Performance of Diversity Schemes for OFDM Systems with Frequency Offset, Phase Noise and Channel Estimation Errors

Ravi Narasimhan
Marvell Semiconductor, Inc.
700 First Avenue, Sunnyvale, CA 94089, USA

***Abstract*— We provide expressions for the bit error rate of various transmit and receive diversity schemes for Orthogonal Frequency-Division Multiplexing (OFDM) systems in the presence of frequency offset, phase noise and channel estimation errors. The derivations are also applicable for a general multiplicative distortion of the received signal. We compare two transmit diversity schemes (space-time and space-frequency diversity) with maximal ratio combining (MRC) receive diversity for frequency-selective Rayleigh fading channels. Our results show that with perfect channel estimates, practical values of the phase noise do not significantly degrade the performance of the various diversity methods for BPSK modulation. In contrast, the transmit diversity schemes for OFDM are much more sensitive to channel estimation errors than MRC receive diversity.**

I. Introduction

Antenna diversity techniques are well known as an effective means of combating multipath fading [1]. Conventional maximum ratio combining (MRC) requires multiple receiver chains that may not be feasible for low-power, low-cost portable devices. Recently, transmit diversity and space-time coding [2], [3] have been proposed as a means to combat multipath fading without requiring multiple receiver chains. Under quasi-static conditions and perfect channel state information at the receiver, the simple transmit diversity method described in [2] with two transmitters and one receiver has the same performance as a system using MRC with one transmitter and two receivers.

The orthogonal transmit diversity techniques described in [2], [3] are directly applicable to narrowband (flat) fading. Orthogonal Frequency-Division Multiplexing (OFDM) is a popular technique for combating frequency-selective fading by dividing the transmission bandwidth into a set of narrowband subchannels [4], [5]. With a cyclic prefix, equalization reduces to compensating for the channel transfer function at each subcarrier [6]. Two transmit diversity schemes for OFDM, space-time OFDM (ST-OFDM) and space-frequency OFDM (SF-OFDM), are described in [7], [8] where the performances of the two methods are evaluated assuming perfect synchronization and channel state information at the receiver.

A drawback of OFDM systems is the sensitivity to carrier frequency offsets and phase noise [9], [10], [11]. Frequency offsets and phase noise result in the loss of orthogonality among the subcarriers and cause intercarrier interference (ICI). Accurate channel estimation is also important for systems using coherent detection. The results given in [12] show that for single carrier transmission in flat Rayleigh fading, the simple space-time transmit diversity method of [2] is more sensitive to channel estimation errors than MRC receive diversity.

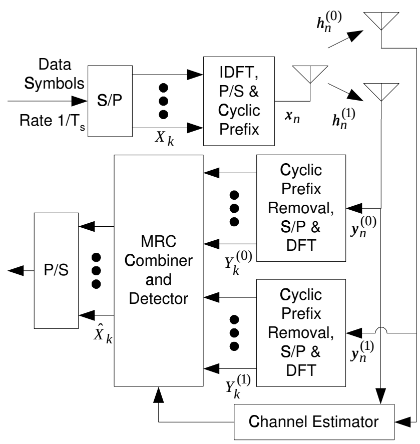

Fig. 1. OFDM system with two-branch MRC receive diversity.

In this paper, we analyze the performance of ST-OFDM, SF-OFDM, and MRC receive diversity in the presence of frequency offset, phase noise and channel estimation errors. Section II describes the system models for the various diversity methods. Expressions for the bit error rate (BER) of the diversity techniques are given in Section III with frequency offset and phase noise present and perfect channel state information at the receiver. In Section IV, BER expressions are given with channel estimation errors present and perfect synchronization (no frequency offset and phase noise). Section V provides performance results for frequency-selective Rayleigh fading channels. Conclusions are given in Section VI.

II. System Models

A. MRC

A block diagram of an OFDM system with two-branch MRC receive diversity is given in Fig. 1. The data symbols enter the serial-to-parallel (S/P) converter at a rate of $1/T_s$, to form a block of N data symbols, $\{X_k\}_{k=0,\ldots,N-1}$, with duration NT_s. Throughout this paper, BPSK modulation ($X_k \in \{-1,1\}$) and an even block length are assumed. The block of data symbols is modulated using an N-point inverse discrete Fourier transform (IDFT), to form a block

0-7803-7400-2/02/$17.00 © 2002 IEEE

of time domain samples, $\{x_n\}_{n=0,\ldots,N-1}$. After parallel-to-serial (P/S) conversion, a cyclic prefix of G samples is inserted to form the transmitted OFDM symbol of $N+G$ samples: $\{x_{N-G},\ldots,x_{N-1},x_0,\ldots,x_{N-1}\}$. The OFDM symbol is transmitted through two independent frequency-selective fading channels whose impulse responses are denoted by $h_n^{(0)}$ and $h_n^{(1)}$ for receiving Antennas 0 and 1, respectively. We assume that the maximum length of the channel impulse responses is less than G samples so that interblock interference (IBI) is avoided and that the channels are quasi-static (channel impulse responses are constant for a few OFDM symbols). The received signal at Antenna i is $y_n^{(i)}$, $i \in \{0,1\}$. For each receiving antenna, the samples corresponding to the cyclic prefix are discarded and the remaining samples enter a serial-to-parallel converter to form a time-domain vector of the received OFDM symbol. The received OFDM symbol is demodulated using an N-point discrete Fourier transform (DFT) to form the frequency-domain vector $\{Y_k^{(i)}\}$. With perfect synchronization, the frequency-domain samples for each antenna are

$$Y_k^{(i)} = H_k^{(i)} X_k + V_k^{(i)}, k = 0,1,\ldots,N-1 \tag{1}$$

where $H_k^{(i)}$ and $V_k^{(i)}$, $i \in \{0,1\}$ denote the DFT's of the channel impulse response and additive white Gaussian noise (AWGN), respectively, at receiving Antenna i. The variance of the real and imaginary parts of $V_k^{(i)}$ is σ_V^2. For each subcarrier k, maximum ratio combining is achieved at the receiver by forming $\tilde{X}_{k,MRC}$, where

$$\tilde{X}_{k,MRC} = H_k^{(0)*} Y_k^{(0)} + H_k^{(1)*} Y_k^{(1)}. \tag{2}$$

The decision statistic for BPSK demodulation is $U_{k,MRC} = \Re\{\tilde{X}_{k,MRC}\}$, where $\Re\{\cdot\}$ denotes the real part; the estimate of the transmitted data is $\hat{X}_{k,MRC} = \mathrm{sgn}(U_{k,MRC})$.

B. ST-OFDM

A block diagram of the two-branch ST-OFDM system described in [7] is given in Fig. 2. Consider two OFDM symbols (Symbol 0 and Symbol 1) for transmission. For Symbol 0 and subcarrier k, $X_{k,0} = s_{k,0}$ and $X_{k,1} = -s_{k,1}$ are transmitted from Antenna 0 and Antenna 1, respectively; for Symbol 1 and subcarrier k, $X_{k,0} = s_{k,1}$ and $X_{k,1} = s_{k,0}$ are transmitted from Antenna 0 and Antenna 1, respectively. With perfect synchronization, the frequency-domain samples corresponding to Symbols 0 and 1 received at the single receiver antenna are given by $Y_{k,0}$ and $Y_{k,1}$, $k = 0,1,\ldots,N-1$, where

$$Y_{k,0} = H_k^{(0)} s_{k,0} - H_k^{(1)} s_{k,1} + V_{k,0} \tag{3}$$

$$Y_{k,1} = H_k^{(1)} s_{k,0} + H_k^{(0)} s_{k,1} + V_{k,1}, \tag{4}$$

$H_k^{(i)}, i \in \{0,1\}$ denotes the DFT of the channel impulse response from transmitting Antenna i to the receiver and $V_{k,0}$ and $V_{k,1}$

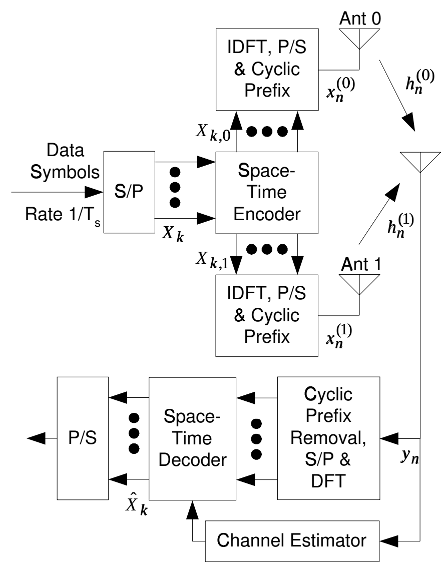

Fig. 2. Two-branch ST-OFDM system.

denote the DFT's of AWGN corresponding to received Symbols 0 and 1, respectively. For each subcarrier k, the receiver forms $\tilde{s}_{k,0}$ and $\tilde{s}_{k,1}$, where

$$\tilde{s}_{k,0} = H_k^{(0)*} Y_{k,0} + H_k^{(1)} Y_{k,1}^* \tag{5}$$

$$\tilde{s}_{k,1} = -H_k^{(1)*} Y_{k,0} + H_k^{(0)} Y_{k,1}^*. \tag{6}$$

The decision statistics for BPSK demodulation are $U_{k,0,ST} = \Re\{\tilde{s}_{k,0}\}$ and $U_{k,1,ST} = \Re\{\tilde{s}_{k,1}\}$; the estimates of the transmitted data are $\hat{s}_{k,0} = \mathrm{sgn}(U_{k,0,ST})$ and $\hat{s}_{k,1} = \mathrm{sgn}(U_{k,1,ST})$.

C. SF-OFDM

A block diagram of the two-branch SF-OFDM system described in [8] is similar to Fig. 2 except the space-time code is replaced by a space-frequency code as explained below. For each OFDM symbol, adjacent subcarriers k and $k+1$ ($k = 0,2,4,\ldots,N-2$) are used in the space-frequency code. For subcarrier k, $X_{k,0} = s_{k,0}$ and $X_{k,1} = -s_{k,1}$ are transmitted from Antenna 0 and Antenna 1, respectively; for subcarrier $k+1$, $X_{k+1,0} = s_{k,1}$ and $X_{k+1,1} = s_{k,0}$ are transmitted from Antenna 0 and Antenna 1, respectively. With perfect synchronization, the DFT outputs at the receiver for subcarriers k and $k+1$ are given by Y_k and Y_{k+1}, where

$$Y_k = H_k^{(0)} s_{k,0} - H_k^{(1)} s_{k,1} + V_k \tag{7}$$

$$Y_{k+1} = H_{k+1}^{(1)} s_{k,0} + H_{k+1}^{(0)} s_{k,1} + V_{k+1} \tag{8}$$

and V_k denotes the DFT of AWGN corresponding to subcarrier k. After conjugating (8), the DFT outputs for subcarriers k and $k+1$ can be written in the matrix notation

$$\begin{pmatrix} Y_k \\ Y_{k+1}^* \end{pmatrix} = \mathcal{H} \begin{pmatrix} s_{k,0} \\ s_{k,1} \end{pmatrix} + \begin{pmatrix} V_k \\ V_{k+1}^* \end{pmatrix} \tag{9}$$

where

$$\mathcal{H} = \begin{pmatrix} H_k^{(0)} & -H_k^{(1)} \\ H_{k+1}^{(1)*} & H_{k+1}^{(0)*} \end{pmatrix}. \quad (10)$$

We consider two methods of data detection. The first method is a zero-forcing (ZF) approach which involves inverting the matrix $\mathcal{H}$ and detecting $s_{k,0}$ and $s_{k,1}$ separately. This detection method will be denoted by SF-OFDM-ZF. In this method, the receiver forms $\tilde{s}_{k,0}$ and $\tilde{s}_{k,1}$ for $k \in \{0,2,\ldots,N-2\}$, where

$$\begin{pmatrix} \tilde{s}_{k,0} \\ \tilde{s}_{k,1} \end{pmatrix} = \mathcal{H}^{-1} \begin{pmatrix} Y_k \\ Y_{k+1}^* \end{pmatrix} \quad (11)$$

The decision statistics for BPSK demodulation are $U_{k,0,SF} = \Re\{\tilde{s}_{k,0}\}$ and $U_{k,1,SF} = \Re\{\tilde{s}_{k,1}\}$; the estimates of the transmitted data are $\hat{s}_{k,0} = \mathrm{sgn}(U_{k,0,SF})$ and $\hat{s}_{k,1} = \mathrm{sgn}(U_{k,1,SF})$.

Since $\mathcal{H}$ is not a scaled unitary matrix in general, correlation of the noise terms is introduced after matrix inversion. The ZF approach is not optimum in the presence of correlated noise. For this reason, a second detection method (denoted by SF-OFDM-ML) is considered based on the maximum likelihood (ML) criterion:

$$\begin{pmatrix} \hat{s}_{k,0} \\ \hat{s}_{k,1} \end{pmatrix} = \arg \min_{\substack{(s_{k,0},s_{k,1})\in \\ \{-1,1\}^2}} \left\| \begin{pmatrix} Y_k \\ Y_{k+1}^* \end{pmatrix} - \mathcal{H} \begin{pmatrix} s_{k,0} \\ s_{k,1} \end{pmatrix} \right\|^2. \quad (12)$$

III. Performance with Frequency Offset and Phase Noise

In this section, we present expressions for the BER of MRC, ST-OFDM and SF-OFDM-ZF with frequency offset and phase noise. Since a simple closed-form expression for the BER of SF-OFDM-ML is not available, we use computer simulations to evaluate the performance of this method. In this section, we assume perfect channel estimates are available at the receiver.

A. MRC

Let the carrier frequency offset be denoted by Δf, the discrete-time phase noise process by ϕ_n and the time-domain samples of the AWGN for Antenna i by $v_n^{(i)}$. In the presence of frequency offset and phase noise, the received signal $y_n^{(i)}$ at Antenna i can be expressed by $y_n^{(i)} = [x_n * h_n^{(i)}]\lambda_n^{(i)} + v_n^{(i)}$, where $\lambda_n^{(i)} = e^{j(2\pi n \Delta f T_s + \phi_n)}$ and $*$ represents convolution. If $\Lambda_k^{(i)}$ denotes the DFT of $\lambda_n^{(i)}$, the received sample for subcarrier k is

$$Y_k^{(i)} = \frac{1}{N} X_k H_k^{(i)} \Lambda_0^{(i)} + \frac{1}{N} \sum_{m=0, m\neq k}^{N-1} X_m H_m^{(i)} \Lambda_{k-m}^{(i)} + V_k^{(i)}. \quad (13)$$

We assume that $\Lambda_0^{(i)}$ (sometimes known as the common phase error) can be estimated perfectly using pilot symbols. Therefore, (2) is modified as

$$\tilde{X}_{k,MRC} = \Lambda_0^{(0)*} H_k^{(0)*} Y_k^{(0)} + \Lambda_0^{(1)*} H_k^{(1)*} Y_k^{(1)}. \quad (14)$$

Substituting (13) into (14) and taking the real part, we obtain the decision statistic

$$U_{k,MRC} = \frac{1}{N}\left(|\Lambda_0^{(0)} H_k^{(0)}|^2 + |\Lambda_0^{(1)} H_k^{(1)}|^2\right) X_k + \frac{1}{N} \sum_{\substack{m=0 \\ m\neq k}}^{N-1} \Re\{\Lambda_0^{(0)*}\Lambda_{k-m}^{(0)} H_k^{(0)*} H_m^{(0)} + \Lambda_0^{(1)*}\Lambda_{k-m}^{(1)} H_k^{(1)*} H_m^{(1)}\} X_m + \Re\{\Lambda_0^{(0)*} H_k^{(0)*} V_k^{(0)} + \Lambda_0^{(1)*} H_k^{(1)*} V_k^{(1)}\}. \quad (15)$$

Proceeding as in [12], we derive the BER conditioned on $\{H_m^{(i)}\}$ and $\{\Lambda_m^{(i)}\}$. The performance is then evaluated in Section V for frequency-selective Rayleigh fading channels and for a phase noise process with a Lorentzian power spectral density [9], [10], [11]. Since the data symbols X_m are independent and identically distributed (i.i.d.) for different subcarriers m, $U_{k,MRC}$ (conditioned on X_k, $\{H_m^{(i)}\}$ and $\{\Lambda_m^{(i)}\}$) approaches a Gaussian random variable for large N (by the central limit theorem). The mean and variance of $U_{k,MRC}$ can be calculated using the relations $E[X_m] = 0, E[X_m^2] = 1$, where $E[\cdot]$ denotes expectation. After simplification, we obtain

$$E[U_{k,MRC}] = a_{k,MRC} X_k \quad (16)$$

$$\sigma^2_{U_{k,MRC}} = \frac{1}{N^2} \sum_{m=0, m\neq k}^{N-1} \Re^2\{\Lambda_0^{(0)*}\Lambda_{k-m}^{(0)} H_k^{(0)*} H_m^{(0)} + \Lambda_0^{(1)*}\Lambda_{k-m}^{(1)} H_k^{(1)*} H_m^{(1)}\} + \left(|\Lambda_0^{(0)} H_k^{(0)}|^2 + |\Lambda_0^{(1)} H_k^{(1)}|^2\right)\sigma_V^2. \quad (17)$$

where $a_{k,MRC} = \frac{1}{N}\left(|\Lambda_0^{(0)} H_k^{(0)}|^2 + |\Lambda_0^{(1)} H_k^{(1)}|^2\right)$. The probability of error for subcarrier k is given by $P_{e,MRC|k} = Q(a_{k,MRC}/\sigma_{U_{k,MRC}})$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$. The BER conditioned on $\{H_m^{(i)}\}$ and $\{\Lambda_m^{(i)}\}$ is obtained by averaging over all subcarriers:

$$P_{e,MRC} = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\frac{a_{k,MRC}}{\sigma_{U_{k,MRC}}}\right). \quad (18)$$

B. ST-OFDM

Let $\Lambda_{k,0}$ and $\Lambda_{k,1}$ denote the DFT's of the frequency offset and phase noise term for OFDM Symbols 0 and 1, respectively. Let $X_{m,n}^{(i)}, (i,n) \in \{0,1\}^2$ denote the BPSK data symbol transmitted from Antenna i in Symbol n on subcarrier m. In the presence of frequency offset and phase noise, the samples for subcarrier k and Symbols 0 and 1 are

$$Y_{k,0} = \frac{1}{N}\left(\Lambda_{0,0} H_k^{(0)} s_{k,0} - \Lambda_{0,0} H_k^{(1)} s_{k,1}\right) + \frac{1}{N} \sum_{m=0, m\neq k}^{N-1} \left(H_m^{(0)} X_{m,0}^{(0)} + H_m^{(1)} X_{m,0}^{(1)}\right)\Lambda_{k-m,0} + V_{k,0} \quad (19)$$

$$Y_{k,1} = \frac{1}{N}\left(\Lambda_{0,1} H_k^{(1)} s_{k,0} + \Lambda_{0,1} H_k^{(0)} s_{k,1}\right)$$

$$+\frac{1}{N}\sum_{m=0,m\neq k}^{N-1}\big(H_m^{(0)}X_{m,1}^{(0)}+H_m^{(1)}X_{m,1}^{(1)}\big)\Lambda_{k-m,1}+V_{k,1}. \quad (20)$$

As in Section III-A, we assume perfect estimation of $\Lambda_{0,0}$ and $\Lambda_{0,1}$. We then obtain expressions for the decision statistics $U_{k,0,ST}$ and $U_{k,1,ST}$, which are Gaussian random variables given $\{H_m^{(i)}\}$, $\{\Lambda_{m,n}\}$, $s_{k,0}$ and $s_{k,1}$. The means and variances of $U_{k,0,ST}$ and $U_{k,1,ST}$ can be calculated by using the fact that the space-time code is applied to each subcarrier and that the following properties for $X_{m,n}^{(i)}$, $i\in\{0,1\}$, $(m,n)\in\{0,1,\ldots,N-1\}\times\{0,1\}$ hold: $E[X_{m,n}^{(0)}X_{m,n}^{(1)}]=0$, $E[X_{m,0}^{(0)}X_{m,1}^{(1)}]=1$, $E[X_{m,1}^{(0)}X_{m,0}^{(1)}]=-1$ and $E[X_{m,0}^{(i)}X_{m,1}^{(i)}]=0$. Define $\alpha_{0,ST}=\Lambda_{0,0}^*|\Lambda_{0,1}|^2\Lambda_{k-m,0}H_k^{(0)*}$, $\beta_{0,ST}=|\Lambda_{0,0}|^2\Lambda_{0,1}\Lambda_{k-m,1}^*H_k^{(1)}$, $\alpha_{1,ST}=-\Lambda_{0,0}^*|\Lambda_{0,1}|^2\Lambda_{k-m,0}H_k^{(1)*}$ and $\beta_{1,ST}=|\Lambda_{0,0}|^2\Lambda_{0,1}\Lambda_{k-m,1}^*H_k^{(0)}$. After simplification, we obtain

$$E[U_{k,0,ST}]=a_{k,ST}s_{k,0} \quad (21)$$

$$\begin{aligned}\sigma^2_{U_{k,0,ST}}=&\frac{1}{N^2}\sum_{\substack{m=0\\m\neq k}}^{N-1}\big[\Re^2\{\alpha_{0,ST}H_m^{(0)}\}+\Re^2\{\alpha_{0,ST}H_m^{(1)}\}\\&+\Re^2\{\beta_{0,ST}H_m^{(0)*}\}+\Re^2\{\beta_{0,ST}H_m^{(1)*}\}\\&+2\Re\{\alpha_{0,ST}H_m^{(0)}\}\Re\{\beta_{0,ST}H_m^{(1)*}\}\\&-2\Re\{\alpha_{0,ST}H_m^{(1)}\}\Re\{\beta_{0,ST}H_m^{(0)*}\}\big]\\&+|\Lambda_{0,0}\Lambda_{0,1}|^2\big(|\Lambda_{0,1}H_k^{(0)}|^2+|\Lambda_{0,0}H_k^{(1)}|^2\big)\sigma_V^2\end{aligned} \quad (22)$$

$$E[U_{k,1,ST}]=a_{k,ST}s_{k,1} \quad (23)$$

$$\begin{aligned}\sigma^2_{U_{k,1,ST}}=&\frac{1}{N^2}\sum_{\substack{m=0\\m\neq k}}^{N-1}\big[\Re^2\{\alpha_{1,ST}H_m^{(0)}\}+\Re^2\{\alpha_{1,ST}H_m^{(1)}\}\\&+\Re^2\{\beta_{1,ST}H_m^{(0)*}\}+\Re^2\{\beta_{1,ST}H_m^{(1)*}\}\\&+2\Re\{\alpha_{1,ST}H_m^{(0)}\}\Re\{\beta_{1,ST}H_m^{(1)*}\}\\&-2\Re\{\alpha_{1,ST}H_m^{(1)}\}\Re\{\beta_{1,ST}H_m^{(0)*}\}\big]\\&+|\Lambda_{0,0}\Lambda_{0,1}|^2\big(|\Lambda_{0,1}H_k^{(1)}|^2+|\Lambda_{0,0}H_k^{(0)}|^2\big)\sigma_V^2\end{aligned} \quad (24)$$

where σ_V^2 is the variance of the real and imaginary parts of $V_{k,n}$ and $a_{k,ST}=\frac{1}{N}|\Lambda_{0,0}\Lambda_{0,1}|^2\big(|H_k^{(0)}|^2+|H_k^{(1)}|^2\big)$. The BER conditioned on $\{H_m^{(i)}\}$ and $\{\Lambda_{m,n}\}$ is given by

$$P_{e,ST-OFDM}=\frac{1}{2N}\sum_{k=0}^{N-1}\Big[Q\Big(\frac{a_{k,ST}}{\sigma_{U_{k,0,ST}}}\Big)+Q\Big(\frac{a_{k,ST}}{\sigma_{U_{k,1,ST}}}\Big)\Big]. \quad (25)$$

C. SF-OFDM-ZF

Let Λ_k denote the DFT of the frequency offset and phase noise term for the OFDM symbol under consideration. Let $X_m^{(i)}, i\in\{0,1\}$ denote the BPSK data symbol transmitted from Antenna i on subcarrier m. With frequency offset and phase noise, the relevant DFT outputs are

$$\begin{aligned}Y_k=&\frac{1}{N}\big[\big(\Lambda_0H_k^{(0)}+\Lambda_{-1}H_{k+1}^{(1)}\big)s_{k,0}+\big(-\Lambda_0H_k^{(1)}+\Lambda_{-1}H_{k+1}^{(0)}\big)s_{k,1}\big]\\&+\frac{1}{N}\sum_{\substack{m=0\\m\neq k,k+1}}^{N-1}\big(H_m^{(0)}X_m^{(0)}+H_m^{(1)}X_m^{(1)}\big)\Lambda_{k-m}+V_k\end{aligned} \quad (26)$$

$$\begin{aligned}Y_{k+1}=&\frac{1}{N}\big[\big(\Lambda_0H_{k+1}^{(1)}+\Lambda_1H_k^{(0)}\big)s_{k,0}+\big(\Lambda_0H_{k+1}^{(0)}-\Lambda_1H_k^{(1)}\big)s_{k,1}\big]\\&+\frac{1}{N}\sum_{\substack{m=0\\m\neq k,k+1}}^{N-1}\big(H_m^{(0)}X_m^{(0)}+H_m^{(1)}X_m^{(1)}\big)\Lambda_{k+1-m}+V_{k+1}.\end{aligned} \quad (27)$$

We proceed as in Sections III-A and III-B to obtain the decision statistics $U_{k,0,SF}$ and $U_{k,1,SF}$ assuming perfect estimation of Λ_0. Since the space-frequency code is applied to all pairs of subcarriers $(k,k+1)$, $k\in\{0,2,\ldots,N-2\}$, we have for any $(m,n)\in\{0,1,\ldots,N-1\}^2$, $E[X_m^{(i)}X_n^{(i)}]=\delta_{n-m}$ and $E[X_m^{(0)}X_n^{(1)}]=\delta_{n-m-1}-\delta_{n-m+1}$, where δ_m is the Kronecker delta function. Define $\alpha_{0,SF}=\Lambda_0^*\Lambda_{k-m}H_{k+1}^{(0)*}$, $\beta_{0,SF}=\Lambda_0\Lambda_{k+1-m}^*H_k^{(1)}$, $\alpha_{1,SF}=-\Lambda_0^*\Lambda_{k-m}H_{k+1}^{(1)*}$ and $\beta_{1,SF}=\Lambda_0\Lambda_{k+1-m}^*H_k^{(0)}$. Given $\{H_m^{(i)}\}$, $\{\Lambda_m\}$, $s_{k,0}$ and $s_{k,1}$, the quantities $U_{k,0,SF}$ and $U_{k,1,SF}$ are Gaussian random variables whose means and variances are given by

$$E[U_{k,0,SF}]=a_{k,0,SF}s_{k,0}+b_{k,0,SF}s_{k,1} \quad (28)$$

$$\begin{aligned}\sigma^2_{U_{k,0,SF}}=&\frac{1}{N^2}\sum_{\substack{m=0\\m\neq k,k+1}}^{N-1}\big[\Re^2\{D_k\big(\alpha_{0,SF}H_m^{(0)}+\beta_{0,SF}H_m^{(0)*}\big)\}\\&+\Re^2\{D_k\big(\alpha_{0,SF}H_m^{(1)}+\beta_{0,SF}H_m^{(1)*}\big)\}\big]\\&+\frac{2}{N^2}\sum_{\substack{m=0\\m\neq k-1,k,k+1}}^{N-2}\Re\{D_k\big(\alpha_{0,SF}H_m^{(0)}+\beta_{0,SF}H_m^{(0)*}\big)\}\\&\times\Re\{D_k\big(\Lambda_0^*\Lambda_{k-1-m}H_{k+1}^{(0)*}H_{m+1}^{(1)}+\Lambda_0\Lambda_{k-m}^*H_k^{(1)}H_{m+1}^{(1)*}\big)\}\\&-\frac{2}{N^2}\sum_{\substack{m=1\\m\neq k,k+1,k+2}}^{N-1}\Re\{D_k\big(\alpha_{0,SF}H_m^{(0)}+\beta_{0,SF}H_m^{(0)*}\big)\}\\&\times\Re\{D_k\big(\Lambda_0^*\Lambda_{k+1-m}H_{k+1}^{(0)*}H_{m-1}^{(1)}+\Lambda_0\Lambda_{k+2-m}^*H_k^{(1)}H_{m-1}^{(1)*}\big)\}\\&+|D_k|^2\big(|\Lambda_0H_{k+1}^{(0)}|^2+|\Lambda_0H_k^{(1)}|^2\big)\sigma_V^2\end{aligned} \quad (29)$$

$$E[U_{k,1,SF}]=a_{k,1,SF}s_{k,1}+b_{k,1,SF}s_{k,0} \quad (30)$$

$$\begin{aligned}\sigma^2_{U_{k,1,SF}}=&\frac{1}{N^2}\sum_{\substack{m=0\\m\neq k,k+1}}^{N-1}\big[\Re^2\{D_k\big(\alpha_{1,SF}H_m^{(0)}+\beta_{1,SF}H_m^{(0)*}\big)\}\\&+\Re^2\{D_k\big(\alpha_{1,SF}H_m^{(1)}+\beta_{1,SF}H_m^{(1)*}\big)\}\big]\\&+\frac{2}{N^2}\sum_{\substack{m=0\\m\neq k-1,k,k+1}}^{N-2}\Re\{D_k\big(\alpha_{1,SF}H_m^{(0)}+\beta_{1,SF}H_m^{(0)*}\big)\}\\&\times\Re\{D_k\big(-\Lambda_0^*\Lambda_{k-1-m}H_{k+1}^{(1)*}H_{m+1}^{(1)}+\Lambda_0\Lambda_{k-m}^*H_k^{(0)}H_{m+1}^{(1)*}\big)\}\end{aligned}$$

$$-\frac{2}{N^2}\sum_{\substack{m=1\\ m\neq k,k+1,k+2}}^{N-1}\Re\{D_k(\alpha_{1,SF}H_m^{(0)}+\beta_{1,SF}H_m^{(0)*})\}$$
$$\times\Re\{D_k(-\Lambda_0^*\Lambda_{k+1-m}H_{k+1}^{(1)*}H_{m-1}^{(1)}+\Lambda_0\Lambda_{k+2-m}^*H_k^{(0)}H_{m-1}^{(1)*})\}$$
$$+|D_k|^2(|\Lambda_0H_{k+1}^{(1)}|^2+|\Lambda_0H_k^{(0)}|^2)\sigma_V^2 \quad (31)$$

where σ_V^2 is the variance of the real and imaginary parts of V_k and V_{k+1} and

$$D_k = H_k^{(0)*}H_{k+1}^{(0)}+H_k^{(1)*}H_{k+1}^{(1)} \quad (32)$$
$$a_{k,0,SF}=\frac{1}{N}\big[|\Lambda_0|^2|D_k|^2 +\Re\{D_k(\Lambda_0^*\Lambda_{-1}H_{k+1}^{(0)*}H_{k+1}^{(1)}+\Lambda_0\Lambda_1^*H_k^{(0)*}H_k^{(1)})\}\big] \quad (33)$$
$$b_{k,0,SF}=\frac{1}{N}\Re\{D_k(\Lambda_0^*\Lambda_{-1}|H_{k+1}^{(0)}|^2-\Lambda_0\Lambda_1^*|H_k^{(1)}|^2)\} \quad (34)$$
$$a_{k,1,SF}=\frac{1}{N}\big[|\Lambda_0|^2|D_k|^2 -\Re\{D_k(\Lambda_0^*\Lambda_{-1}H_{k+1}^{(0)}H_{k+1}^{(1)*}+\Lambda_0\Lambda_1^*H_k^{(0)}H_k^{(1)*})\}\big] \quad (35)$$
$$b_{k,1,SF}=\frac{1}{N}\Re\{D_k(-\Lambda_0^*\Lambda_{-1}|H_{k+1}^{(1)}|^2+\Lambda_0\Lambda_1^*|H_k^{(0)}|^2)\}. \quad (36)$$

The BER conditioned on $\{H_m^{(i)}\}$ and $\{\Lambda_m\}$ is then given by

$$P_{e,SF-OFDM-ZF}=\frac{1}{2N}\sum_{\substack{k=0\\ k\text{ even}}}^{N-2}\sum_{i=0}^{1}\Big[Q\Big(\frac{a_{k,i,SF}+b_{k,i,SF}}{\sigma_{U_{k,i,SF}}}\Big) +Q\Big(\frac{a_{k,i,SF}-b_{k,i,SF}}{\sigma_{U_{k,i,SF}}}\Big)\Big]. \quad (37)$$

IV. Performance with Channel Estimation Errors

In this section, expressions for the BER of MRC, ST-OFDM and SF-OFDM-ZF are given in the presence of channel estimation errors. We assume perfect synchronization such that there is neither a frequency offset nor phase noise. We assume that the channel estimates can be written as $\hat{H}_k^{(i)}=H_k^{(i)}+\varepsilon_k^{(i)},\ \ i\in\{0,1\},\ \ k\in\{0,1,\ldots,N-1\}$, where $\varepsilon_k^{(i)}$ represent the estimation errors.

A. MRC

With channel estimation errors, (2) becomes $\tilde{X}_{k,MRC}=\hat{H}_k^{(0)*}Y_k^{(0)}+\hat{H}_k^{(1)*}Y_k^{(1)}$. As in Section III, we compute the BER conditioned on $H_k^{(i)}$ and $\varepsilon_k^{(i)}$; in Section V, we present performance results for frequency-selective Rayleigh fading channels and for i.i.d. Gaussian channel estimation errors $\varepsilon_k^{(i)}$ [12]. Given X_k, $H_k^{(i)}$ and $\varepsilon_k^{(i)}$, the decision statistic $U_{k,MRC}$ is a Gaussian random variable whose mean and variance are given by

$$E[U_{k,MRC}]=a_{k,MRC}X_k \quad (38)$$
$$\sigma^2_{U_{k,MRC}}=(|\hat{H}_k^{(0)}|^2+|\hat{H}_k^{(1)}|^2)\sigma_V^2 \quad (39)$$

where $a_{k,MRC}=|H_k^{(0)}|^2+|H_k^{(1)}|^2+\Re\{\varepsilon_k^{(0)*}H_k^{(0)}+\varepsilon_k^{(1)*}H_k^{(1)}\}$. Equations (38) and (39) are generalizations of the results given in [12] to OFDM. The BER conditioned on $\{H_k^{(i)}\}$ and $\{\varepsilon_k^{(i)}\}$ is then given by (18) with (38) and (39).

B. ST-OFDM

Replacing $H_k^{(i)}$ by $\hat{H}_k^{(i)}$ in (5) and (6) and taking the real part, we obtain the decision statistics $U_{k,0,ST}$ and $U_{k,1,ST}$. By symmetry, it suffices to consider $U_{k,0,ST}$ to compute the BER. We have

$$E[U_{k,0,ST}]=a_{k,0,ST}s_{k,0}+b_{k,0,ST}s_{k,1} \quad (40)$$
$$\sigma^2_{U_{k,0,ST}}=(|\hat{H}_k^{(0)}|^2+|\hat{H}_k^{(1)}|^2)\sigma_V^2 \quad (41)$$

where $a_{k,0,ST}=|H_k^{(0)}|^2+|H_k^{(1)}|^2+\Re\{\varepsilon_k^{(0)*}H_k^{(0)}+\varepsilon_k^{(1)}H_k^{(1)*}\}$ and $b_{k,0,ST}=\Re\{\varepsilon_k^{(1)}H_k^{(0)*}-\varepsilon_k^{(0)*}H_k^{(1)}\}$. Equations (40) and (41) are generalizations of the results given in [12] to OFDM. The conditional BER is then

$$P_{e,ST-OFDM}=\frac{1}{2N}\sum_{k=0}^{N-1}\Big[Q\Big(\frac{a_{k,0,ST}+b_{k,0,ST}}{\sigma_{U_{k,0,ST}}}\Big) +Q\Big(\frac{a_{k,0,ST}-b_{k,0,ST}}{\sigma_{U_{k,0,ST}}}\Big)\Big]. \quad (42)$$

We note that $P_{e,ST-OFDM}>P_{e,MRC}$ if $a_{k,0,ST}>0$, which is true for $|\varepsilon_k^{(i)}|<|H_k^{(i)}|, i=0,1$.

C. SF-OFDM-ZF

We proceed as in Section IV-B to obtain the means and variances of $U_{k,0,SF}$ and $U_{k,1,SF}$ (given $\{H_m^{(i)}\}$ and $\{\varepsilon_m^{(i)}\}$):

$$E[U_{k,0,SF}]=a_{k,0,SF}s_{k,0}+b_{k,0,SF}s_{k,1} \quad (43)$$
$$\sigma^2_{U_{k,0,SF}}=|\hat{D}_k|^2(|\hat{H}_{k+1}^{(0)}|^2+|\hat{H}_k^{(1)}|^2)\sigma_V^2 \quad (44)$$
$$E[U_{k,1,SF}]=a_{k,1,SF}s_{k,1}+b_{k,1,SF}s_{k,0} \quad (45)$$
$$\sigma^2_{U_{k,1,SF}}=|\hat{D}_k|^2(|\hat{H}_{k+1}^{(1)}|^2+|\hat{H}_k^{(0)}|^2)\sigma_V^2 \quad (46)$$

where

$$\hat{D}_k=\hat{H}_k^{(0)*}\hat{H}_{k+1}^{(0)}+\hat{H}_k^{(1)*}\hat{H}_{k+1}^{(1)} \quad (47)$$
$$a_{k,0,SF}=\Re\{\hat{D}_k[\hat{H}_{k+1}^{(0)*}H_k^{(0)}+\hat{H}_k^{(1)}H_{k+1}^{(1)*}]\} \quad (48)$$
$$b_{k,0,SF}=\Re\{\hat{D}_k(-\varepsilon_{k+1}^{(0)*}H_k^{(1)}+\varepsilon_k^{(1)}H_{k+1}^{(0)*})\} \quad (49)$$
$$a_{k,1,SF}=\Re\{\hat{D}_k[\hat{H}_{k+1}^{(1)*}H_k^{(1)}+\hat{H}_k^{(0)}H_{k+1}^{(0)*}]\} \quad (50)$$
$$b_{k,1,SF}=\Re\{\hat{D}_k(-\varepsilon_{k+1}^{(1)*}H_k^{(0)}+\varepsilon_k^{(0)}H_{k+1}^{(1)*})\}. \quad (51)$$

The conditional BER is given by (37) using (43)–(51).

V. Simulation Results

The average BER of each OFDM system can be obtained by averaging the conditional BER expressions given in Sections III and IV over the distributions for the phase noise, channel gains and channel estimation errors. For example, the BER of the MRC system can be computed as follows. Let $\mathbf{\Lambda} = [\Lambda_0^{(0)} \cdots \Lambda_{N-1}^{(0)} \ \Lambda_0^{(1)} \cdots \Lambda_{N-1}^{(1)}]^T$, $\mathbf{H} = [H_0^{(0)} \cdots H_{N-1}^{(0)} \quad H_0^{(1)} \cdots H_{N-1}^{(1)}]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_0^{(0)} \cdots \varepsilon_{N-1}^{(0)} \ \varepsilon_0^{(1)} \cdots \varepsilon_{N-1}^{(1)}]^T$. The average BER of the MRC system with frequency offsets and phase noise is given by

$$P_{e,MRC,\text{avg.}} = \int P_{e,MRC}\, p(\mathbf{\Lambda},\mathbf{H})\, d\mathbf{\Lambda}\, d\mathbf{H} \qquad (52)$$

where $p(\mathbf{\Lambda},\mathbf{H})$ is the joint probability density function (PDF) of $(\mathbf{\Lambda},\mathbf{H})$ and $P_{e,MRC}$ is the conditional BER given by (18). Similarly, the average BER of the MRC system with channel estimation errors is given by

$$P_{e,MRC,\text{avg.}} = \int P_{e,MRC}\, p(\boldsymbol{\varepsilon},\mathbf{H})\, d\boldsymbol{\varepsilon}\, d\mathbf{H} \qquad (53)$$

where $p(\boldsymbol{\varepsilon},\mathbf{H})$ is the joint PDF of $(\boldsymbol{\varepsilon},\mathbf{H})$ and $P_{e,MRC}$ is the conditional BER given in Section IV-A. The average BER's for ST-OFDM and SF-OFDM are computed in a similar fashion.

The average BER is evaluated for each system semianalytically by calculating the conditional BER followed by Monte Carlo simulation for frequency-selective Rayleigh fading channels. The system parameters, listed in Table I, are based on the physical layer specifications of the HIPERLAN/2 and IEEE 802.11a wireless LAN standards [13], [14]. Without any overhead for pilot symbols, these parameters imply an uncoded data rate of 16 Mbps. The average signal-to-noise ratio (SNR) is defined by $\text{SNR} = E[|H_k|^2]/\sigma_V^2$ and the channel estimation error-to-signal ratio (ESR) is defined by $\text{ESR} = E[|\varepsilon_k^{(i)}|^2]/E[|H_k^{(i)}|^2]$. We assume that the channel is quasi-static (constant impulse response for a few OFDM symbols and no Doppler spread) and that the channel estimation errors are i.i.d. Gaussian random variables. The second assumption is justified by considering the following least squares channel estimator. In the HIPERLAN/2 system, the frequency domain training sequence X_k for channel estimation is a BPSK signal. From (1), the least squares channel estimates are given by $\hat{H}_k^{(i)} = Y_k^{(i)}/X_k = H_k^{(i)} + V_k^{(i)}/X_k$. Since $|X_k| = 1$, the channel estimation errors are i.i.d. Gaussian variables with variance σ_V^2.

TABLE I

System parameters.

Parameter	Value
Power Delay Profile	Exponential
RMS Delay Spread	100 ns
Sampling Rate $(1/T_s)$	20 MHz
Number of Subcarriers (N)	64
Cyclic Prefix (G)	16

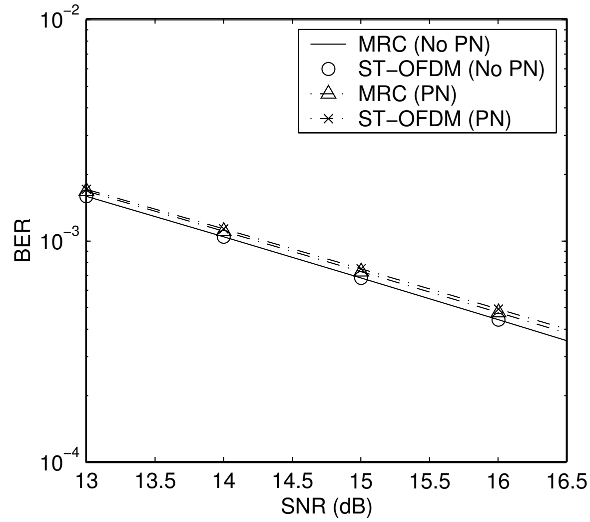

Fig. 3. BER performance of MRC and ST-OFDM. The curves labeled by (No PN) are results without frequency offset and phase noise; the curves labeled by (PN) are results for $\Delta f = 10$ kHz and rms phase noise 5^o with 3-dB frequency of 50 kHz.

In obtaining the conditional BER expressions in Section III, the ICI due to frequency offsets and phase noise was assumed to be Gaussian distributed. This assumption was verified by comparing the average BER's obtained using the semianalytical approach with the results of exact simulations for each system. The calculations and simulations are seen to agree. Since the Gaussian ICI assumption is accurate, the results to follow are obtained using the semianalytical approach, except for the SF-OFDM-ML system for which the results are obtained from exact simulations.

Figs. 3 and 4 are plots of the BER versus SNR for the various diversity techniques. It can be seen that without frequency offset or phase noise (curves labeled by (No PN)), the performances of MRC and ST-OFDM are identical, as is expected for transmit diversity and receive diversity with perfect channel estimates and an ideal receiver. In contrast, there is a loss of 1.3 dB at $\text{BER} = 10^{-3}$ for the SF-OFDM-ZF method with respect to MRC and ST-OFDM. As discussed in Section II-C, the SF-OFDM-ZF method is not optimal for channels with nonzero delay spread since the channel matrix $\mathcal{H}$ is not a scaled unitary matrix, and the matrix inversion introduces correlation of the noise terms on adjacent subcarriers. The performance of the SF-OFDM method is improved significantly by using the maximum likelihood criterion (SF-OFDM-ML); the loss of SF-OFDM-ML with respect to MRC at $\text{BER} = 10^{-3}$ is 0.15 dB. The curves labeled by (PN) represent the performance for a residual frequency offset of $\Delta f = 10$ kHz and a Lorentzian phase noise process with rms value of 5^o and 3-dB frequency of 50 kHz. It can be seen that the degradation is small for practical values of the residual frequency offset and phase noise;

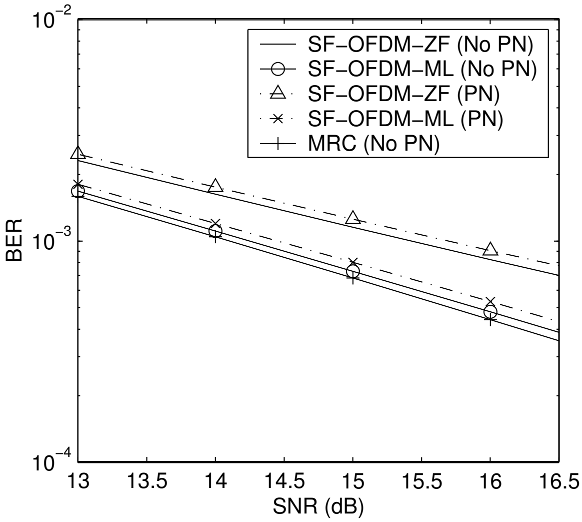

Fig. 4. BER performance of SF-OFDM-ZF and SF-OFDM-ML. The curves labeled by (No PN) are results without frequency offset and phase noise; the curves labeled by (PN) are results for $\Delta f = 10$ kHz and rms phase noise 5^o with 3-dB frequency of 50 kHz. The curve for MRC is included for reference.

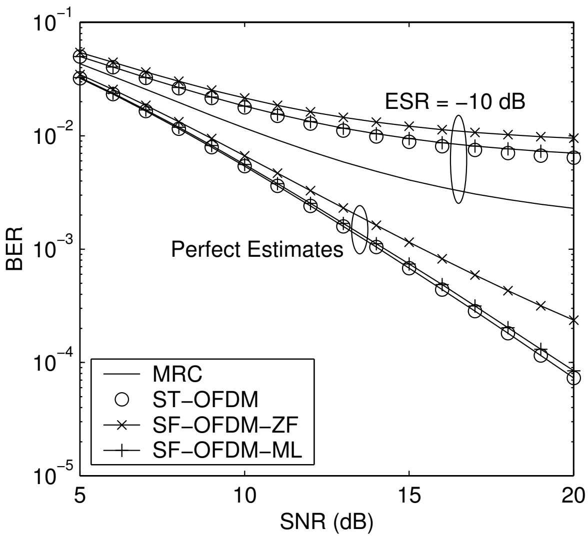

Fig. 5. BER performance of diversity techniques with channel estimation errors.

at BER $= 10^{-3}$, the loss is 0.14 dB for MRC, 0.2 dB for ST-OFDM, 0.26 dB for SF-OFDM-ZF, and 0.2 dB for SF-OFDM-ML. The degradation would be larger for higher order constellations.

Fig. 5 is a plot of the BER versus SNR in the presence of channel estimation errors. From the plot, we see that the transmit diversity methods for OFDM are much more sensitive to channel estimation errors than MRC receive diversity. For ESR $= -10$ dB, the degradation at BER $= 10^{-2}$ is 2.25 dB for MRC, 5.45 dB for ST-OFDM, 9.5 dB for SF-OFDM-ZF, and 5.9 dB for SF-OFDM-ML.

VI. Conclusions

Expressions have been derived for the BER of various transmit and receive diversity methods for OFDM systems using BPSK with carrier frequency offset, phase noise and channel estimation errors. The derivations with frequency offset and phase noise can be applied to a general multiplicative distortion of the received signal. Simulation results for frequency-selective multipath channels show a small loss in performance due to frequency offset and phase noise. For the SF-OFDM transmit diversity method, it is seen that SF-OFDM-ZF incurs a loss with respect to ST-OFDM and MRC due to the variation in the channel transfer function between adjacent subcarriers; using SF-OFDM-ML improves the performance significantly at the expense of increased complexity. The results also indicate that the transmit diversity methods are much more sensitive to channel estimation errors than the MRC receive diversity technique.

References

[1] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1974.

[2] S. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–58, Oct. 1998.

[3] V. Tarokh, H. Jafarkhani and A. R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. on Inform. Theory*, vol. 45, no. 5, pp. 1456–67, Jul. 1999.

[4] L. J. Cimini, Jr., “Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing,” *IEEE Trans. on Commun.*, vol. COM-33, no. 7, pp. 665–75, Jul. 1985.

[5] J. Bingham, “Multicarrier modulation for data transmission: an idea whose time has come,” *IEEE Commun. Mag.*, vol. 28, no. 5, pp. 5–14, May 1990.

[6] E. Feig, “Practical aspects of DFT-based frequency division multiplexing for data transmission,” *IEEE Trans. on Commun.*, vol, 38, no. 7, pp. 929–32, Jul. 1990.

[7] K. F. Lee and D. B. Williams, “A space-time coded transmitter diversity technique for frequency selective fading channels,” in *Proc. 2000 IEEE Sensor Array and Multichannel Signal Processing Workshop*, Cambridge, MA, Mar. 2000, pp. 149–152.

[8] K. F. Lee and D. B. Williams, “A space-frequency transmitter diversity technique for OFDM systems,” in *Proc. 2000 Global Telecommunications Conf.*, San Francisco, CA, Nov. 2000, pp. 1473–7.

[9] A. R. S. Bahai and B. R. Saltzberg, *Multi-carrier Digital Communications: Theory and Applications of OFDM*, New York: Kluwer Academic/Plenum, 1999.

[10] T. Pollet, M. van Bladel and M. Moeneclaey, “BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise,” *IEEE Trans. on Commun.*, vol. 43, no. 2/3/4, pp. 191–3, Feb./Mar./Apr. 1995.

[11] L. Tomba, “On the effect of Wiener phase noise in OFDM systems,” *IEEE Trans. on Commun.*, vol. 46, no. 5, pp. 580–3, May 1998.

[12] X. Feng and C. Leung, “Performance sensitivity comparison of two diversity schemes,” *Elect. Lett.*, vol. 36, no. 9, pp. 838–9, Apr. 2000.

[13] ETSI, “Broadband radio access networks (BRAN); HIPERLAN type 2; physical (PHY) layer,” document DTS/BRAN-0023003, Eur. Telecomm. Stds. Inst., Sophia Antipolis Cedex, France, Apr. 2000.

[14] IEEE, “Part 11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications: high-speed physical layer in the 5 GHz band,” IEEE Std 802.11a-1999, New York, NY, Sep. 1999.