Single User Random Beamforming in Gaussian MIMO Broadcast Channels

Youngjae Kim[1], Kee-Bong Song, Ravi Narasimhan and John M. Cioffi
Department of Electrical Engineering
Stanford University
Stanford, California 94305
Email: {youngjae, kbsong, nkravi, cioffi}@stanford.edu

***Abstract*— This paper proposes a new transmitting scheme for a multi-antenna (MIMO) Gaussian broadcast channel. The transmitter exploits multiuser diversity using a small amount of feedback information from the receivers. The feedback information consists of the maximum achievable rate for each user and the necessary power distribution profile across the spatial dimensions that the transmitter should use to achieve that rate. The receivers use the iterative water-filling algorithm to maximize the achievable rate. The new scheme serves a single user at each transmission, hence the name Single User Random Beamforming (SUBF) scheme. The proposed scheme maximizes the average sum rate under a TDMA environment (i.e., supporting a single user at a time) with the partial channel state information (CSI) feedback constraint.**

I. INTRODUCTION

The use of multiple antennas in Gaussian broadcast channels has been widely studied. It was recently discovered that the use of Dirty Paper Coding (DPC) at the transmitter can achieve the sum capacity in a general case where the numbers of transmit or receive antennas are arbitrary. Although DPC performs optimally, it requires full channel knowledge at the transmitter and is computationally intensive; hence, it is very hard to implement. Natural questions follow on how effective it is to feedback only partial channel state information (CSI) to the transmitter and how off the sub-optimal transmitting schemes are from the optimal DPC scheme. Suppose n is the number of transmit antennas, r is the number of receive antennas at the mobile of each user and K is the number of users in a cell. In [1], it was shown that as $K \rightarrow \infty$, the sum capacity asymptotically grows as $n \log\log(r \cdot K)$ when the full channel knowledge is available at the transmitter. With no channel knowledge at the transmitter, the sum capacity becomes independent of K and scales as only $\log n + O(1)$ for $r = 1$. With partial CSI feedback such as signal-to-interference-plus-noise-ratio (SINR), however, the sum capacity grows as $n \log\log(r \cdot K) + O(\log\log\log n)$, which has the same asymptotic growth rate as the case of the full channel knowledge. Therefore, in a MIMO broadcast channel, even partial CSI can be very beneficial.

This phenomenon basically stems from the so-called multiuser diversity. Consider a wireless system which consists of one base station and K users. At any point of time, there always exists a user who has the best channel. This information partially indicates the current channel state and is very valuable if it is available at the base station because it enables the base station to select the best channel at that time. The more users in the cell, the larger set of fading channels to select an even better channel. This multiuser diversity effect can be exploited more if there are larger fluctuations in the channels. Thus, in slow-fading channels, multiuser diversity may be smaller.

In [2], the authors addressed this issue and proposed to use 'dumb' multiple antennas at the base station that induce artificial channel variations. By applying a random vector with unit power across multiple antennas, the base station generated an effective single-antenna channel with more variations. The multiple antennas were used in a so-called 'dumb' way such that the same data is transmitted from all transmit antennas. Therefore, this technique can also be viewed as beamforming data to one user with multiple transmit antennas. In this scheme, only the case of multiple transmit antennas and a single receive antenna was considered, i.e., a Multiple-Input Single-Output (MISO) system.

In this paper, this result is extended to a system with multiple receive antennas, i.e. a Multiple-Input Multiple-Output (MIMO) system. As in [2], only one user will be served at each transmission by the base station (i.e., Time-Division Multiple Access (TDMA)) even though the base station is equipped with multiple antennas. Thus, the scheme is called Single User Beamforming (SUBF) in a MIMO broadcast channel. The same asymptotical behavior will be observed with much higher data rates due to the array and multiplexing gains.

A similar effort of utilizing multiple receive antennas in a multi-antenna broadcast channel was reported in [3]. In order to capture the effect of crosstalk between antennas, authors defined an Effective Signal to Noise Ratio (ESNR) that was computed for each receive antenna by each user and fed back to the base station. Based on ESNRs, waterfilling power-and-rate-allocation across spatial dimensions was performed at the base station to maximize the data rate. While this transmission method improved the overall rate of the system, due to the use of ESNRs it did not truly maximize the data rate with partial CSI and TDMA.

The proposed scheme in this paper maximizes this data rate. The key idea is that once the base station randomly beamforms the reference signal vector, the effective MIMO

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channel estimated by each user can be seen as a MIMO Multiple Access Channel (MAC). The capacity of a MIMO MAC and the corresponding rate-and-power allocation method called Iterative Water-Filling (IWF) is well studied by Yu et al. [4],[5]. Following this observation, each user computes his own maximum data rate and the corresponding transmit power distribution profile using the IWF algorithm. This data rate and the power profile are fed back to the base station through a low-rate feedback channel. Upon receiving this information from all users, the base station can select the best user and send data only to that user. Since IWF achieves the capacity of the effective channel of each user, the proposed scheme maximizes the achievable data rate naturally extending the result in [2].

The paper is organized as follows. Starting with introducing the system model in Section II, we review the sum capacity of MIMO broadcast channel in Section III-A, multiuser diversity and proportional fair scheduling in Section III-B and IWF in a MAC in Section III-C. In Section IV, SUBF is proposed for Gaussian MIMO broadcast channels and simulation results are provided in Section V.

II. System Model

The Gaussian MIMO broadcast system model consists of one base station with n transmit antennas and K users, each of whom has r receive antennas. The channel is assumed to be quasi-static which means that it is fixed within one packet transmission time and varies independently from one packet to another due to channel fading. This model is also called the block fading channel model and can be described as follows :

$$\underline{y}_k(t) = \mathbf{H_k}(t)\,\underline{x}(t) + \underline{z}_k(t) \quad (k = 1, 2, \cdots, K) \tag{1}$$

where $\underline{y}_k$ is an $r \times 1$ vector denoting a received signal of the k^{th} user, $\underline{x}$ an $n \times 1$ vector denoting a transmitted signal by the base station, $\mathbf{H_k}$ is a $r \times n$ channel matrix, $\underline{z}_k$ is an additive noise to the k^{th} user and t denotes the time index. Throughout the paper, underlined lower case letters represent vectors, boldface capital letters represent matrices and plain lower case letters represent scalars.

III. Preliminary Results

A. Sum Capacity of a Gaussian MIMO Broadcast Channel

The sum capacity of a Gaussian MIMO broadcast channel is defined as the maximum sum of achievable data rates of all users. When $n = 1$ and $r = 1$, i.e., both the number of transmit and receive antenna are one, the broadcast channel becomes degraded [6]; specifically, if the users are ordered according to their respective channel gains, the received signal of each user can be considered as the noisy version of its predecessor. In this case, it is known that the maximum sum rate strategy is to transmit data only to the user with the best channel gain [7]. In other words, TDMA-based transmission is optimal for a single antenna system. However, when the number of transmit antennas is more than one, the channel is no longer degraded (regardless of the number of receive antennas). Consequently, selecting one best user does not necessarily guarantee the maximization of the sum rate. The insight behind this fact is that the system can achieve a higher sum rate by serving more than one user at each transmission because multiple antennas at the base station can support multiple data streams simultaneously (i.e., Spatial-Division Multiple Access (SDMA)). More exactly, the sum capacity of a Gaussian MIMO broadcast channel is generally achieved with the DPC scheme at the base station [8], [9] and the DPC scheme becomes equivalent to TDMA-based transmission when $n = 1$ and $r = 1$.

Despite its sub-optimality in a multiple transmit antenna case from a sum-capacity point of view, TDMA-based transmission that serves only one user at a time can be still beneficial for other reasons. In fact, the DPC scheme that simultaneously transmits multiuser data streams in an optimal way is very difficult to implement in practice. Furthermore, in some applications, a peak data rate of each user rather than the sum of all users could be a more important measure. Finally, an important aspect of TDMA with multiple antennas is multiuser diversity that is explained in the next subsection.

For comparison, this paper also shows the sum rate of a sub-optimal SDMA scheme, where the transmitter sends $\min\{r, n\}$ orthogonal beams with equal powers. Then, each receiver performs MMSE equalization to extract each beam's SINR and feeds them back to the transmitter with their beam indices. Upon receiving these SINRs from the receivers, the transmitter assigns each beam to the user who reported the highest SINR. This scheme is the same as the one introduced in [1] except that the scheme in [1] treats each receive antenna independently ignoring the coordination capability at the receiver. Meanwhile, similar sub-optimal SDMA schemes in MIMO systems were proposed in [10] with the round robin scheduling and in [11] with the proportional fair scheduling.

B. Multiuser Diversity and Proportional Fair Scheduling

Denoting $R_k(t)$ as the requested data rate by the k^{th} user at a certain time t, multiuser diversity can be exploited when the base station selects a user who requested the highest $R_k(t)$. Since the user with an inferior channel might not receive any data from this scheme, the idea of feeding back $\frac{R_k(t)}{T_k(t)}$ instead of $R_k(t)$ was proposed, where $T_k(t)$ is the average throughput that the k^{th} user has achieved until time t. This idea is called Proportional Fair Scheduling (PFS) and it balances the sum rate of the system and fairness among users. (See [2], [12] for the details).

This paper extends the result in [2] to the case where the number of receive antennas are more than one. From (1), the signal received by the k^{th} user is represented by

$$\underline{y}_k = \mathbf{H_k}\,\underline{x} + \underline{z}_k \quad (k = 1, 2, \cdots, K) \tag{2}$$

where the time index t is omitted for simplicity. Using a singular value decomposition (SVD), the channel matrix $\mathbf{H_k}$ can be decomposed as

$$\mathbf{H_k} = \mathbf{U_k \Sigma_k V_k^*} \tag{3}$$

where $\mathbf{U_k} \in \mathcal{C}^{r\times r}$, $\mathbf{V_k} \in \mathcal{C}^{n\times n}$ with $\mathbf{U_k^* U_k} = \mathbf{I}, \mathbf{V_k^* V_k} = \mathbf{I}$, $[\ ]^*$ represents a conjugate-transpose operation on the matrix and $\mathbf{I}$ denotes the identity matrix. $\mathbf{\Sigma_k}$ is an $r \times n$ matrix whose diagonal elements are $\mathbf{H_k}$'s singular values and off-diagonal elements are all zeros, i.e., $\mathbf{\Sigma_k}(i,j) = 0,\, i \neq j$. When the beamforming matrix $\mathbf{V}_b \in \mathcal{C}^{n \times n}$, where $\mathbf{V_b^* V_b} = \mathbf{I}$, is multiplied at the base station before each transmission, the transmitted signal $\underline{x}$ becomes $\mathbf{V}_b \underline{u}$. Here, the vector $\underline{u}$ represents n independent spatial data streams with the covariance matrix $\mathbf{R}_{uu}$ being diagonal. Note that there is no correlation among the elements of $\underline{u}$ and that each element of $\underline{u}$ may have a different power allocation. Then, combined with (3), (2) becomes

$$\underline{y}_k = \mathbf{U_k \Sigma_k V_k^* V}_b \underline{u} + \underline{z}_k \quad (k = 1, 2, \cdots, K) \tag{4}$$

If $\mathbf{V_k^* V}_b = \mathbf{I}$, the k^{th} user is said to be in *the true beamforming configuration*. In this situation, the interferences from other antennas are completely canceled at the transmitter side as if the base station had perfect channel knowledge and precoded the transmitted signal using a matrix $\mathbf{V}_b$.

Suppose that the signal transmission in (4) supports only one particular user (i.e., TDMA). This implies that the power profile in $\underline{u}$ is active only for data streams of one serving user. As discussed in section III-A, the sum capacity is not achieved under this constraint. Instead, in a slowly fading environment, the beamforming matrix $\mathbf{V}_b$ can be viewed as creating an *artificial* fast fading channel that will manifest multiuser diversity. The resulting sum rate is expected to be higher than the single antenna case because of the array gains from the multiple receive antennas and the multiplexing gains from the multiple data streams.

When $r = 1$, the computation of $R_k(t)$ that is necessary in the PFS algorithm is similar to the capacity of a scalar Gaussian channel and the corresponding power profile is to activate only one element of $\underline{u}$ related to the k^{th} user. If $r > 1$, however, the computation of $R_k(t)$ requires the optimization of the power allocation across the corresponding user's data streams because now multiple antennas can support up to $\min\{r, n\}$ multiple data streams for each user. This optimization procedure is facilitated due to the capacity results of MAC as explained in the next section.

The asymptotic behavior of each user's average throughput is worthy of attention when the number of users K increases. Suppose that the channel fades very slowly so that all channel coefficients can be regarded as constants during many transmit time slots. In each time slot, the base station randomly beamforms data using a different $\mathbf{V_b}$ and selects a user using the PFS algorithm. Then, as the number of users grows in the cell, it is very likely that the user who is in *the true beamforming configuration* is selected. More specifically, let $T_k^{(K)}$ denote the average throughput of the k^{th} user in the cell and R_k^{bf} denote the beamformed data rate for the k^{th} user defined as $R_k^{bf} = \sum_i (\log(1 + \kappa_i \lambda_i))^\dagger, \kappa_i = (\mu - 1/\lambda_i)^\dagger, \sum_i \kappa_i = P$, where λ_i is the channel gain for i^{th} data stream, μ is the water level, P is the total power constraint and $(x)^\dagger = \max(0, x)$. Then, the following relation is conjectured and verified by simulation results: With very high probability,

$$\lim_{K \to \infty} K T_k^{(K)} = R_k^{bf} \tag{5}$$

This phenomenon can be explained as follows. Since the base station selects a user based on the ratio $\frac{R_k(t)}{T_k(t)}$, as $K \to \infty$, it is likely that the user who is near his own peak will be selected. The user in *the true beamforming configuration* is clearly in his own peak and his maximum achievable data rate can be found by performing waterfilling at the transmitter. The proof for $n > 1$ and $r = 1$ case is shown in [2, Appendix A].

C. MAC sum capacity with a sum power constraint

Starting with (4), let $\mathbf{H_{eff,k}}=\mathbf{U_k \Sigma_k V_k^* V}_b$ be called the *effective channel* for the k^{th} user. Then, (4) becomes

$$\underline{y}_k = \mathbf{H_{eff,k}} \underline{u} + \underline{z}_k \quad (k = 1, 2, \cdots, K) \tag{6}$$

Since there is no correlation between the elements of $\underline{u}$, it can be assumed that each data stream of $\underline{u}$ is sent from an independent user. Then, this situation can be viewed as a MAC where the n independent users at the base station side send data to the k^{th} user. In addition, the transmit power constraint at the base station now becomes a sum power constraint of these n users. Therefore, the maximum achievable rate for the k^{th} user is equivalent to the sum capacity of the MAC with the sum power constraint. This observation leads to the key concept of our scheme.

In order to simplify the notation, omit k from (6) and let $\mathbf{H_{eff}} = [\underline{h}_1\ \underline{h}_2\ \ldots\ \underline{h}_n]$ and $\underline{u} = [u_1\ u_2\ \ldots\ u_n]^T$. Then, (6) becomes

$$\underline{y} = \sum_{i=1}^{n} \underline{h}_i u_i + \underline{z} \tag{7}$$

Then, the maximum achievable rate for the k^{th} user can be found by solving the following optimization problem :

$$\textit{Maximize} \quad \log \Big| \sum_{i=1}^{n} P_i \underline{h}_i \underline{h}_i^* + \mathbf{I} \Big| \tag{8}$$

$$\textit{Subject to} \quad \sum_{i=1}^{n} P_i \leq P_{tot} \quad (i = 1, 2, \cdots, n) \tag{9}$$

$$P_i \geq 0 \tag{10}$$

where $|\cdot|$ represents a determinant of a matrix, $P_i = E[|u_i|^2]$ and P_{tot} is the total power constraint at the base station. Therefore, the covariance matrix of $\underline{u}$ can be written as $\mathbf{R}_{uu} = diag(P_1, P_2, \cdots, P_n)$. The covariance matrix of noise $\underline{z}$ is assumed to be identity without loss of generality.

This optimization problem can be solved efficiently by IWF in [13] or IWF with a dual decompostion approach [14]. The steps to find the optimal distribution of P_i are as follows:

1. Initialize $P_i(0) = 0$ for $\forall i$.

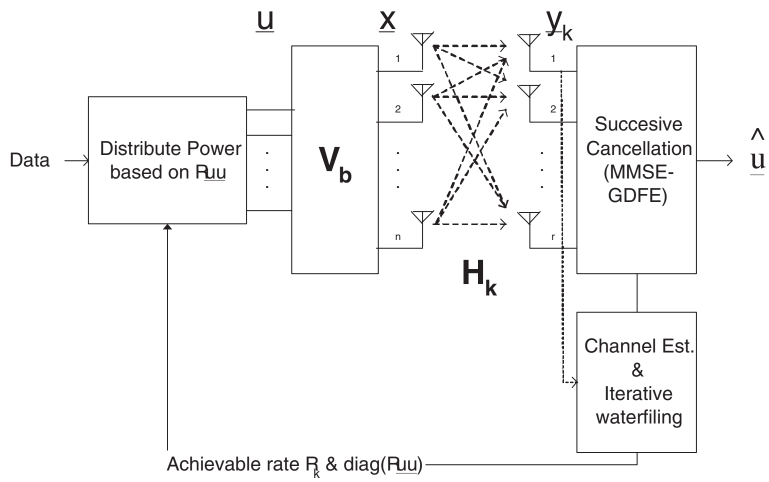

Fig. 1. The proposed scheme

2. For iteration l : Consider the interferences as noise and generate effective channels

$$\underline{h}_i^{eff} = \left(\mathbf{I} + \sum_{j=1, j\neq i}^{n} P_i \underline{h}_j \underline{h}_j^* \right)^{-\frac{1}{2}} \underline{h}_i \quad (11)$$

3. Treat these effective channels as parallel, non-interfering channels, perform waterfilling with P_{tot} and obtain the new set of $P_i(l)$ $(i = 1, \cdots, n)$. i.e.,

$$\{P_i(l)\}_{i=1}^{n} = \arg \max_{X_i, \cdots, X_n} \sum_{i=1}^{n} \log \left| \mathbf{I} + X_i \underline{h}_i^{eff} \underline{h}_i^{eff\ *} \right| \quad (12)$$

$$= \arg \max_{X_i, \cdots, X_n} \sum_{i=1}^{n} \log \left(1 + X_i \| \underline{h}_i^{eff} \|^2 \right) \quad (13)$$

where, $X_i \geq 0$ and $\sum_{i=1}^{n} X_i \leq P_{tot}$.

4. Repeat from step 2 until desired accuracy is achieved.

The detailed proof of convergence of this algorithm can be found in [13] and [14].

Let $\mathbf{R}_{uu}^{\star} = diag(P_1^{\star}, P_2^{\star}, \cdots, P_n^{\star})$ be the optimized covariance matrix. Having the full channel estimate of $\mathbf{H_{eff}}$, each user calculates the MAC sum capacity which becomes the user's requested rate R_k. The R_k is fed back to the transmitter with the associated $P_i^{\star}$'s $(i = 1, \cdots, n)$. This optimized power distribution tells the transmitter how to distribute its total power in each data stream in order to achieve R_k. The transmitter selects the user who has the highest rate and sends data to that user. The same multiuser diversity concept in Section III-B applies from now on. Note that it is assumed that the total power constraint P_{tot} has already been revealed to each user.

At the receiver side, the Minimum Mean Squared Error-Generalized Decision Feedback Equalization (MMSE-GDFE) structure that performs the successive cancellation across antenna streams is used since it was shown to achieve this sum capacity of the MAC channel [15], [16].

IV. Proposed Single User Random Beamforming Scheme for MIMO Systems

The proposed scheme uses the multiple antennas to support multiple data streams and it utilizes the partial CSI which is fed back from the users. Each user performs IWF to calculate the maximum achievable data rate. The procedure is shown in Fig. 1 and is detailed below:

1. The transmitter loads a training sequence to $\underline{u}$, multiplies it by a random beamforming matrix $\mathbf{V}_b$ and sends it to all users. The training sequence is used for channel estimation and assumed to be known to all users.

2. Each user estimates the effective channel ($\mathbf{H_{eff,k}}$) and calculates its maximum achievable data rate using IWF.

3. The requested rates are fed back to the transmitter.

4. The transmitter selects the user who has the highest rate and broadcast this information to all users.

5. The selected user feeds back the power distribution profile.

6. The transmitter sends data to the selected user using the received power distribution.

7. The selected user performs successive cancelation (MMSE-GDFE) to decode each data stream.

8. Repeat steps 1 through 7 for each channel realization.

V. Simulation Results

In Fig. 2, the multiuser diversity in MIMO systems is verified using 300 realizations of i.i.d Rayleigh fading channels. The average sum rates are compared while fairness is not considered. For comparison, the sum capacity from the optimal DPC scheme and the sum rate achieved by the SDMA scheme were also drawn. Note that when $K = 1$, SUBF is better than SDMA because this case is equivalent to a point-to-point link and the SUBF is using an optimal strategy to achieve the capacity. The multiuser diversity effect can be also observed when receivers feed back only the achievable rates and the transmitter uses a uniform power profile. Its result is drawn on the same graph, whose difference from SUBF shows the benefit of the optimal power profile feedback.

The conjecture made in (5) is verified in Fig. 3. In this simulation, the channel is realized once and kept constant. Only the random beamforming matrix changes for each calculation. As the number of users in the system increases, the data rate of each user multiplied by K converges to the beamformed rate R_k^{bf} of (5). This phenomenon is demonstrated with different SNRs, i.e., 0dB, 5dB and 10dB. Note that $K \cdot T_1^{(K)}$ converges to the beamformed data rate quickly as the number of users increases. However, as more antennas are added to the transmitter and the receivers, the convergence speed reduces. Fig. 4 shows the result when $n = 3$ and $r = 3$. This phenomenon can be explained as follows: More antennas mean more spatial degrees of freedom which makes it more difficult for the base station to randomly select the correct beamforming matrix.

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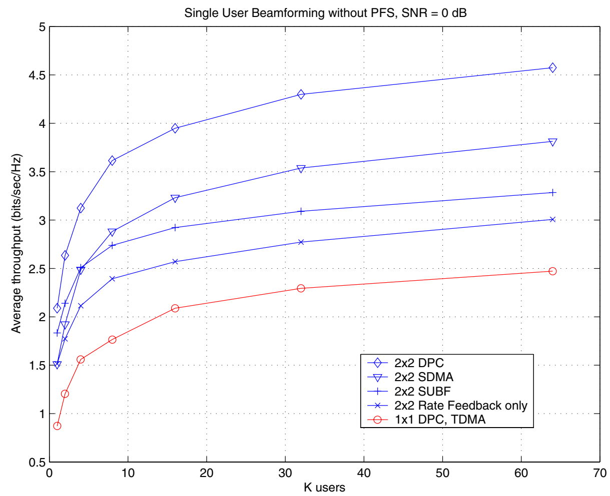

Fig. 2. The effect of multiuser diversity in 2x2 MIMO systems compared with the 1x1 system. SNR = 0dB.

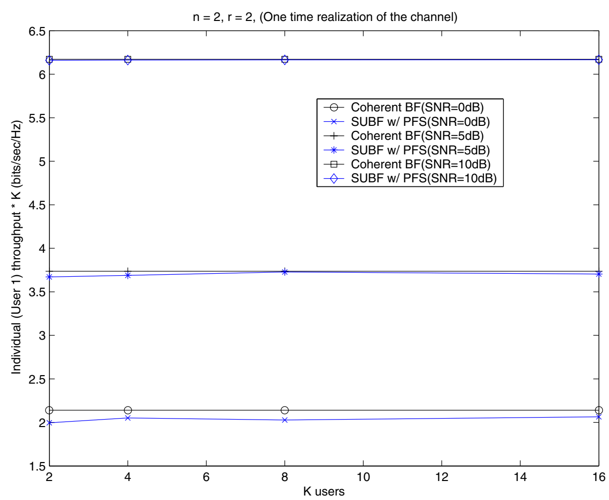

Fig. 3. The convergence of $K \cdot T_1^{(K)}$ in 2x2 MIMO systems when SNR = 0dB, 5dB and 10dB.

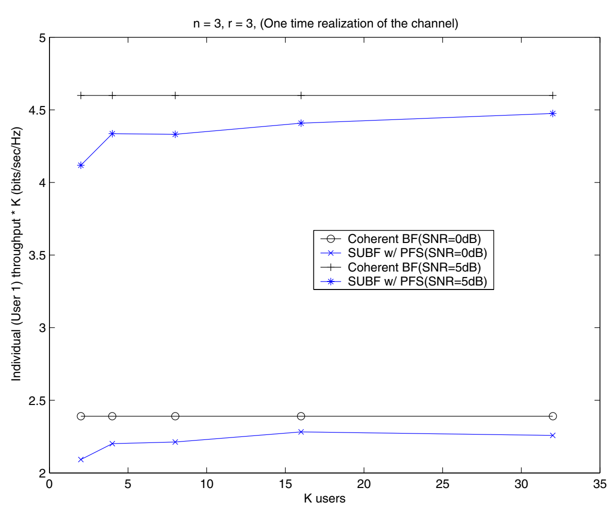

Fig. 4. The convergence of $K \cdot T_1^{(K)}$ in 3x3 MIMO systems when SNR = 0dB and 5dB.