# Finite-SNR Diversity-Multiplexing Tradeoff of Space-Time Codes

Ravi Narasimhan
Department of Electrical Engineering
University of California, Santa Cruz, CA 95064
Email: ravi@soe.ucsc.edu

Amal Ekbal and John M. Cioffi
Department of Electrical Engineering
Stanford University, Stanford, CA 94305
Email: {ekbal,cioffi}@stanford.edu

***Abstract*— A novel framework is presented to characterize the tradeoff between diversity and multiplexing of space-time codes at finite signal-to-noise ratios (SNR's). The diversity gain of a space-time code is defined by the slope at a particular SNR of the outage probability versus SNR curve for a multiplexing gain defined by the ratio of the system spectral efficiency to the capacity of an additive white Gaussian noise (AWGN) channel. The finite-SNR diversity-multiplexing tradeoff is evaluated for orthogonal space-time block codes and spatial multiplexing with horizontal encoding. The tradeoff curves provide a characterization of achievable diversity and multiplexing gains for a given space-time code at SNR's encountered in practice. It is seen that the achievable diversity gains at finite SNR are significantly lower than the asymptotic values given in the literature.**

## I. Introduction

Multiple-input multiple-output (MIMO) processing has emerged as a promising technique to achieve high capacities in wireless communication links. Traditionally, multiple antennas at the transmitter and receiver have been used to provide either enhanced reliability through spatial diversity techniques or increased data rates using spatial multiplexing. Recently, a framework was presented in [1] that provides a unified view of the fundamental diversity-multiplexing tradeoff in MIMO systems. This framework bridges the two endpoints of pure diversity gain or pure multiplexing gain. The diversity-multiplexing tradeoff curve of a given space-time code enables one to determine the set of diversity and multiplexing gains that can be obtained simultaneously.

The diversity-multiplexing tradeoff computed in [1] is an asymptotic result as the signal-to-noise ratio (SNR) approaches infinity. Asymptotic analysis was also used in [2] to obtain the well-known rank and determinant criteria for space-time code design. Recently, several papers have noted that codes designed for maximum performance at high SNR in independent and identically distributed (i.i.d.) Rayleigh fading channels are not optimal at low to medium SNR, especially in the presence of correlated fading [3], [4], [5], [6]. The literature to date has not adequately addressed the issue of the diversity-multiplexing tradeoff in the non-asymptotic, finite-SNR regime. Thus, it is of interest to characterize this tradeoff for finite SNR to gain new insights into space-time code design.

This paper develops a new framework to characterize the diversity-multiplexing tradeoff of space-time codes as a function of SNR. In contrast to previous asymptotic work, the diversity gain *at each SNR* is obtained from the slope of the outage probability versus SNR curve. The outage probability is the block error probability in quasi-static MIMO channels when capacity achieving codes are used over a block of time containing a single channel realization. The significance of this definition for system design is that the diversity gain at a particular operating SNR provides an indication of the additional power required to decrease the error probability by a specified amount. The multiplexing gain is measured as the ratio of the spectral efficiency of the MIMO system to the capacity of an additive white Gaussian noise (AWGN) channel. As in [1], this definition implies that for a constant multiplexing gain, the spectral efficiency increases with SNR. The multiplexing gain can be interpreted in the context of rate adaptation in which the data rate is adapted as a function of SNR. These definitions of diversity and multiplexing gains are generalizations for finite SNR of the corresponding definitions given in [1].

The remainder of the paper is organized as follows. Section II introduces the system model and the definitions of diversity and multiplexing gains as functions of SNR. The diversity and multiplexing gains are then computed for orthogonal space-time block codes (OSTBC) and spatial multiplexing with horizontal encoding (SM-HE) in Section III. In Section IV, outage probability and diversity-multiplexing tradeoff curves are presented. The results are also compared with asymptotic results in the literature as $\mathrm{SNR} \to \infty$. It is seen that the achievable diversity at operational SNR's is significantly lower than the asymptotic value. Conclusions are given in Section V.

## II. System Model and Definitions

Consider a narrowband MIMO system with $M_T$ transmit and $M_R$ receive antennas and a rich scattering Rayleigh fading environment such that the $M_R \times M_T$ channel matrix $\mathbf{H}$ is a matrix of i.i.d. complex Gaussian random variables with zero mean and unit variance. The received vector is corrupted by additive white Gaussian noise. Quasi-static fading is assumed in which the channel is constant over one coding block and varies independently across blocks. Capacity achieving codes are used in each block such that the probability of block error is equal to the channel outage probability, i.e., the prob-

0-7803-8939-5/05/$20.00 (C) 2005 IEEE

ability that the mutual information between the transmitted and received signals is less than the target data rate. Such a block fading model is applicable in several wireless systems. For example, the channel in a wireless local area network is constant for the duration of a packet and varies from packet to packet. Thus, with capacity achieving codes applied to each packet, the packet error rate is determined by the outage probability.

The conventional definitions of multiplexing and diversity gains of a MIMO system refer to asymptotic quantities as the SNR approaches infinity. For instance, consider a space-time system with average SNR per receive antenna $\rho$, spectral efficiency $R$ (bps/Hz) and corresponding outage probability $P_{\text{out}}$. The asymptotic multiplexing and diversity gains defined in [1] are given by

$$r_{\text{asymptotic}} = \lim_{\rho\to\infty} \frac{R}{\log_2 \rho} \tag{1}$$

$$d_{\text{asymptotic}} = -\lim_{\rho\to\infty} \frac{\log_2 P_{\text{out}}}{\log_2 \rho}. \tag{2}$$

These definitions capture the *high-SNR* tradeoff of data rate, represented by the multiplexing gain, and reliability, represented by the diversity gain. Note that in environments with high spatial correlation or line-of-sight components, the asymptotic results are useful only at very large SNR's, high data rates and correspondingly low outage probabilities. However, for system design, it is desirable to characterize the diversity-multiplexing tradeoff at operational SNR's, data rates and outage probabilities.

These considerations motivate the new non-asymptotic definitions introduced in this paper. The multiplexing gain $r$ is defined as the ratio of $R$ to the capacity of an AWGN channel at SNR $\rho$:

$$r = \frac{R}{\log_2(1+\rho)}. \tag{3}$$

Note that for a constant multiplexing gain $r$, the spectral efficiency must increase as the SNR increases. The multiplexing gain $r$ provides an indication of the sensitivity of a rate adaptation algorithm as the SNR changes. As the value of $r$ increases, a more sensitive rate adaptation strategy is used in which a moderate change in SNR can result in a significant change in the data rate.

As illustrated in Fig. 1, the diversity gain $d(r,\rho)$ of a system with multiplexing gain $r$ at SNR $\rho$ is defined by the negative slope of the log-log plot of outage probability versus SNR:

$$d(r,\rho) = -\frac{\rho}{P_{\text{out}}(r,\rho)} \cdot \frac{\partial P_{\text{out}}(r,\rho)}{\partial \rho} \tag{4}$$

where $P_{\text{out}}(r,\rho)$ is the outage probability as function of the multiplexing gain and SNR. The significance of this definition for system design is that the diversity gain at a particular operating SNR can be used to estimate the additional SNR required to decrease the outage probability by a specified amount, for a given rate adaptation strategy represented by the multiplexing gain.

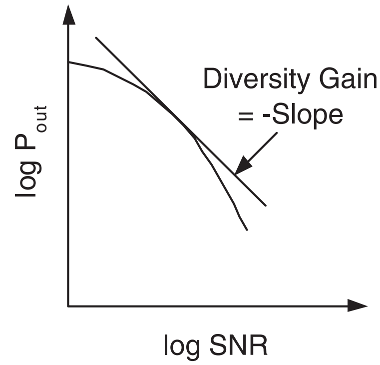


Fig. 1. Definition of diversity gain as a function of SNR.

## III. Computation of Finite-SNR Diversity-Multiplexing Tradeoff

The definitions given in Section II are now used to compute the diversity-multiplexing tradeoff as a function of SNR for OSTBC and SM-HE.

### A. OSTBC

Orthogonal space-time block codes have the attractive property that the maximum diversity gain of $M_T M_R$ can be achieved at high SNR in rich scattering using linear processing at the receiver. Given a channel realization $\mathbf{H}$, the mutual information for OSTBC with spatial code rate $r_s$ is given by [7]

$$I_{\text{OSTBC}} = r_s \log_2\Big(1 + \frac{\rho}{M_T}\|\mathbf{H}\|_F^2\Big) \tag{5}$$

where $\|\mathbf{H}\|_F^2$ is the squared Frobenius norm of $\mathbf{H}$. The spatial code rate $r_s$ is equal to the average number of independent symbols transmitted per channel use through the $M_T$ transmit antennas. For example, $r_s = 1$ for the Alamouti space-time code [8], and $r_s = 1/2$ or $r_s = 3/4$ for the complex orthogonal designs given in [9]. From the fact that the squared Frobenius norm of the i.i.d. complex Gaussian channel matrix $\mathbf{H}$ is a gamma random variable with parameters $(M_T M_R, 1)$ [7], the outage probability for OSTBC is computed as follows:

$$\begin{aligned} P_{\text{out,OSTBC}}(r,\rho) &= \Pr\{I_{\text{OSTBC}} < r\log_2(1+\rho)\} \\ &= \Pr\Big\{\|\mathbf{H}\|_F^2 < \frac{M_T}{\rho}\Big[(1+\rho)^{r/r_s} - 1\Big]\Big\} \\ &= \frac{\gamma\Big(M_T M_R, \frac{M_T}{\rho}\Big[(1+\rho)^{r/r_s}-1\Big]\Big)}{(M_T M_R - 1)!} \end{aligned} \tag{6}$$

where $\gamma(a,x) = \int_0^x t^{a-1}e^{-t}\,dt$ is the incomplete gamma function. Now, from (4), the diversity gain is

$$\begin{aligned} d_{\text{OSTBC}}(r,\rho) = &\frac{\frac{M_T}{\rho}\Big(\frac{M_T}{\rho}\Big[(1+\rho)^{r/r_s}-1\Big]\Big)^{M_T M_R - 1}}{\gamma\Big(M_T M_R, \frac{M_T}{\rho}\Big[(1+\rho)^{r/r_s}-1\Big]\Big)} \\ &\cdot e^{-\frac{M_T}{\rho}[(1+\rho)^{r/r_s}-1]}\Big[(1+\rho)^{r/r_s} - 1 - \frac{r\rho}{r_s}(1+\rho)^{r/r_s-1}\Big]. \end{aligned} \tag{7}$$

### B. SM-HE

Spatial multiplexing with horizontal encoding is a simple technique to achieve high spectral efficiencies in MIMO systems. In SM-HE, a high rate data stream is demultiplexed into $M_T$ substreams that are then independently encoded. In this paper, zero forcing (ZF) or minimum mean squared error (MMSE) linear processing is assumed at the receiver with $M_R \geq M_T$. The mutual information for SM-HE is limited by the smallest post-processing SNR among the $M_T$ substreams. Therefore, the mutual information of SM-HE given the channel $\mathbf{H}$ is [7]

$$I_{\text{SM-HE}} = M_T \log_2\Big(1 + \min_{i\in\{1,\ldots,M_T\}} \eta_i\Big) \quad (8)$$

where $\eta_i$ is the post-processing SNR of $i$-th substream. The substream SNR $\eta_i$ is given by

$$\eta_i = \begin{cases} \frac{\rho/M_T}{[(\mathbf{H}^*\mathbf{H})^{-1}]_{i,i}}, & \text{ZF} \\ \frac{1}{[(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^*\mathbf{H})^{-1}]_{i,i}} - 1, & \text{MMSE} \end{cases} \quad (9)$$

where $\mathbf{I}_{M_T}$ is the $M_T \times M_T$ identity matrix and the superscript $^*$ denotes conjugate transposition. Note that the $\eta_i$ are not independent since the noise terms among the substreams are correlated. The outage probability is given by

$$\begin{aligned} P_{\text{out,SM-HE}}(r,\rho) &= \Pr\{I_{\text{SM-HE}} < r\log_2(1+\rho)\} \\ &= \Pr\Big\{\min_{i\in\{1,\ldots,M_T\}} \eta_i < (1+\rho)^{r/M_T} - 1\Big\} \end{aligned} \quad (10)$$

Due to the correlation of $\eta_i$ for $i = 1,\ldots,M_T$, direct computation of (10) is complicated. Simple upper and lower bounds for $P_{\text{out,SM-HE}}(r,\rho)$ can be used to gain insight into the diversity-multiplexing tradeoff. An upper bound is computed using the following inequality [10]: $\min_{i\in\{1,\ldots,M_T\}} \eta_i \geq \frac{\rho}{M_T}\lambda_{\min}(\mathbf{H}^*\mathbf{H})$ , where $\lambda_{\min}(\mathbf{A})$ is the minimum eigenvalue of a square matrix $\mathbf{A}$. Thus,

$$P_{\text{out,SM-HE}} \leq \Pr\Big\{\lambda_{\min}(\mathbf{H}^*\mathbf{H}) < \frac{M_T}{\rho}[(1+\rho)^{r/M_T} - 1]\Big\} \quad (11)$$

where the dependence of $P_{\text{out,SM-HE}}$ on $(r,\rho)$ has been suppressed for notational convenience. The probability density function (pdf) of $\lambda_{\min}(\mathbf{H}^*\mathbf{H})$ has been computed in [11] and is given by

$$f_{\lambda_{\min}}(\lambda) = \alpha_{M_R,M_T}\lambda^{M_R-M_T}\, e^{-M_T\lambda}\sum_{k=0}^{\nu} a_k\lambda^k,\ \lambda \geq 0 \quad (12)$$

where the normalization constant $\alpha_{M_R,M_T}$ is given by

$$\alpha_{M_R,M_T} = \left[\sum_{k=0}^{\nu} a_k \frac{(M_R - M_T + k)!}{M_T^{M_R-M_T+k+1}}\right]^{-1} \quad (13)$$

and $\nu = (M_R - M_T)(M_T - 1)$. The positive coefficients $a_k$ can be determined numerically from recursion relations. The upper bound is simplified further using the following lemma:

*Lemma :* Let $X$ be a gamma random variable with parameters $(M_R - M_T + 1, 1/M_T)$. Then,

$$F_{\lambda_{\min}}(\lambda) \leq F_X(\lambda) \quad (14)$$

where $F_{\lambda_{\min}}(\cdot)$ and $F_X(\cdot)$ denote the cumulative distribution functions (cdf's) of $\lambda_{\min}(\mathbf{H}^*\mathbf{H})$ and $X$, respectively.

*Proof:* The proof is given in the Appendix. ■

From (14), the following upper bound is obtained for the outage probability:

$$P_{\text{out,SM-HE}} \leq \frac{\gamma\Big(M_R - M_T + 1, \frac{M_T^2}{\rho}[(1+\rho)^{r/M_T} - 1]\Big)}{(M_R - M_T)!} \quad (15)$$

A lower bound for $P_{\text{out,SM-HE}}$ can be obtained by considering the post-processing SNR of any substream (e.g., the first substream). Thus,

$$P_{\text{out,SM-HE}} \geq \Pr\big\{\eta_1 < (1+\rho)^{r/M_T} - 1\big\}. \quad (16)$$

Since the distribution of $\eta_i$ for the MMSE receiver is not known in closed form, the lower bound for the ZF receiver is obtained. For the ZF receiver, $\eta_1$ follows a gamma distribution with parameters $(M_R - M_T + 1, \rho/M_T)$. Thus, the lower bound for the outage probability of SM-HE using the ZF receiver is given by

$$P_{\text{out,SM-HE}} \geq \frac{\gamma\Big(M_R - M_T + 1, \frac{M_T}{\rho}[(1+\rho)^{r/M_T} - 1]\Big)}{(M_R - M_T)!}. \quad (17)$$

Note that the upper and lower bounds for $P_{\text{out,SM-HE}}$ differ only by a factor of $M_T$ in the second argument of the incomplete gamma function.

From (4) and the bounds for $P_{\text{out,SM-HE}}$, the following bounds are derived for the diversity gain $d_{\text{SM-HE}}(r,\rho)$:

$$\begin{aligned} d_{\text{SM-HE}}(r,\rho) \geq{}& \frac{\frac{M_T^2}{\rho}\Big(\frac{M_T^2}{\rho}\Big[(1+\rho)^{r/M_T} - 1\Big]\Big)^{M_R-M_T}}{\gamma\Big(M_R - M_T + 1, \frac{M_T^2}{\rho}\Big[(1+\rho)^{r/M_T} - 1\Big]\Big)} \\ &\cdot e^{-\frac{M_T^2}{\rho}[(1+\rho)^{r/M_T}-1]}\Big[(1+\rho)^{r/M_T} - 1 - \frac{r\rho}{M_T}(1+\rho)^{r/M_T-1}\Big] \end{aligned} \quad (18)$$

$$\begin{aligned} d_{\text{SM-HE}}(r,\rho) \leq{}& \frac{\frac{M_T}{\rho}\Big(\frac{M_T}{\rho}\Big[(1+\rho)^{r/M_T} - 1\Big]\Big)^{M_R-M_T}}{\gamma\Big(M_R - M_T + 1, \frac{M_T}{\rho}\Big[(1+\rho)^{r/M_T} - 1\Big]\Big)} \\ &\cdot e^{-\frac{M_T}{\rho}[(1+\rho)^{r/M_T}-1]}\Big[(1+\rho)^{r/M_T} - 1 - \frac{r\rho}{M_T}(1+\rho)^{r/M_T-1}\Big]. \end{aligned} \quad (19)$$

Note that while both (18) and (19) hold for the ZF receiver, only the lower bound (18) on the diversity gain is valid for the MMSE receiver.

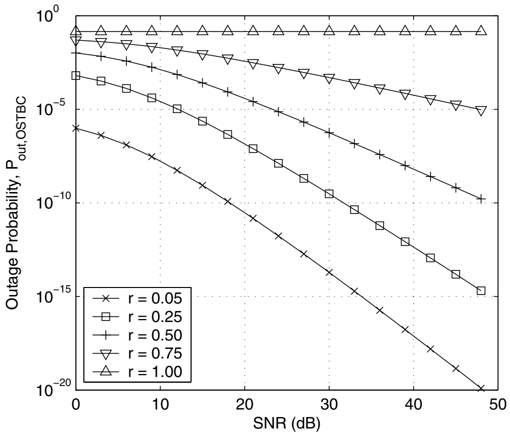


Fig. 2. Outage probability of OSTBC for different multiplexing gains $r$.

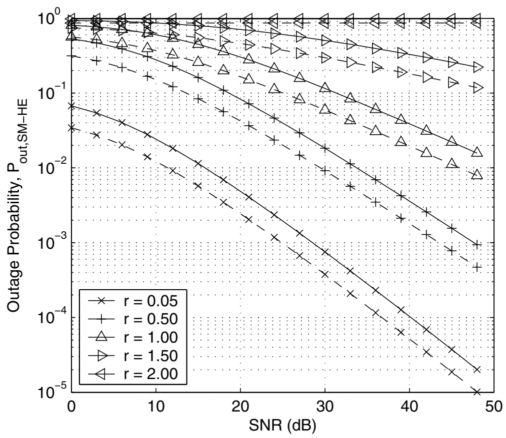


Fig. 3. Bounds on outage probability of SM-HE for different multiplexing gains $r$ (upper bounds: solid, lower bounds: dashed).

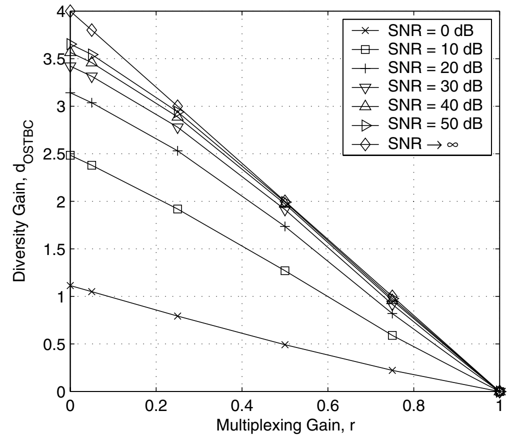


Fig. 4. Diversity-multiplexing tradeoff curves for OSTBC and different SNR's.

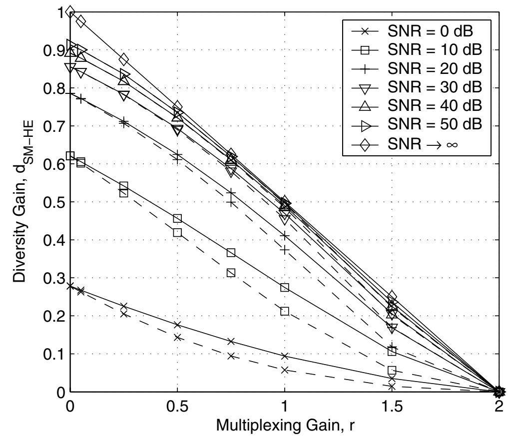


Fig. 5. Bounds on diversity-multiplexing tradeoff curves for SM-HE and different SNR's (upper bounds: solid, lower bounds: dashed).

## IV. Numerical Results

This section provides some numerical results on the outage probabilities and diversity-multiplexing tradeoff curves, including a comparison with asymptotic results. Let $d_{\max}$ denote the maximum diversity gain for each space-time code. Then,

$$d_{\max} = \begin{cases} M_T M_R, & \text{OSTBC} \\ M_R - M_T + 1, & \text{SM-HE} \end{cases} . \qquad (20)$$

The high-SNR limits of the diversity gains for OSTBC and SM-HE are

$$\lim_{\rho \to \infty} d_{\text{OSTBC}}(r, \rho) = d_{\max}\left(1 - \frac{r}{r_s}\right), \ 0 \le r \le r_s \qquad (21)$$

$$\lim_{\rho \to \infty} d_{\text{SM-HE}}(r, \rho) = d_{\max}\left(1 - \frac{r}{M_T}\right), \ 0 \le r \le M_T \qquad (22)$$

which agree with the results of [1].

Consider a MIMO system with $M_T = M_R = 2$ and the Alamouti code for OSTBC. The outage probabilities versus SNR are plotted for OSTBC and SM-HE in Figs. 2 and 3, respectively. An interpretation of these outage probability curves is that they provide a measure of system performance for a particular sensitivity of the rate adaptation strategy represented by the multiplexing gain $r$. Since OSTBC depends on the Frobenius norm of the channel rather than the channel singular values, the outage probability of OSTBC, in contrast to SM-HE, decreases significantly even for moderate sensitivities of the rate adaptation (*e.g.*, $r = 0.5$).

The diversity-multiplexing tradeoff curves for various SNR's are plotted in Figs. 4 and 5, for OSTBC and SM-HE, respectively. It can be seen from the tradeoff curves that the available diversity is significantly lower than the high-SNR theoretical limit for practical values of SNR. For instance, the Alamouti code provides a maximum diversity gain of only around 2.5 at SNR = 10 dB, instead of $d_{\max} = 4$. From Fig. 5, the upper and lower bounds for $d_{\text{SM-HE}}$ coincide at the endpoints $r = 0$ and $r = M_T$. Furthermore, the bounds converge as the SNR increases.

The convergence of the tradeoff curves to the asymptote as $\text{SNR} \to \infty$ is nonuniform in that the convergence is faster for higher multiplexing gains. In fact, even for $\text{SNR} = 50$ dB, neither OSTBC nor SM-HE reach a diversity gain of $d_{\max}$. For system design purposes, it is useful to study the maximum

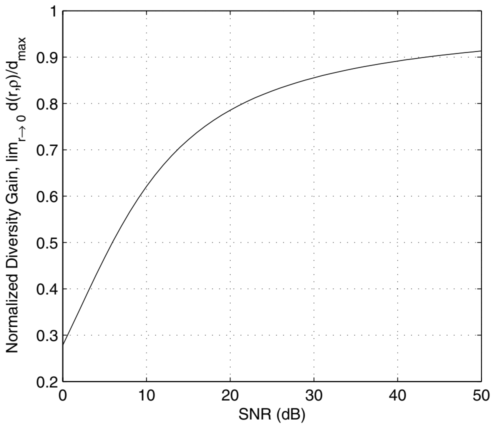


Fig. 6. Fraction of maximum achievable diversity gain as a function of SNR.

achievable diversity gain at finite SNR (i.e., the diversity gain for $r \to 0$). From (7), (18) and (19), the limit as $r \to 0$ of the diversity gain as a function of SNR is given by

$$\lim_{r\to 0} d(r,\rho) = d_{\max}\left[1 - \frac{\rho}{(1+\rho)\ln(1+\rho)}\right] \tag{23}$$

where the corresponding $d_{\max}$ is used for OSTBC and SM-HE. The fraction of the maximum achievable diversity gain is plotted as a function of SNR in Fig. 6. Note that for SNR's in the range of 10 dB – 20 dB, the achievable diversity is only around 60% – 80% of the asymptotic value $d_{\max}$. This insight is useful in predicting system performance at the operating SNR.

## V. Conclusion

A new framework is presented in this paper to characterize the diversity-multiplexing tradeoff of space-time codes at finite SNR. Non-asymptotic multiplexing and diversity gains are defined to characterize the achievable performance of space-time systems at operational SNR's. The diversity-multiplexing tradeoff analysis for OSTBC and SM-HE shows that achievable diversity gains are significantly lower at operational SNR's than the conventional asymptotic values for $\mathrm{SNR} \to \infty$. This finite-SNR analysis provides new insight to design space-time codes optimized for realistic operating and channel conditions.

## Appendix
## Proof of (14)

From the cdf's of the random variables $X$ and $\lambda_{\min}(\mathbf{H}^*\mathbf{H})$,

$$\begin{aligned}F_X(\lambda) - F_{\lambda_{\min}}(\lambda) &= \frac{\gamma(M_R - M_T + 1, M_T\lambda)}{(M_R - M_T)!} - \\ &\alpha_{M_R,M_T} \sum_{k=0}^{\nu} \frac{a_k \gamma(M_R - M_T + k + 1, M_T\lambda)}{M_T^{M_R - M_T + k + 1}} \\ &= \alpha_{M_R,M_T}\left\{\frac{\gamma(M_R - M_T + 1, M_T\lambda)}{\alpha_{M_R,M_T}(M_R - M_T)!} - \right. \\ &\left. \sum_{k=0}^{\nu} \frac{a_k \gamma(M_R - M_T + k + 1, M_T\lambda)}{M_T^{M_R - M_T + k + 1}}\right\} \\ &= \alpha_{M_R,M_T}\left\{\sum_{k=0}^{\nu} \frac{a_k}{M_T^{M_R - M_T + k + 1}}\right. \\ &\cdot \left[\frac{(M_R - M_T + k)!}{(M_R - M_T)!}\gamma(M_R - M_T + 1, M_T\lambda)\right. \\ &\left.\left. - \gamma(M_R - M_T + k + 1, M_T\lambda)\right]\right\}. \end{aligned} \tag{24}$$

Now, from the fact that for integer $m$,

$$\gamma(m,x) = (m-1)!\left(1 - e^{-x}\sum_{n=0}^{m-1}\frac{x^n}{n!}\right) \tag{25}$$

we have

$$\begin{aligned}&F_X(\lambda) - F_{\lambda_{\min}}(\lambda) = \alpha_{M_R,M_T}\sum_{k=1}^{\nu}\frac{a_k}{M_T^{M_R - M_T + k + 1}} \\ &\cdot\left[(M_R - M_T + k)!\left(1 - e^{-M_T\lambda}\sum_{n=0}^{M_R - M_T}\frac{(M_T\lambda)^n}{n!}\right)\right. \\ &\left. - (M_R - M_T + k)!\left(1 - e^{-M_T\lambda}\sum_{n=0}^{M_R - M_T + k}\frac{(M_T\lambda)^n}{n!}\right)\right] \\ &= \alpha_{M_R,M_T}\sum_{k=1}^{\nu}\frac{a_k e^{-M_T\lambda}(M_R - M_T + k)!}{M_T^{M_R - M_T + k + 1}} \\ &\cdot\sum_{n=M_R - M_T + 1}^{M_R - M_T + k}\frac{(M_T\lambda)^n}{n!} \\ &\geq 0 \end{aligned} \tag{26}$$

since $a_k > 0$ and $\lambda \geq 0$.

## References


[1] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1073–1096, May 2003.

[2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[3] H. El Gamal, "On the robustness of space-time coding," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 2417–2428, Oct. 2002.

[4] S. Siwamogsatham and M. P. Fitz, "Robust space-time codes for correlated Rayleigh fading channels," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 2408–2416, Oct. 2002.

[5] B. Clerckx, L. Vandendope, D. Vanhoenacker-Janvier and A. J. Paulraj, "On the "high SNR" assumption in space-time codes designs," in *Proc. IEEE ICC 2004*, Paris, France, June 2004.

[6] M. Vajapeyam, J. Geng and U. Mitra, "Low SNR design of space-time block codes based on union bound and indecomposable error patterns," in *Proc. IEEE ICC 2004*, Paris, France, June 2004.

[7] A. Paulraj, R. Nabar and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge Univ. Press, 2003.

[8] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select Areas in Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[9] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.

[10] R. Narasimhan, "Spatial multiplexing with transmit antenna and constellation selection for correlated MIMO fading channels," *IEEE Trans. on Signal Proc.*, vol. 51, pp. 2829–2838, Nov. 2003.

[11] A. Edelman, *Eigenvalues and Condition Numbers of Random Matrices*, Ph.D. Thesis, Massachusetts Institute of Technology, 1989.