Selection of Transmit Antennas, Constellations and Powers for Correlated MIMO Multiple Access Channels

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***Abstract*— A novel algorithm is presented to select the transmit antennas, constellations and distribution of total power of the users in a correlated multiple-input multiple-output (MIMO) multiple access channel. Given a target probability of symbol error, the spectral efficiency for each user and estimates of the channel correlation matrices, the transmit antennas, constellations and user power distribution are determined such that the required signal-to-noise ratio (SNR) is minimized for linear and Vertical Bell Laboratories Layered Space-Time (V-BLAST) receivers. This selection algorithm results in significant performance gains not only for correlated channels but also for independently fading channels.**

I. Introduction

The high capacities of multiple-input multiple-output (MIMO) wireless systems [1], [2] have inspired significant research over the past few years. Recently, the capacity results for single-user links have been extended to MIMO multiple access channels where the users and the base station have multiple antennas [1], [3]. The promise of high data rates in a multiuser environment has motivated research into efficient algorithms for signaling and detection.

If all users have knowledge of the complete multiple access channel, precoding techniques can be applied. In [4], transmit precoders for the users are determined iteratively to minimize the system-wide mean squared error. The signal-to-interference-plus-noise-ratio (SINR) is another criterion used to design precoders for MIMO multiple access channels. Codeword optimization for code-division multiple access (CDMA) systems based on maximizing SINR is presented in [5]. An algorithm for SINR balancing (i.e., ensuring the SINR of each link be greater than a threshold) using instantaneous channel estimates is described in [6]. For rapidly fading channels, an approximate algorithm is proposed in [6] by adding a fading margin to the SINR threshold.

It is desirable to derive signaling techniques that are optimized for fast fading by not requiring tracking of the fading MIMO channel. As an alternative to tracking the fading channel, it may be feasible to use channel statistics due to their slow variation. Optimal precoding using channel statistics may require different powers across the transmit antennas of a given user. In many cases, it is preferable to allocate equal power to the transmit antennas of a user, for example to avoid needing precise gain and phase calibration of the radio frequency (RF) chains. To this end, a transmit antenna and constellation selection algorithm with uniform power allocation has recently been proposed for single-user spatial multiplexing in correlated MIMO channels [7]. It is shown that the performance can be enhanced significantly by uniform power allocation across a *subset* of all transmit antennas.

In this paper, an algorithm is presented for selecting the transmit antennas, constellations and distribution of total power of the users in a correlated MIMO multiple access channel. Given a target probability of symbol error and the spectral efficiency of each user, the selection algorithm minimizes the required signal-to-noise ratio (SNR) for linear and Vertical Bell Laboratories Layered Space-Time (V-BLAST) receivers using estimates of the channel correlation matrices. Several previous MIMO techniques (e.g., [8], [9], [10]) have used knowledge of correlation matrices due to their slowly varying nature. The slow variation allows for a more accurate estimation of the correlation matrices than the instantaneous fading channel realization. Furthermore, a low-rate feedback broadcast channel is sufficient to provide the users with the selection results. In addition to not requiring instantaneous channel knowledge, a key advantage of this algorithm is that for a particular user, a uniform constellation and power allocation across a subset of the available antennas results in a simple selection algorithm and a reduced signaling overhead. While uncoded transmission is considered in this paper, the selection algorithm can be easily extended to include channel coding. The performance of the selection algorithm for MIMO multiple access channels is seen to be superior to a system in which all available transmit antennas are used.

The paper is organized as follows. In Section II, a model for a correlated MIMO multiple access channel is described. The algorithm to select the transmit antennas, constellations and powers for minimization of the required received SNR is presented in Section III. In Section IV, error rate results are provided for MIMO multiple access channels with two users to demonstrate the performance of the selection algorithm. Conclusions are given in Section V.

II. System Model

A narrowband MIMO multiple access channel is considered with N users. For $n = 1, \ldots, N$, User n has K_n antennas and transmits power P_n to a base station receiver that has K_R antennas. Multiuser spatial multiplexing is used with

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$K_R \geq \sum_{n=1}^{N} K_n$. As a result of antenna selection, $M_n \leq K_n$ antennas are used for transmission from User n; each antenna transmits an independent substream of data with a power of (P_n/M_n). Note that the transmit power per antenna for User n is allowed to vary depending on the number of active antennas M_n. Let $\mathbf{x}_n = \sqrt{P_n/M_n}\,\mathbf{s}_n$ denote the transmitted vector from User n, where $\mathbf{s}_n = [s_{n,1} \ \cdots \ s_{n,M_n}]^T$ and $s_{n,i}$ are symbols drawn from a unit-energy constellation. Further, let $\mathbf{H}_n$ denote the $K_R \times M_n$ MIMO channel matrix from User n (after antenna selection) to the base station receiver and $\mathbf{y} = [y_1 \ \cdots \ y_{K_R}]^T$ denote the received vector. Defining

$$\mathbf{D} = \begin{bmatrix} \sqrt{P_1/M_1}\,\mathbf{I}_{M_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sqrt{P_N/M_N}\,\mathbf{I}_{M_N} \end{bmatrix} \quad (1)$$

where $\mathbf{I}_{M_n}$ denotes the $M_n \times M_n$ identity matrix, we obtain the system equation for the MIMO multiple access channel

$$\mathbf{y} = \mathbf{HDs} + \mathbf{w} \quad (2)$$

where $\mathbf{H} = [\mathbf{H}_1 \ \cdots \ \mathbf{H}_N]$, $\mathbf{s} = [\mathbf{s}_1^T \ \cdots \ \mathbf{s}_N^T]^T$ and $\mathbf{w}$ is a complex additive white Gaussian noise (AWGN) vector with covariance $N_0\mathbf{I}_{K_R}$. For all users, the total number of active transmit antennas (after selection) is $M_{\text{tot}} = \sum_{n=1}^{N} M_n$.

A model for the $K_R \times M_{\text{tot}}$ matrix $\mathbf{H}$ that takes into account antenna correlation is given by [9], [11], [12]

$$\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{H}_w\mathbf{R}_T^{1/2} \quad (3)$$

where $\mathbf{R}_T$ is the $M_{\text{tot}} \times M_{\text{tot}}$ transmit correlation matrix (among the active transmit antennas of all users), $\mathbf{R}_R$ is the $K_R \times K_R$ receive correlation matrix, and $\mathbf{H}_w$ is a $K_R \times M_{\text{tot}}$ matrix whose elements are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Similar to (3), letting $\mathbf{H}_n = \mathbf{R}_{R,n}^{1/2}\mathbf{H}_{w,n}\mathbf{R}_{T,n}^{1/2}$, one can show that

$$\mathbf{R}_T = \begin{bmatrix} \mathbf{R}_{T,1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{R}_{T,N} \end{bmatrix} \quad (4)$$

$$\mathbf{R}_R = \sum_{n=1}^{N} \left(\frac{P_n}{P_{\text{tot}}}\right)\mathbf{R}_{R,n} \quad (5)$$

where $P_{\text{tot}} = \sum_{n=1}^{N} P_n$ and $\mathbf{H}_{w,n}$ is a $K_R \times M_n$ i.i.d. complex Gaussian matrix. The $M_n \times M_n$ matrix $\mathbf{R}_{T,n}$ and the $K_R \times K_R$ matrix $\mathbf{R}_{R,n}$ are correlation matrices determined from the geometry of the active antennas, the angles of departure and arrival, the angle spreads and the power azimuth spectra between User n and the base station receiver.

Based on knowledge of $\mathbf{R}_T$ and $\mathbf{R}_R$ for all combinations of active antennas, the base station selects the desired combination of active antennas and transmit powers and conveys the selection results to the users by means of a low-rate feedback broadcast channel. We consider linear and V-BLAST processing at the base station receiver, which has estimates of $\mathbf{H}$. For linear processing, the receiver estimates the transmitted symbols s by forming $\hat{\mathbf{s}} = \mathbf{Gy}$, where $\mathbf{G}$ is the $M_{\text{tot}} \times K_R$ receive matrix. For the zero-forcing (ZF) receiver, $\mathbf{G} = \mathbf{D}^{-1}(\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*$, and for the minimum mean squared error (MMSE) receiver, $\mathbf{G} = \mathbf{D}^{-1}(\mathbf{H}^*\mathbf{H} + N_0\mathbf{D}^{-2})^{-1}\mathbf{H}^*$. The V-BLAST receiver uses ordered successive interference cancellation and nulling based on either the ZF (for ZF V-BLAST) or MMSE (for MMSE V-BLAST) criterion [13].

III. Selection of Transmit Antennas, Constellations and Powers

An algorithm is now presented to select the transmit antennas, constellations and total power distribution of the users in a correlated MIMO multiple access channel. The goal of the selection algorithm is to minimize the average required SNR per receive antenna (P_{tot}/N_0) given the target probability of error and the spectral efficiency of each user. Let $\text{Pr}_{e,n}$ and $b_{T,n}$ denote the average probability of symbol error and the spectral efficiency (in bps/Hz) of User n, respectively. We assume that the set of spectral efficiencies $\{b_{T,1}, \ldots, b_{T,N}\}$ lies within the achievable rate region of the MIMO multiple access channel (for a given power constraint and channel coding). Now, let $\text{SNR}_{n,l}$ denote the post-processing SNR corresponding to the l-th substream of User n. Using the nearest neighbor union bound [14] for uncoded transmission, we have

$$\text{Pr}_{e,n} \leq \frac{N_{e,n}}{M_n}\sum_{l=1}^{M_n} E\left[Q\left(\sqrt{\frac{c_n\text{SNR}_{n,l}}{2^{b_{T,n}/M_n}-1}}\right)\right] \quad (6)$$

where $N_{e,n}$ and c_n are constants that depend on the constellation of User n, $E[\cdot]$ denotes expectation, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}\,dt$. The constants $N_{e,n}$ and c_n for specific constellations are given in Section IV. Note that given $b_{T,n}$ and M_n, the transmit constellation for User n is uniquely determined and has $2^{b_{T,n}/M_n}$ points. Using the Chernoff bound $Q(x) \leq \exp(-x^2/2)$, we have

$$\begin{aligned} \text{Pr}_{e,n} &\leq \frac{N_{e,n}}{M_n}\sum_{l=1}^{M_n} E\left[\exp\left(-\frac{c_n\text{SNR}_{n,l}}{2(2^{b_{T,n}/M_n}-1)}\right)\right] \\ &\leq N_{e,n}E\left[\exp\left(-\frac{c_n\text{SNR}_{n,\min}}{2(2^{b_{T,n}/M_n}-1)}\right)\right] \end{aligned} \quad (7)$$

where $\text{SNR}_{n,\min} = \min_l \text{SNR}_{n,l}$.

We now have the following lower bound for $\text{SNR}_{n,\min}$.

Theorem 1: For a MIMO multiple access channel with ZF, MMSE, ZF V-BLAST, or MMSE V-BLAST reception,

$$\text{SNR}_{n,\min} \geq \frac{P_n}{M_nN_0}\lambda_{\min}(\mathbf{H}^*\mathbf{H}) \quad (8)$$

where $\lambda_{\min}(\mathbf{A})$ is the minimum eigenvalue of a square matrix $\mathbf{A}$ and the superscript * denotes conjugate transpose.

Proof: The proof is a straightforward extension of the analysis in [7] to a MIMO multiple access channel. ■

From Appendix II of [7], we have

$$\lambda_{\min}(\mathbf{H}^*\mathbf{H}) \geq \lambda_{\min}(\mathbf{R}_T)\lambda_{\min}(\mathbf{R}_R)\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w). \quad (9)$$

Substituting (8) and (9) into (7), we have

$$\text{Pr}_{e,n} \leq N_{e,n}E\left[e^{-\frac{c_nP_n\lambda_{\min}(\mathbf{R}_T)\lambda_{\min}(\mathbf{R}_R)\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)}{2M_nN_0(2^{b_{T,n}/M_n}-1)}}\right]. \quad (10)$$

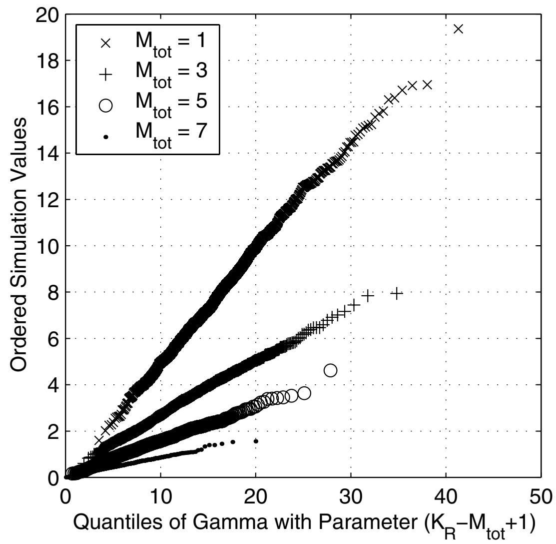

Fig. 1. Gamma probability plots of $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ with $K_R = 8$.

In order to compute the expectation in (10), we need the distribution of the minimum eigenvalue of the complex Wishart matrix $\mathbf{H}_w^*\mathbf{H}_w$. From [15], the probability density function (pdf) of $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ is given by

$$f(\lambda) = \alpha_{K_R,M_{\text{tot}}}\lambda^{K_R-M_{\text{tot}}}\, e^{-M_{\text{tot}}\lambda} F_{K_R,M_{\text{tot}}}(\lambda),\ \lambda \geq 0 \quad (11)$$

where $\alpha_{K_R,M_{\text{tot}}}$ is a normalization constant and $F_{K_R,M_{\text{tot}}}(\lambda)$ is a polynomial of degree $\nu = (K_R - M_{\text{tot}})(M_{\text{tot}} - 1)$ with positive coefficients. We note that for $M_{\text{tot}} = 1$ or $M_{\text{tot}} = K_R$, $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ is proportional to a gamma random variable with parameter K_R or 1, respectively. Since the diversity order using a linear or V-BLAST receiver is $(K_R - M_{\text{tot}} + 1)$, the distribution of $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ for $1 < M_{\text{tot}} < K_R$ can be approximated by a scaled gamma random variable with parameter $(K_R - M_{\text{tot}} + 1)$. In Fig. 1, gamma probability plots are given for 1000 simulated values of $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ for $K_R = 8$ and various M_{tot}. From the figure, we see that the linearity of the probability plots for $M_{\text{tot}} > 1$ is similar to the linearity for $M_{\text{tot}} = 1$ (which corresponds to an exact gamma random variable). The probability plot correlation coefficients (PPCC's) are given in Table I. Since the PPCC's are close to unity, the gamma distribution is a good approximation to the distribution of $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ for the values of K_R and M_{tot} of interest here. Motivated by this observation, we now obtain an upper bound for (10) using the following result.

Theorem 2: Define the random variable X by $X = a\Gamma$, where $a > 0$ and Γ is a gamma random variable with parameter $(K_R - M_{\text{tot}} + 1)$. Let $\mathcal{L}_X(s) = E[e^{-sX}]$ and $\mathcal{L}_\lambda(s) = E[e^{-s\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)}]$. If $a \leq 1/M_{\text{tot}}$,

$$\mathcal{L}_\lambda(s) \leq \mathcal{L}_X(s),\ s \geq 0. \quad (12)$$

TABLE I

PROBABILITY PLOT CORRELATION COEFFICIENTS WITH $K_R = 8$.

M_{tot}							
1	2	3	4	5	6	7	8
0.9991	0.9988	0.9987	0.9984	0.9989	0.9991	0.9989	0.9974

Proof: The proof is given in the Appendix. ■

We note that Theorem 2 holds independently of whether $\lambda_{\min}(\mathbf{H}_w^*\mathbf{H}_w)$ is approximated by a scaled gamma random variable. Using (12) with $a = 1/M_{\text{tot}}$ and the fact that $\mathcal{L}_X(s) = (1 + as)^{-(K_R - M_{\text{tot}}+1)}$ for $s \geq 0$, we have

$$\text{Pr}_{e,n} \leq \frac{N_{e,n}}{(1 + \beta_n\rho_n)^{K_R - M_{\text{tot}}+1}} \quad (13)$$

where

$$\beta_n = \frac{c_n\lambda_{\min}(\mathbf{R}_T)\lambda_{\min}(\mathbf{R}_R)}{2M_nM_{\text{tot}}(2^{b_{T,n}/M_n} - 1)} \quad (14)$$

and $\rho_n = \frac{P_n}{N_0}$ is the transmit SNR for User n.

For a target probability of symbol error of $\text{Pr}_{e,\text{target}}$, we obtain a conservative value for the required transmit SNR for User n by equating $\text{Pr}_{e,\text{target}}$ with the right hand side of (13) and solving for ρ_n:

$$\rho_{n,\text{target}} = \frac{1}{\beta_n}\left[\left(\frac{N_{e,n}}{\text{Pr}_{e,\text{target}}}\right)^{\frac{1}{K_R - M_{\text{tot}}+1}} - 1\right]. \quad (15)$$

With the average SNR per receive antenna given by $\text{SNR} = P_{\text{tot}}/N_0$, the required received SNR is

$$\text{SNR}_{\text{required}} = \sum_{n=1}^{N}\rho_{n,\text{target}} \quad (16)$$

and the corresponding transmit power of User n is

$$P_n = \left(\frac{\rho_{n,\text{target}}}{\text{SNR}_{\text{required}}}\right)P_{\text{tot}}. \quad (17)$$

We select the transmit powers, antennas and constellations such that $\text{SNR}_{\text{required}}$ is minimized. For each valid set $\{M_1, \ldots, M_N\}$ of the number of active transmit antennas, the active antenna elements are chosen to maximize β_n, $n = 1, \ldots, N$, given the channel correlation matrices. The values of $\rho_{n,\text{target}}$ are then computed according to (15). From $\rho_{n,\text{target}}$ for each user, the required received SNR is computed using (16). The set $\{M_1, \ldots, M_N\}$ that yields the smallest required received SNR is chosen for transmission with User n's fraction of the total transmit power (P_n/P_{tot}) determined from (17). We note that the feasible set for this optimization problem is $\{(M_1, \ldots, M_N, P_1, \ldots, P_N) : M_n \in \{1, \ldots, K_n\},\ P_n \in [0, P_{\text{tot}}],\ n = 1, \ldots, N,\ \sum_{n=1}^{N} P_n = P_{\text{tot}}\}$. Once the optimum values of M_n and P_n for $n = 1, \ldots, N$ are obtained, the corresponding constellation for User n is chosen such that a spectral efficiency of $(b_{T,n}/M_n)$ bps/Hz is transmitted from each of the M_n active transmit antennas.

IV. Simulation Results

We now evaluate the performance of the selection algorithm for correlated MIMO multiple access channels. We consider $N = 2$ users with $K_1 = K_2 = 4$ transmit antennas and a base station receiver with $K_R = 8$ antennas. The target probability of symbol error is $\text{Pr}_{e,\text{target}} = 10^{-3}$, and the user spectral efficiencies are $b_{T,1} = 12$ bps/Hz and $b_{T,2} = 8$ bps/Hz. Uniform linear arrays with half-wavelength antenna separation are assumed for the transmit and receive arrays. Two scenarios of transmit correlation are considered: (1)

TABLE II

SELECTION ALGORITHM RESULTS.

		M_n	Constellation	P_n/P_{tot}
Scenario 1	User 1	3	16-QAM	0.6
	User 2	2	16-QAM	0.4
Scenario 2	User 1	2	64-QAM	0.3273
	User 2	1	256-QAM	0.6727

independent fading with $\mathbf{R}_T = \mathbf{I}$ and (2) correlated fading with two transmit scattering clusters for User 1 and one transmit scattering cluster for User 2. In Scenario 2, a uniform power azimuth spectrum is used with angles of departure for User 1's clusters of 30^o and 60^o from broadside and angle spreads of 30^o and 20^o, respectively; for User 2's cluster, the angle of departure is 120^o with angle spread of 20^o. In both scenarios, the receive correlation matrix is $\mathbf{R}_R = \mathbf{I}$. Uncoded square quadrature amplitude modulation (QAM) and eight-point phase-shift keying (8-PSK) constellations with a Gray bit mapping are used to transmit a spectral efficiency of $(b_{T,n}/M_n)$ bps/Hz from each active antenna of User n. For these constellations, the constants $N_{e,n}$ and c_n in (6) are given by [14]

$$N_{e,n} = \begin{cases} 4(1 - 2^{-b_{T,n}/(2M_n)}), & \text{square QAM} \\ 2, & \text{8-PSK} \end{cases} \quad (18)$$

$$c_n = \begin{cases} 3, & \text{square QAM} \\ 2\sin^2 \frac{\pi}{8}, & \text{8-PSK.} \end{cases} \quad (19)$$

The results of the antenna, constellation and power selection are given in Table II. The bit error rate (BER) performance of the selection algorithm is evaluated for linear MMSE and MMSE V-BLAST receivers. For Scenario 1, Figs. 2 and 3 are plots of the BER versus average SNR per receive antenna for Users 1 and 2, respectively. The corresponding BER plots for Scenario 2 are given in Figs. 4 and 5. Since the spectral efficiency of User 1 is greater than that of User 2, $M_1 > M_2$ for both scenarios. As expected, the presence of antenna correlation in Scenario 2 results in fewer active antennas and higher order constellations than in Scenario 1. Furthermore, the transmit power distribution is selected such that the BER's of both users are similar. This distribution results in the minimum $\text{SNR}_{\text{required}}$. From Figs. 2 and 3, we see that the selection algorithm provides significant gains over using all available antennas even for i.i.d. fading channels. The gains are much larger for correlated fading channels, as seen in Figs. 4 and 5. We also note that as expected, the gain of MMSE V-BLAST over linear MMSE detection is larger for larger M_n.

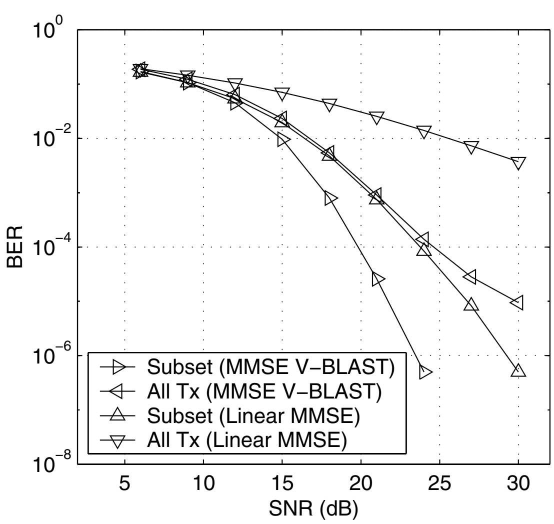

Fig. 2. BER versus SNR for User 1 and Scenario 1 (i.i.d. fading) using all transmit antennas (labeled by "All Tx") and the selected subset of transmit antennas (labeled by "Subset"). All $K_R = 8$ receive antennas at the base station are active.

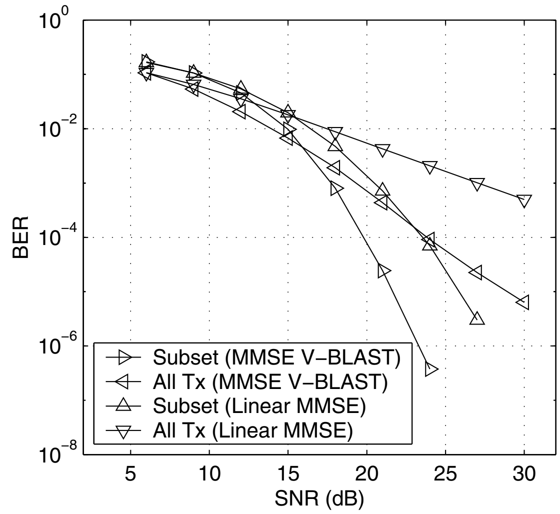

Fig. 3. BER versus SNR for User 2 and Scenario 1 (i.i.d. fading).

V. CONCLUSION

A new algorithm is presented to select the transmit antennas, constellations and powers of users in correlated MIMO multiple access channels. Given the channel correlation matrices, the desired user spectral efficiencies and the target symbol

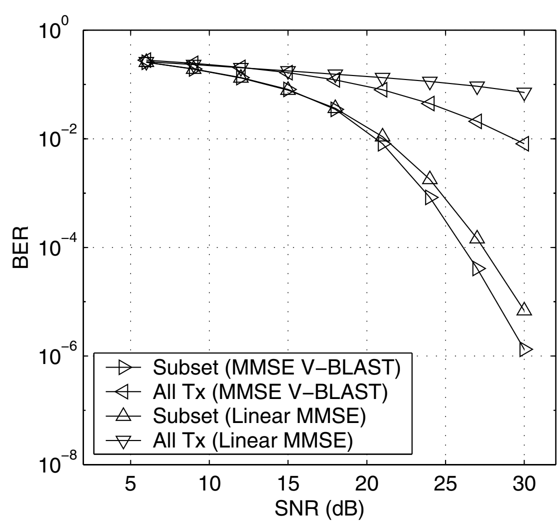

Fig. 4. BER versus SNR for User 1 and Scenario 2 (correlated fading).

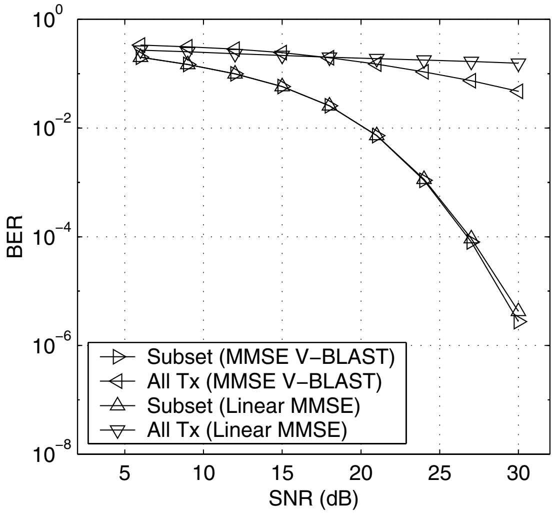

Fig. 5. BER versus SNR for User 2 and Scenario 2 (correlated fading).

error rate, the selection algorithm minimizes the required SNR for linear and V-BLAST receivers. Simulation results demonstrate significant performance gains using the selection algorithm for both correlated and independently fading MIMO multiple access channels.

APPENDIX

Let the polynomial $F_{K_R,M_{\rm tot}}(\lambda)$ be defined by $F_{K_R,M_{\rm tot}}(\lambda) = \sum_{k=0}^{\nu}\gamma_k\lambda^k$ where $\gamma_k > 0$. From (11), we have

$$f(\lambda) = \alpha_{K_R,M_{\rm tot}} e^{-M_{\rm tot}\lambda}\sum_{k=0}^{\nu}\gamma_k\lambda^{K_R-M_{\rm tot}+k} \qquad (20)$$

and

$$\begin{aligned}\mathcal{L}_\lambda(s) &= \alpha_{K_R,M_{\rm tot}}\sum_{k=0}^{\nu}\gamma_k\int_0^\infty \lambda^{K_R-M_{\rm tot}+k}\, e^{-(s+M_{\rm tot})\lambda}\, d\lambda \\ &= \alpha_{K_R,M_{\rm tot}}\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(s+M_{\rm tot})^{K_R-M_{\rm tot}+k+1}},\;\; s\geq 0. \qquad (21)\end{aligned}$$

Using the fact that $\mathcal{L}_\lambda(0) = 1$, we determine the normalization constant $\alpha_{K_R,M_{\rm tot}}$:

$$\alpha_{K_R,M_{\rm tot}} = \left[\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right]^{-1}. \qquad (22)$$

Thus,

$$\begin{aligned}&\mathcal{L}_X(s) - \mathcal{L}_\lambda(s) \\ &= (1+as)^{-(K_R-M_{\rm tot}+1)} - \left[\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right]^{-1}\times \\ &\quad \sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(s+M_{\rm tot})^{K_R-M_{\rm tot}+k+1}} \\ &= \left[(1+as)^{K_R-M_{\rm tot}+1}\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right]^{-1}\times \\ &\quad \left[\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}} - \right. \\ &\quad \left.\sum_{k=0}^{\nu}\gamma_k\frac{(1+as)^{K_R-M_{\rm tot}+1}(K_R-M_{\rm tot}+k)!}{(s+M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right] \\ &= \left[(1+as)^{K_R-M_{\rm tot}+1}\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right]^{-1}\times \\ &\quad \left[\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\times\right. \\ &\quad \left.\left(1-\frac{(1+as)^{K_R-M_{\rm tot}+1}(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}{(s+M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right)\right] \\ &= \left[(1+as)^{K_R-M_{\rm tot}+1}\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\right]^{-1}\times \\ &\quad \left[\sum_{k=0}^{\nu}\gamma_k\frac{(K_R-M_{\rm tot}+k)!}{(M_{\rm tot})^{K_R-M_{\rm tot}+k+1}}\times\right. \\ &\quad \left.\left(1-\left(\frac{1+as}{1+\frac{s}{M_{\rm tot}}}\right)^{K_R-M_{\rm tot}+1}\frac{1}{(1+\frac{s}{M_{\rm tot}})^k}\right)\right] \\ &\geq 0 \qquad (23)\end{aligned}$$

since $a \leq 1/M_{\rm tot}$ and $s \geq 0$.

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