac{\beta_{\mathcal{S}, \min}}{\beta_j}\right)^i \right] \delta_{k+1-i, \mathcal{S}} \quad (12) \\ & \quad k = 0, 1, \dots \end{aligned}$$

Hence,

$$P(\mathcal{O}_{\mathcal{S}}; r_1, \dots, r_K, \rho) = P\left(\log_2(1 + \rho G_{\mathcal{S}}) < r_{\mathcal{S}} \log_2(1 + \rho)\right) = F_{G_{\mathcal{S}}}(\alpha_{\mathcal{S}, \rho}) \quad (13)$$

where $\alpha_{\mathcal{S}, \rho} = [(1 + \rho)^{r_{\mathcal{S}}} - 1]/\rho$. Thus, upper and lower bounds on the outage probability are given by, respectively,

$$P_{\text{out}}(r_1, \dots, r_K, \rho) \leq \sum_{\mathcal{S} \subseteq \{1, \dots, K\}} F_{G_{\mathcal{S}}}(\alpha_{\mathcal{S}, \rho}) \quad (14)$$

$$P_{\text{out}}(r_1, \dots, r_K, \rho) \geq F_{G_{\mathcal{S}_m}}(\alpha_{\mathcal{S}_m, \rho}). \quad (15)$$

B. Diversity Gain Estimates

Estimates of the finite-SNR diversity gain can be obtained by substituting the right hand sides of (14) and (15), respectively, into (7):

$$\begin{aligned} \hat{d}_1(r_1, \dots, r_K, \rho) &= \frac{\sum_{\mathcal{S} \subseteq \{1, \dots, K\}} F'_{G_{\mathcal{S}}}(\alpha_{\mathcal{S}, \rho}) [\alpha_{\mathcal{S}, \rho} - r_{\mathcal{S}}(1 + \rho)^{r_{\mathcal{S}} - 1}]}{\sum_{\mathcal{S} \subseteq \{1, \dots, K\}} F_{G_{\mathcal{S}}}(\alpha_{\mathcal{S}, \rho})} \quad (16) \end{aligned}$$

$$\begin{aligned} \hat{d}_2(r_1, \dots, r_K, \rho) &= \frac{F'_{G_{\mathcal{S}_m}}(\alpha_{\mathcal{S}_m, \rho})}{F_{G_{\mathcal{S}_m}}(\alpha_{\mathcal{S}_m, \rho})} [\alpha_{\mathcal{S}_m, \rho} - r_{\mathcal{S}_m}(1 + \rho)^{r_{\mathcal{S}_m} - 1}]. \quad (17) \end{aligned}$$

An important condition is the case of symmetric multiplexing gain in which $r_i = r$, $i = 1, \dots, K$. In this case, the bounds on outage probability and diversity gain estimates are computed using (14), (15), (16), and (17) with $r_{\mathcal{S}} = |\mathcal{S}|r$.

The high-SNR diversity gain estimates can be computed by taking the limits of (16) and (17) as $\rho \rightarrow \infty$. For instance, it can be seen that terms with $r_{\mathcal{S}} \geq 1$ tend to zero. For $0 < r_{\mathcal{S}} < 1$ and $\rho \rightarrow \infty$, $\alpha_{\mathcal{S}, \rho} \approx 1/\rho^{1-r_{\mathcal{S}}}$ and $F_{G_{\mathcal{S}}}(\alpha_{\mathcal{S}, \rho}) \approx K_{\mathcal{S}}/\rho^{|\mathcal{S}|(1-r_{\mathcal{S}})}/\Gamma(|\mathcal{S}| + 1)$, where $K_{\mathcal{S}}$ contains constants that depend on \mathcal{S} . For the same asymptotic conditions, $\mathcal{S}_m \rightarrow \operatorname{argmin}_{\mathcal{S} \subseteq \{1, \dots, K\}} [|\mathcal{S}|(1 - r_{\mathcal{S}})^+]$. Thus,

$$\begin{aligned} \lim_{\rho \rightarrow \infty} \hat{d}_2(r_1, \dots, r_K, \rho) &= \lim_{\rho \rightarrow \infty} \frac{|\mathcal{S}_m| K_{\mathcal{S}_m} / \rho^{(1-r_{\mathcal{S}_m})(|\mathcal{S}_m|-1)}}{K_{\mathcal{S}_m} / \rho^{(1-r_{\mathcal{S}_m})|\mathcal{S}_m|}} \left[\frac{1}{\rho^{1-r_{\mathcal{S}_m}}} - \frac{r_{\mathcal{S}_m}}{\rho^{1-r_{\mathcal{S}_m}}} \right] \\ &= |\mathcal{S}_m| (1 - r_{\mathcal{S}_m}), \quad 0 < r_{\mathcal{S}_m} < 1 \\ &= \min_{\mathcal{S} \subseteq \{1, \dots, K\}} [|\mathcal{S}|(1 - r_{\mathcal{S}})^+]. \quad (18) \end{aligned}$$

It can be shown that the same result is obtained for the asymptotic diversity gain estimate $\lim_{\rho \rightarrow \infty} \hat{d}_1(r_1, \dots, r_K, \rho)$. Note that the asymptotic diversity gain estimate (18) is exactly equal to (5). Hence, the finite-SNR framework presented here generalizes previous asymptotic analysis for the multiple access channel.

Another limit of interest is the diversity gain as the symmetric multiplexing gain r tends to zero. In this limit, $\alpha_{\mathcal{S}, \rho} \approx r|\mathcal{S}|[\ln(1 + \rho)]/\rho$,

$$F_{G_{\mathcal{S}}}(\alpha_{\mathcal{S}, \rho}) \approx \frac{K_{\mathcal{S}}}{\Gamma(|\mathcal{S}| + 1)} \left(\frac{r|\mathcal{S}| \ln(1 + \rho)}{\rho} \right)^{|\mathcal{S}|},$$

and

$$\begin{aligned} |\mathcal{S}_m| &\rightarrow \left| \operatorname{argmax}_{\mathcal{S} \subseteq \{1, \dots, K\}} \frac{K_{\mathcal{S}}}{\Gamma(|\mathcal{S}| + 1)} \left(\frac{r|\mathcal{S}| \ln(1 + \rho)}{\rho} \right)^{|\mathcal{S}|} \right| \\ &\rightarrow 1 \text{ as } r \rightarrow 0. \end{aligned}$$

Hence, the limiting diversity gain estimate is

$$\begin{aligned} & \lim_{r \rightarrow 0} \hat{d}_2(r, \dots, r, \rho) \\ &= \lim_{r \rightarrow 0} \frac{K_{\mathcal{S}_m}}{K_{\mathcal{S}_m} r [\ln(1 + \rho)] / \rho} \left[r [\ln(1 + \rho)] / \rho - \frac{r}{1 + \rho} \right] \\ &= 1 - \frac{\rho}{(1 + \rho) \ln(1 + \rho)}. \end{aligned} \quad (19)$$

This limiting diversity gain, which agrees with the results given in [6], [7] for point-to-point multi-antenna channels, is also equal to $\lim_{r \rightarrow 0} \hat{d}_1(r, \dots, r, \rho)$.

Note that the limiting diversity gain does not depend on the relative user powers. In order to obtain insight into the effect of different user powers, it is useful to study the diversity gain for a low (nonzero) symmetric multiplexing gain. In this condition, known as the lightly loaded regime, single-user performance is achieved [3]. As $\rho \rightarrow \infty$, single-user performance is achieved for $r \leq 1/(K + 1)$.

In the lightly loaded regime, the user with the minimum transmit power dominates the outage performance, i.e., $|\mathcal{S}_m| = 1$ and $\mathcal{S}_m = \operatorname{argmin}_{i \in \{1, \dots, K\}} \beta_i$. Hence, the outage probability in this regime is

$$\begin{aligned} P_{\text{out, light load}}(r, \dots, r, \rho) &= \gamma \left(1, \frac{(1 + \rho)^r - 1}{\beta_{\min} \rho} \right) \\ &= 1 - \exp \left(- \frac{(1 + \rho)^r - 1}{\beta_{\min} \rho} \right) \end{aligned} \quad (20)$$

where $\beta_{\min} = \min_{i \in \{1, \dots, K\}} \beta_i$. The corresponding diversity gain is then

$$d_{\text{light load}}(r, \dots, r, \rho) = \frac{1}{\beta_{\min}} \cdot \frac{\frac{(1 + \rho)^r - 1}{\rho} - r(1 + \rho)^{r-1}}{\exp \left(\frac{(1 + \rho)^r - 1}{\beta_{\min} \rho} \right) - 1}. \quad (21)$$

There is a simple relation between β_{\min} and the slope with respect to r of the finite-SNR diversity gain in the limit as $r \rightarrow 0$. By differentiating (21) with respect to r and using L'Hôpital's rule, one can show that

$$\begin{aligned} & \lim_{r \rightarrow 0} \frac{\partial d_{\text{light load}}(r, \dots, r, \rho)}{\partial r} \\ &= \frac{1}{2} \left[\frac{1}{\beta_{\min}(1 + \rho)} - \frac{\rho}{1 + \rho} - \frac{\ln(1 + \rho)}{\beta_{\min} \rho} \right]. \end{aligned} \quad (22)$$

Hence, although the limiting diversity gain (19) does not depend on relative user powers, the limiting slope (22) with respect to r of the diversity gain has a simple relation with the fraction of received SNR contributed by the weakest user.

IV. NUMERICAL RESULTS

In this section, numerical results are given to illustrate the impact of finite SNR and different user powers on the outage performance in multiple access channels. Consider a multiple access channel with $K = 2$ users and (1) $\beta_1 = \beta_2$ and (2) $\beta_1 = \beta_2/100$. Scenario 1 corresponds to a symmetric user distribution. Scenario 2 corresponds to the situation in which

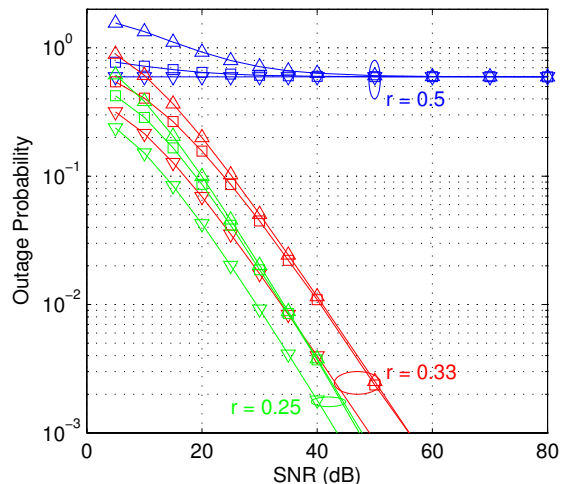


Fig. 3. Outage probability of Scenario 1 versus SNR for various symmetric multiplexing gains r . Legend: \triangle \rightarrow upper bound (14), ∇ \rightarrow lower bound (15), \square \rightarrow exact simulation results.

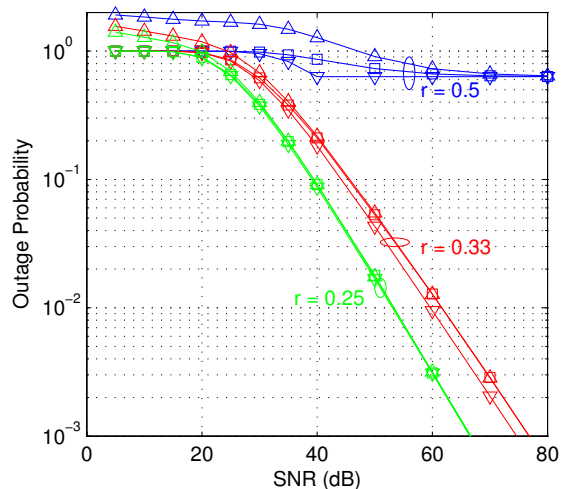


Fig. 4. Outage probability of Scenario 2 versus SNR for various symmetric multiplexing gains r . Legend: \triangle \rightarrow upper bound (14), ∇ \rightarrow lower bound (15), \square \rightarrow exact simulation results.

the SNR contributed by user 1 is 20 dB below that of user 2. Even with power control, this condition can occur in WMANs with a near/far imbalance of user positions [10].

In Fig. 3, the outage probability for Scenario 1 is plotted versus SNR for different symmetric multiplexing gains. The upper and lower bounds of the outage probability given in (14) and (15) are compared to exact Monte Carlo simulations. It can be seen that for low to moderate SNR, the union bound (14) is quite loose (sometimes greater than unity). In contrast, the lower bound (15) is close to the actual outage probability at low SNR. A similar plot of outage probability for Scenario 2 is given in Fig. 4. The presence of a weak user significantly degrades the outage performance at low to moderate SNR. Note that this effect is not captured by the asymptotic diversity gain as the SNR tends to infinity.

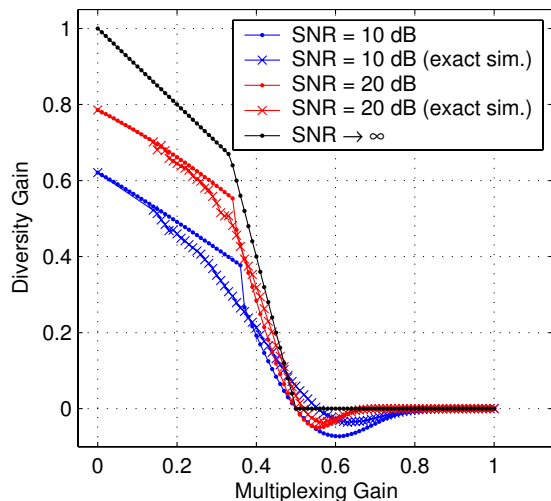


Fig. 5. Finite-SNR diversity-multiplexing tradeoff curves for Scenario 1. The actual diversity gains from Monte Carlo simulations (labeled by “exact sim.”) are also plotted.

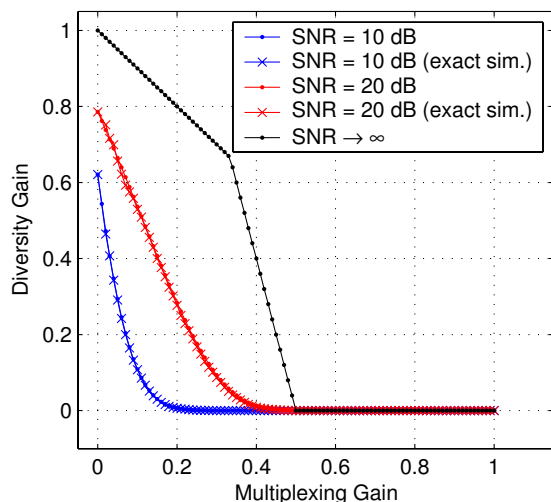


Fig. 6. Finite-SNR diversity-multiplexing tradeoff curves for Scenario 2. The actual diversity gains from Monte Carlo simulations (labeled by “exact sim.”) are also plotted.

Since the lower bound (15) is closer to the actual outage probability than the upper bound (14), especially for low SNR, the diversity gain (for symmetric multiplexing gain) is estimated using $\hat{d}_2(r, r, \rho)$. This estimate is used to plot finite-SNR diversity-multiplexing tradeoff curves in Figs. 5 and 6 for Scenarios 1 and 2, respectively. Tradeoff curves obtained from Monte Carlo simulations are also plotted.

It can be seen that the estimated diversity gains are close to the simulated results at SNRs of 10 dB and 20 dB, especially for Scenario 2. The jump in $\hat{d}_2(r, r, \rho)$ near $r = 1/3$ for Scenario 1 is due to the transition between the lightly and heavily loaded regimes. The simulated diversity gain has a smoother transition at finite SNR. For Scenario 2, the transition is smooth for $\hat{d}_2(r, r, \rho)$ since the weak user dominates

performance. The nonzero diversity gain observed for Scenario 1 for $r \geq 1/2$ is due to slight changes in the outage probability as the SNR increases. This effect can be seen in the $r = 1/2$ curves in Figs. 3 and 4. The finite-SNR tradeoff curves are significantly lower than the asymptotic (high-SNR) tradeoff. Because of the weak user in Scenario 2, the tradeoff curve at a given SNR is much lower for Scenario 2 than for Scenario 1. As predicted by (19), the limiting diversity gain is the same for both scenarios. However, the drop in diversity gain is much greater in Scenario 2 as r increases from zero. This drop in the finite-SNR diversity gain can be estimated using the limiting slope of diversity gain with respect to r given in (22).

V. CONCLUSION

A nonasymptotic framework for outage probability and diversity gain is essential to characterize the achievable performance in rate-adaptive fading multiple access channels. The finite-SNR analysis presented in this paper allows accurate performance estimation at realistic SNRs with disparate user powers. Upper and lower bounds on the common outage probability are derived for rate-adaptive multiple access channels. These bounds are used to estimate the diversity gain as a function of the user multiplexing gains and SNR. This nonasymptotic diversity gain, viewed as a reliability measure of a wireless system, is well suited to a perturbation analysis. At realistic SNR, the diversity gain is significantly lower than asymptotic results. Further degradation of the finite-SNR diversity gain occurs when there is an imbalance of user powers. Such insight is useful to design rate adaptation algorithms for multiple access channels with different user transmit powers and channel qualities.

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