

# Adaptive Local Exponential Bases for Estimation of Linear Time-Varying Channels

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**Abstract**— A new non-parametric method is presented for the estimation of linear time-varying (LTV) channels. A basis of smooth local exponentials, which are approximate eigenfunctions of underspread LTV channels, is chosen to select time intervals during which the time variation of the channel is small. The time-varying transfer function is expanded in the basis of smooth local exponentials to obtain an estimate of the time-varying impulse response. Simulations for mobile radio channels demonstrate robust tracking of the time-varying channel for a large range of mobile speeds.

## I. INTRODUCTION

Multipath propagation in mobile radio systems causes severe amplitude variation, or fading, of the received signal. In addition, wideband signals are susceptible to intersymbol interference (ISI) due to multipath delay spread. The mobility of subscribers causes the multipath environment to vary with time. Estimates of the time-varying mobile radio channel are needed to mitigate the effects of ISI.

Adaptive estimation of a communication channel is often accomplished using variations of the least mean square (LMS) or recursive least squares (RLS) algorithms [1], [2], [3]. For mobile radio channels, the rate of adaptation must be chosen to balance the error due to misadjustment and the error due to the lag in tracking a time-varying channel. In [4], a maximum-likelihood estimate is obtained for a finite impulse response (FIR) channel whose taps vary according to a first order autoregressive (AR) process. This method involves estimating and inverting the covariance matrix of a block of output samples when a training sequence is transmitted. The estimation error of the output covariance matrix depends on the block size and the rate of variation of the channel taps. Application of the channel estimation method described in [4] is limited by the complexity of the matrix inversion.

An important characteristic for estimation of mobile radio channels is the underspread property [5]. Estimator performance can be improved using approximate eigenfunctions of underspread linear time-varying (LTV) channels. In [6], optimal precoding for underspread LTV channels is described using approximate singular functions. The singular functions are computed under the assumption that perfect knowledge of the LTV channel is available at the transmitter. In [7], it is shown that the ambiguity function of an approximate eigenfunction is nearly constant over the time-frequency support of the channel spreading function. Structured bases of approximate eigenfunctions and fast algorithms are difficult to obtain using the approaches described in the literature mentioned above.

In this paper, adaptive bases of smooth local complex exponentials, which are approximate eigenfunctions of underspread LTV systems, are used to estimate mobile radio channels. The adaptive bases are selected using best basis methods [8], [9], [10] and related techniques to partition the time axis into vari-

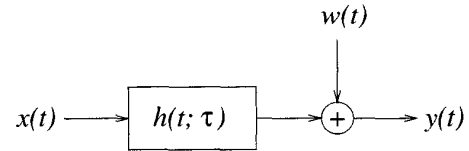


Fig. 1. Baseband equivalent model for mobile radio channel.

able duration segments. The time segments are chosen such that the channel time variation is small within each segment. An estimate of the time-varying transfer function is obtained using the basis expansion coefficients of a training sequence and the signal at the output of a mobile radio channel. The time-varying impulse response is then estimated by the expansion of the time-varying transfer function in the selected basis.

The paper is organized as follows. In Section II, a model for the mobile radio channel is presented. In Section III, smooth local exponentials are shown to be approximate eigenfunctions of underspread LTV channels. Section IV describes a method, using adaptive local exponential bases, to estimate the time-varying transfer function and the impulse response of an LTV channel. Section V presents a performance comparison of the estimation method described in Section IV with an estimation method using temporal windows of fixed duration for mobile radio channels with various mobile speeds. Conclusions are given in Section VI.

## II. CHANNEL MODEL

A baseband equivalent model for the mobile radio channel is illustrated in Fig. 1. The time-varying impulse response  $h(t; \tau)$  represents the channel response at time  $t$  due to an impulse at time  $t - \tau$ . For a transmitted signal  $x(t)$ , the received signal  $y(t)$  is given by

$$y(t) = \int_{-\infty}^{\infty} h(t; \tau) x(t - \tau) d\tau + w(t) \quad (1)$$

where  $w(t)$  represents additive white Gaussian noise (AWGN) with variance  $\sigma_w^2$ . The Gaussian wide-sense stationary, uncorrelated scattering (GWSSUS) model is used for the channel [11]. The multipath spread, maximum Doppler frequency, and coherence time of the channel are denoted by  $T_m$ ,  $f_{\max}$ , and  $T_c$ , respectively. The coherence time is inversely proportional to  $f_{\max}$ . For underspread channels,  $T_m f_{\max} < 1$ .

For the discrete-time channel model needed in Section IV, the signals in (1) are sampled at frequency  $f_s > 2(B_x + f_{\max})$ , where  $B_x$  is the bandwidth of the transmitted signal  $x(t)$ . The discrete-time equivalent of (1) is then given by

$$y[n] = \sum_{m=-\infty}^{\infty} h[n, m] x[n - m] + w[n]. \quad (2)$$

The discrete-time channel  $h[n, m]$  is modeled as a time-varying tapped delay line with  $M$  taps, *i.e.*,  $h[n, m] = 0$  for  $m < 0$  or  $m \geq M$ , where  $M = \lceil T_m f_s \rceil$ . For a given value of  $m$ ,  $h[n, m]$  is a wide-sense stationary complex Gaussian process that represents the time-varying channel response at delay  $m/f_s$ . Under the GWSSUS model, the cross correlation of  $h[n, m]$  can be written as

$$\frac{1}{2} E\{h[n, m] h^*[n', m']\} = P[m] \rho_m[n - n'] \delta[m - m'] \quad (3)$$

where  $P[m]$  is the multipath intensity profile,  $\rho_m[n - n']$  is the normalized Doppler autocorrelation of  $h[n, m]$  (for a fixed  $m$ ), and  $\delta[m - m']$  is the Kronecker delta function.

### III. APPROXIMATE EIGENFUNCTIONS OF LTV CHANNELS

In this section, smooth local complex exponentials are shown to be approximate eigenfunctions of underspread LTV channels. Using a framework similar to that of [8], we consider an LTV operator  $\mathcal{H}$  defined for any function  $x \in L^2(\mathbf{R})$ :

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} h(t; \tau) x(t - \tau) d\tau \quad (4)$$

where  $h(t; \tau)$  is the LTV impulse response defined in Section II. For underspread LTV systems, the following conditions hold:

$$h(t; \tau) \approx h(t'; \tau), \quad t \in \left[ t' - \frac{T_c}{2}, t' + \frac{T_c}{2} \right] \quad (5)$$

$$h(t; \tau) \approx 0, \quad \tau \notin [0, T_m]. \quad (6)$$

It is useful to define a time-varying transfer function  $H(t; \nu)$  of the LTV system by

$$H(t; \nu) = \int_{-\infty}^{\infty} h(t; \tau) e^{-j2\pi\nu\tau} d\tau. \quad (7)$$

Since  $h(t; \tau)$  is approximately delay-limited in  $\tau$  to an interval of size  $T_m$ ,  $H(t; \nu)$  is approximately constant in  $\nu$  over intervals of size  $1/T_m$ . Furthermore, since the time variation of  $h(t; \tau)$  is negligible over the interval  $t \in [t' - \frac{T_c}{2}, t' + \frac{T_c}{2}]$ , for any  $\xi \in \mathbf{R}$ , we have  $H(t; \nu) \approx H(t'; \xi)$ ,  $(t, \nu) \in [t' - \frac{T_c}{2}, t' + \frac{T_c}{2}] \times [\xi - \frac{1}{2T_m}, \xi + \frac{1}{2T_m}]$ .

We now show that smooth local complex exponentials are approximate eigenfunctions of the LTV operator  $\mathcal{H}$ . Let

$$\phi_{t', \xi}(t) = b(t) e^{j(2\pi\xi t + \theta)} \quad (8)$$

where  $b(t)$  is a smooth window function with support  $[t' - \frac{T_c}{2}, t' + \frac{T_c}{2}]$ , and  $\xi$  and  $\theta$  are the frequency and phase of the complex exponential, respectively. The result of applying the LTV operator  $\mathcal{H}$  to  $\phi_{t', \xi}(t)$  is given by

$$\mathcal{H}\phi_{t', \xi}(t) = \int_{-\infty}^{\infty} h(t; \tau) \phi_{t', \xi}(t - \tau) d\tau \quad (9)$$

$$= \int_{-\infty}^{\infty} H(t; \nu) \Phi_{t', \xi}(\nu) e^{j2\pi\nu t} d\nu \quad (10)$$

where  $\Phi_{t', \xi}(\nu)$  is the Fourier transform of  $\phi_{t', \xi}(t)$ , and (10) follows from Parseval's identity. Since  $b(t)$  is a smooth window, most of the energy of  $\Phi_{t', \xi}(\nu)$  is concentrated in the

interval  $\nu \in [\xi - \frac{1}{2T_c}, \xi + \frac{1}{2T_c}]$ . From the relations  $H(t; \nu) \approx H(t; \xi)$ ,  $\nu \in [\xi - \frac{1}{2T_m}, \xi + \frac{1}{2T_m}]$  and  $T_m < T_c$  (for underspread channels), we have

$$\begin{aligned} \mathcal{H}\phi_{t', \xi}(t) &\approx \int_{-\infty}^{\infty} H(t; \xi) \Phi_{t', \xi}(\nu) e^{j2\pi\nu t} d\nu \quad (11) \\ &= H(t; \xi) \phi_{t', \xi}(t). \quad (12) \end{aligned}$$

Finally, we use the fact that  $H(t; \xi) \approx H(t'; \xi)$  over the support of  $\phi_{t', \xi}(t)$  to obtain

$$\mathcal{H}\phi_{t', \xi}(t) \approx H(t'; \xi) \phi_{t', \xi}(t). \quad (13)$$

Thus,  $\phi_{t', \xi}(t)$  is an approximate eigenfunction of  $\mathcal{H}$ . In Section IV, adaptive bases of approximate eigenfunctions are used to estimate the time-varying transfer function and the impulse response of an LTV channel.

### IV. ESTIMATION OF LTV CHANNELS USING ADAPTIVE BASES OF LOCAL EXPONENTIALS

In this section, an estimation method for discrete-time underspread LTV channels is presented. It is useful to define the discrete-time LTV operator  $\mathcal{H}_d$  by

$$\mathcal{H}_d x[n] = \sum_{m=0}^{M-1} h[n, m] x[n - m], \quad n = 0, 1, \dots, N - 1 \quad (14)$$

where  $N$  is the size of a block of output samples. Let  $\mathcal{B} = \{\phi_i\}_{0 \leq i \leq N-1}$  be an orthonormal basis of  $\mathbf{C}^N$ . The goal of the LTV channel estimation is to determine the coefficients

$$a_{i,j} = \langle \phi_i, \mathcal{H}_d \phi_j \rangle \quad (15)$$

where  $\langle f, g \rangle = \sum_{n=0}^{N-1} f^*[n] g[n]$ . In order to reduce the number of coefficients to be estimated, it is desirable to select a basis  $\mathcal{B}$  that diagonalizes (or nearly diagonalizes)  $\mathcal{H}_d$ . Using such a basis, we estimate the matrix  $[a_{i,j}]$  with a diagonal matrix  $[\hat{a}_{i,j}]$  (*i.e.*,  $\hat{a}_{i,j} = 0$  for  $i \neq j$ ). Therefore, the Hilbert-Schmidt (HS) error of the channel estimation is

$$\begin{aligned} \|\mathcal{H}_d - \hat{\mathcal{H}}_d\|_{HS}^2 &= \sum_{i=0}^{N-1} |a_{i,i} - \hat{a}_{i,i}|^2 + \sum_{i,j=0, i \neq j}^{N-1} |a_{i,j}|^2 \\ &= \sum_{i=0}^{N-1} |a_{i,i} - \hat{a}_{i,i}|^2 + \|\mathcal{H}_d\|_{HS}^2 - \sum_{i=0}^{N-1} |a_{i,i}|^2. \quad (16) \end{aligned}$$

In order to minimize the estimation error, the quantity  $\sum_{i=0}^{N-1} |a_{i,i}|^2$  must be maximized by selecting a "best" basis among those that diagonalize  $\mathcal{H}_d$ .

Using the results of Section III, one can show that there exist smooth local exponential bases that nearly diagonalize  $\mathcal{H}_d$ . Thus, bases of smooth local exponentials are used for channel estimation. We process blocks of the channel output and consider a typical block of  $N$  samples, where  $N$  is a dyadic length (power of two). Fast selection of a basis for diagonalization involves a recursive dyadic partition of the interval  $I_{0,0} = \{0, 1, \dots, N - 1\}$  [10]. For each  $p$  satisfying  $0 \leq p \leq P$ , dyadic subintervals of  $I_{0,0}$  with length  $N2^{-p}$  are given by

$$I_{p,q} = \{N2^{-p}q, N2^{-p}q + 1, \dots, N2^{-p}(q+1) - 1\}, \quad 0 \leq q < 2^p. \quad (17)$$

The choice of  $P$  is discussed below. A recursive dyadic partition of  $I_{0,0}$  is represented by  $\mathcal{P}^\zeta = \{I_{p,q} : (p,q) \in \zeta\}$ , where  $I_{0,0} = \bigcup_{(p,q) \in \zeta} I_{p,q}$  and  $I_{p,q} \cap I_{p',q'} = \emptyset$  for  $(p,q) \neq (p',q')$ .

Associated with each subinterval  $I_{p,q}$  are two smooth window functions  $b_+^{p,q}[n]$  and  $b_-^{p,q}[n]$ . The window functions are nonzero over an interval consisting of  $I_{p,q}$  extended by  $\varepsilon \in \mathbf{Z}^+$  samples at each endpoint. Smooth local exponential functions associated with  $I_{p,q}$  are defined by

$$\Psi^{p,q,k}[n] = b_+^{p,q}[n] \sqrt{\frac{1}{N2^{-p}}} \exp \left\{ j \frac{2\pi k(n+1/2-N2^{-p}q)}{N2^{-p}} \right\} + b_-^{p,q}[n] \sqrt{\frac{1}{N2^{-p}}} \exp \left\{ -j \frac{2\pi k(n+1/2-N2^{-p}q)}{N2^{-p}} \right\} \quad (18)$$

where  $-N2^{-p-1} \leq k < N2^{-p-1}$ . The window functions  $b_+^{p,q}[n]$  and  $b_-^{p,q}[n]$  are chosen such that if  $P \leq \log_2 \left( \frac{N}{2\varepsilon} \right)$ , then  $\mathcal{B}^\zeta = \{\Psi^{p,q,k}[n]\}_{(p,q) \in \zeta, -N2^{-p-1} \leq k < N2^{-p-1}}$  forms an orthonormal basis for discrete signals having compact support in  $[\varepsilon, N - \varepsilon]$ . The ratio  $\|b_-^{p,q}\|^2 / (\|b_+^{p,q}\|^2 + \|b_-^{p,q}\|^2)$  is kept small such that most of the energy of  $\Psi^{p,q,k}[n]$  is contained in the first term of (18). The second term of (18) is needed to overcome the Balian-Low obstruction for orthonormal bases using windowed exponentials [12], [13]. The library of bases  $\mathcal{B}^\zeta$  that correspond to different recursive dyadic partitions of  $I_{0,0}$  is denoted by  $\mathcal{L}$ .

In order to minimize the channel estimation error, we require the time-varying transfer function coefficients  $H^{p,q,k} = \langle \Psi^{p,q,k}, \mathcal{H}_d \Psi^{p,q,k} \rangle$ . These coefficients are estimated using a training sequence and the following observation: the channel is approximately time invariant over each subinterval of length  $N2^{-p}$ , where  $p \geq \bar{p}$  for some  $\bar{p} \geq 0$ . Equivalently, the coherence time satisfies  $T_c > N2^{-\bar{p}}/f_s$ , where  $f_s$  is the sampling frequency. Therefore, smooth local complex exponentials  $\Psi^{p,q,k}$  approximately diagonalize the channel  $\mathcal{H}_d$ . This fact is used to estimate  $H^{p,q,k}$ . Let  $x^{p,q,k} = \langle \Psi^{p,q,k}, x \rangle$  denote the coefficients of the local exponential transform of the training sequence  $x[n]$ . Similarly, let  $y^{p,q,k} = \langle \Psi^{p,q,k}, y \rangle$  denote the transform coefficients of the noisy channel output  $y[n]$ . An estimate of the channel impulse response at partition depth  $p = \bar{p}$  is given by

$$\hat{h}^{\bar{p}}[n, m] = \sum_{q=0}^{2^{\bar{p}-1}} \sum_{k=-N2^{-\bar{p}-1}}^{N2^{-\bar{p}-1}-1} H_{nz}^{\bar{p},q,k} \Psi^{\bar{p},q,k}[m + N2^{-\bar{p}}q] \times \mathbf{1}_{I_{\bar{p},q}}[n] \mathbf{1}_{[0, M-1]}[m] \quad (19)$$

where

$$H_{nz}^{\bar{p},q,k} = \begin{cases} \frac{x^{\bar{p},q,k}}{x^{\bar{p},q,k}}, & x^{\bar{p},q,k} \neq 0 \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

and

$$\mathbf{1}_I[n] = \begin{cases} 1, & n \in I \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

The coefficients  $H^{p,q,k}$  are then estimated by

$$\hat{H}^{p,q,k} = \langle \Psi^{p,q,k}, \hat{\mathcal{H}}_d^{\bar{p}} \Psi^{p,q,k} \rangle \quad (22)$$

where

$$\hat{\mathcal{H}}_d^{\bar{p}} \Psi^{p,q,k}[n] = \sum_{m=0}^{M-1} \hat{h}^{\bar{p}}[n, m] \Psi^{p,q,k}[n - m]. \quad (23)$$

The “best” basis  $\mathcal{B}^\zeta$  is chosen to minimize the Hilbert-Schmidt error of the channel estimation among all bases  $\{\mathcal{B}^\zeta\}_{\zeta \in \Gamma}$  in the library  $\mathcal{L}$ . Therefore,

$$\sum_{(p,q) \in \zeta} \|\hat{H}^{p,q}\|^2 = \max_{\zeta \in \Gamma} \sum_{(p,q) \in \zeta} \|\hat{H}^{p,q}\|^2 \quad (24)$$

where

$$\|\hat{H}^{p,q}\|^2 = \sum_{k=-N2^{-p-1}}^{N2^{-p-1}-1} \left| \hat{H}^{p,q,k} \right|^2. \quad (25)$$

The number of bases  $\mathcal{B}^\zeta \in \mathcal{L}$  is greater than  $2^{2^{p-1}}$ , where  $P$  is the maximum partition depth. However, there exists a fast dynamic programming algorithm [10] that selects the best orthonormal basis  $\mathcal{B}^\zeta$  in  $O(N \log_2 N)$  operations. The basic step of this algorithm is a comparison of  $\|\hat{H}^{p,q}\|^2$ , associated with the “parent” subinterval  $I_{p,q}$ , with  $(\|\hat{H}^{p+1,2q}\|^2 + \|\hat{H}^{p+1,2q+1}\|^2)$ , associated with the two “children” subintervals  $I_{p+1,2q}$  and  $I_{p+1,2q+1}$ . Starting with the smallest subintervals and proceeding to the root interval  $I_{0,0}$ , the algorithm recursively removes the children subintervals from the partition  $\mathcal{P}^\zeta$  if  $\|\hat{H}^{p,q}\|^2 \geq (\|\hat{H}^{p+1,2q}\|^2 + \|\hat{H}^{p+1,2q+1}\|^2)$ ; otherwise, the children intervals are included in the partition and the value  $(\|\hat{H}^{p+1,2q}\|^2 + \|\hat{H}^{p+1,2q+1}\|^2)$  is associated with  $I_{p,q}$  in future steps of the algorithm.

For robustness to noise, an alternate adaptive basis selection method that prefers larger subintervals over smaller ones is also evaluated for channel estimation. In this method, children subintervals are removed if  $\|\hat{H}^{p,q}\|^2 \geq (\|\hat{H}^{p+1,2q}\|^2 + \|\hat{H}^{p+1,2q+1}\|^2)/\alpha$ , where  $\alpha > 1$ . Once  $\mathcal{B}^\zeta$  is chosen using one of the adaptive basis selection methods, the estimate of the channel impulse response is obtained by

$$\hat{h}[n, m] = \sum_{(p,q) \in \zeta} \sum_{k=-N2^{-p-1}}^{N2^{-p-1}-1} \hat{H}^{p,q,k} \Psi^{p,q,k}[m + N2^{-p}q] \times \mathbf{1}_{I_{p,q}}[n] \mathbf{1}_{[0, M-1]}[m]. \quad (26)$$

The following section presents results of simulations that illustrate the performance of the LTV channel estimation method using adaptive bases of smooth local exponentials.

## V. SIMULATION RESULTS

The LTV channel estimation method of Section IV is compared to a method that uses a windowed Fourier transform over signal blocks of a fixed duration. Performance is evaluated using the estimation signal-to-noise ratio (SNR) defined by

$$\text{SNR}_e = \frac{E[|y[n] - w[n]|^2]}{E[|y[n] - \hat{y}[n]|^2]} \quad (27)$$

where  $y[n]$  and  $w[n]$  are the received signal and additive noise, respectively, as defined in (2), and

$$\hat{y}[n] = \sum_{m=0}^{M-1} \hat{h}[n, m] x[n - m]. \quad (28)$$

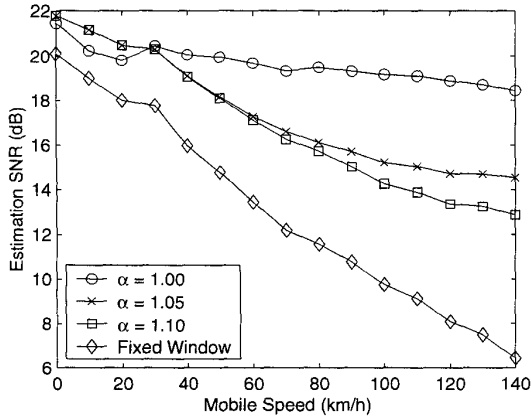


Fig. 2. Estimation SNR versus mobile speed for exponential profile ( $\text{SNR}_w = 20$  dB).

The normalized Doppler autocorrelation function of the channels used in the simulations is given by [14]

$$\rho_m[n - n'] = J_0\left(\frac{2\pi f_{\max}(n - n')}{f_s}\right), \quad m = 0, 1, \dots, M - 1 \quad (29)$$

where  $J_0(\cdot)$  denotes the zeroth-order Bessel function of the first kind and  $f_{\max} = v f_c / c$  is the maximum Doppler frequency for mobile speed  $v$ , carrier frequency  $f_c$ , and speed of light  $c$ . The carrier frequency and sampling frequency used in the simulations are  $f_c = 1.92$  GHz and  $f_s = 500$  kHz, respectively. The size of the signal block is chosen such that the duration of the signal block is less than the coherence time  $T_c$  for the mobile speeds of interest. For the Doppler correlation given by (29),  $T_c \approx 0.4 / f_{\max}$ ; thus, a signal block of  $N = 512$  samples has a block duration of 1.024 ms that is less than  $T_c$  for  $v < 220$  km/h.

Two multipath intensity profiles  $P[m]$  are considered: an exponential profile (truncated to  $M = 6$  samples) with r.m.s. delay spread of  $2 \mu\text{s}$  and a two-path profile with multipath spread  $T_m = 12 \mu\text{s}$ . The training sequence  $x[n]$  is generated using a pseudo-random binary sequence (PRBS) with polynomial  $1 + D + D^{16}$ , where  $D$  denotes a delay by one sample. The signal-to-additive noise ratio is defined by  $\text{SNR}_w = E[|x[n]|^2] / \sigma_w^2$ , where  $\sigma_w^2$  is the variance of  $w[n]$ . The duration of the smallest subintervals in the partition is chosen to be 0.128 ms, which corresponds to  $P = 3$ ; in the simulations, we let  $\bar{p} = P$ .

Fig. 2 is a plot of the estimation SNR versus mobile speed for the exponential multipath intensity profile. The results represent averages over 120 simulation runs with  $\text{SNR}_w = 20$  dB. The curve for the best basis channel estimator described in Section IV is labeled by  $\alpha = 1$ . The adaptive basis channel estimators that prefer larger subintervals are represented by the curves labeled by  $\alpha > 1$ . For purposes of comparison, a curve, labeled by "Fixed Window," is included in Fig. 2 for a channel estimator that uses smooth windowed Fourier transforms of the training sequence and the channel output over fixed intervals of 1.024 ms.

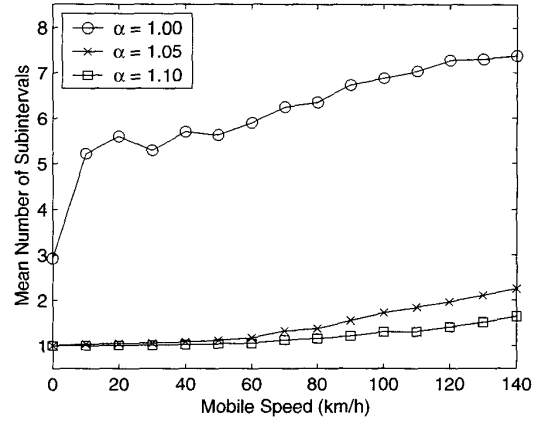


Fig. 3. Mean number of subintervals versus mobile speed for exponential profile ( $\text{SNR}_w = 20$  dB).

Fig. 2 indicates that among the estimation methods considered, the adaptive basis methods with  $\alpha > 1$  perform slightly better than the best basis estimator ( $\alpha = 1$ ) for speeds less than 30 km/h. This behavior is due to the fact that at low mobile speeds, the received signal is approximately stationary over larger subintervals. Therefore, channel estimators that use larger subintervals provide increased frequency resolution and improved noise averaging compared to estimators that use smaller subintervals. For higher mobile speeds, the best basis estimator performs the best since smaller windows can track the changes in the statistics of the received signal. In addition, the best basis estimator suffers the least performance degradation as the mobile speed increases. For the mobile speeds of interest, the adaptive basis estimators perform significantly better than the estimator that uses fixed windows.

For a given channel and as  $\alpha$  increases, the probability that a subinterval is an element of the partition  $\mathcal{P}^{\hat{\zeta}}$  increases for larger subintervals and decreases for smaller subintervals. As a consequence, the mean number of subintervals in  $\mathcal{P}^{\hat{\zeta}}$  decreases as  $\alpha$  increases. In addition, as the mobile speed increases, the mean number of subintervals in  $\mathcal{P}^{\hat{\zeta}}$  increases in order to track the channel time variation. Fig. 3 is a plot of the mean number of subintervals in  $\mathcal{P}^{\hat{\zeta}}$  versus mobile speed for various values of  $\alpha$ . For  $P = 3$ , the maximum number of subintervals is  $2^P = 8$ . Fig. 3 shows that as the mobile speed increases, the mean number of subintervals increases dramatically for  $\alpha = 1$  and slightly for  $\alpha > 1$ .

Fig. 4 is a plot of the estimation SNR versus signal-to-additive noise ratio ( $\text{SNR}_w$ ) for the exponential profile and for a mobile speed of 70 km/h. At high  $\text{SNR}_w$ , the estimation SNR saturates since the performance is dominated by the time variation of the channel.

In Fig. 5, the estimation SNR is plotted versus mobile speed for the two-path profile. The adaptive basis methods suffer a performance degradation for the two-path profile in comparison to the exponential profile results of Fig. 2. This effect is due to errors in estimating zero-valued taps in the time-varying

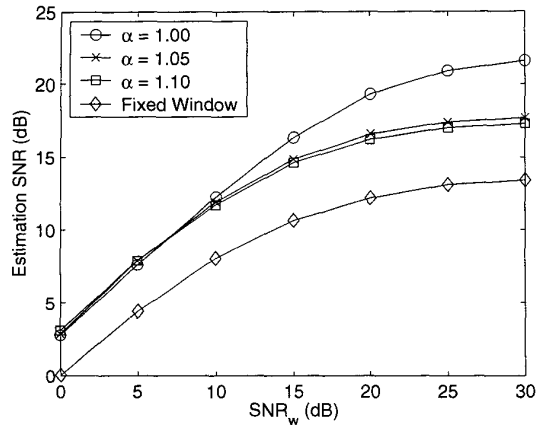


Fig. 4. Estimation SNR versus signal-to-additive noise ratio ( $SNR_w$ ) for exponential profile ( $v = 70$  km/h).

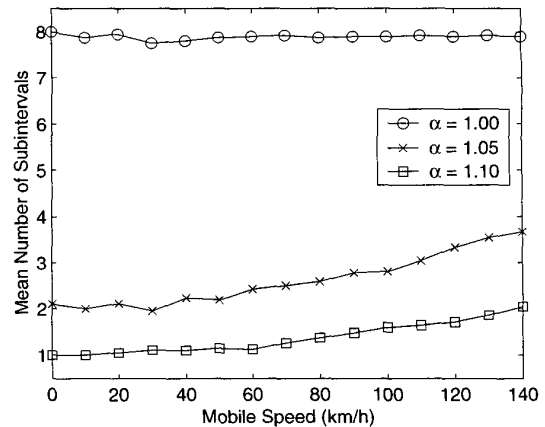


Fig. 6. Mean number of subintervals versus mobile speed for two-path profile ( $SNR_w = 20$  dB).

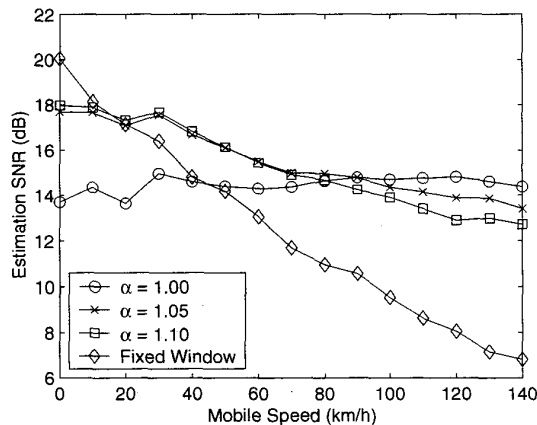


Fig. 5. Estimation SNR versus mobile speed for two-path profile ( $SNR_w = 20$  dB).

impulse response. Fig. 5 also shows that the adaptive basis estimators with  $\alpha > 1$  perform better than the best basis estimator for speeds less than 80 km/h. As shown in Fig. 6, this behavior is due to the fact that the smallest subintervals are almost always present in the best basis partition. The variation across disjoint subintervals in the errors in estimating the zero-valued taps causes the best basis algorithm to select the smallest subintervals even for low mobile speeds. The use of larger subintervals reduces this variation and yields better performance at low mobile speeds. As in the exponential profile case, the best basis method performs better than the other estimators for higher mobile speeds. The results of the estimation SNR versus  $SNR_w$  for the two-path profile and for the exponential profile are qualitatively similar.

## VI. CONCLUSION

A new technique using adaptive local exponential bases is presented for estimation of linear time-varying channels. The motivation for using such bases is the approximate eigenfunction property of smooth local complex exponentials. From a

library of bases, a best basis is selected such that the Hilbert-Schmidt error of the channel estimation is minimized. Other adaptive bases that prefer smooth local exponentials of longer duration are also studied to provide robustness to noise. Simulations for mobile radio channels demonstrate that for a large range of mobile speeds, adaptive basis techniques track the time-varying impulse response and perform significantly better than a method that uses a Fourier transform over windows of a fixed duration.

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