Effect of Channel Estimation Errors on Diversity-Multiplexing Tradeoff in Multiple Access Channels

Ravi Narasimhan
Department of Electrical Engineering
University of California, Santa Cruz, CA 95064, USA
Email: ravi@soe.ucsc.edu

***Abstract*— In this paper, multiple access channels are considered in which user data rates increase with the signal-to-noise ratio (SNR). To account for channel estimation errors, a finite-SNR diversity-multiplexing tradeoff framework is formulated. The multiplexing gain is defined for each user as the ratio of the user's data rate to the capacity of an additive white Gaussian noise (AWGN) channel. For a set of user multiplexing gains, a finite-SNR diversity gain is defined by the slope of the common outage probability as a function of SNR for quasi-static fading. These definitions have been used in previous work to characterize a finite-SNR diversity-multiplexing tradeoff in multiple access channels and point-to-point multiantenna channels. Inner and outer bounds on the feasible rate region of a multiple access channel with channel estimation errors are used to obtain estimates of the common outage probability. The outage probability is then used to estimate the diversity gain as a function of user multiplexing gains and SNR. It is shown that a finite-SNR formulation is essential to characterizing the impact of channel estimation errors on the diversity-multiplexing tradeoff.**

I. Introduction

The diversity and multiplexing gains of quasi-static multiantenna point-to-point channels [1] and multiple access channels [2] have been studied as the system signal-to-noise ratio (SNR) tends to infinity. These diversity-multiplexing tradeoffs quantify the diversity and multiplexing gains that can be achieved simultaneously. The high-SNR asymptotic formulation yields tractable results that can be interpreted using dimension counting arguments. However, several realistic propagation conditions, such as line-of-sight components and spatially correlated fading, do not affect the performance as the SNR approaches infinity. In addition, differences in path loss, shadowing, and transmit power in multiple access channels can be observed only at finite SNR. To account for these effects, finite-SNR formulations of diversity-multiplexing tradeoffs have been introduced for point-to-point multiantenna channels [3] and multiple access channels [4].

In this paper, the effect of channel estimation errors is studied on the diversity-multiplexing tradeoff in multiple access channels. In order to focus on channel estimation errors, the users and access point are each assumed to have a single antenna. Since channel estimation errors typically decrease as the SNR increases, a finite-SNR framework is adopted, as in [4]. Furthermore, as noted in [5], it is difficult to determine the exact capacity region of a multiple access channel with channel estimation errors. For this reason, the computations in this paper utilize the inner and outer bounds on the feasible rate region given in [5]. A quasi-static fading model is assumed such that the fading is constant for one coding block (or packet) and varies independently from packet to packet. The focus of this paper is on the common outage probability of the multiple access channel. With capacity-achieving codes applied per packet, the common packet error rate (PER) is equal to the common outage probability, defined as the probability that a rate vector lies outside the achievable region of the quasi-static fading multiple access channel.

The finite-SNR diversity-multiplexing tradeoff is computed using finite-SNR definitions of multiplexing and diversity gains. The finite-SNR multiplexing gain for each user is the ratio of the user's data rate to the capacity of an additive white Gaussian noise (AWGN) channel. Given a set of user multiplexing gains, the finite-SNR diversity gain of the multiple access channel is the negative slope of the common outage probability versus SNR on a log-log scale. The performance impact of channel estimation errors is studied through the resulting finite-SNR diversity-multiplexing tradeoff.

The remainder of the paper is organized as follows. Section II introduces the system model and definitions of finite-SNR multiplexing and diversity gains. Inner and outer bounds of the feasible rate region in a multiple access channel with channel estimation errors are also presented. Since exact calculation of the common outage probability in a general rate-adaptive multiple access channel is not tractable, upper and lower outage probability bounds are derived in Section III for both the inner and outer rate regions. Hence, four estimates of the outage probability are obtained, which are used to estimate the diversity gain as a function of the user multiplexing gains and SNR. Section IV presents numerical results illustrating the performance with channel estimation errors. Comparisons to high-SNR asymptotic results are also included. Conclusions are given in Section V.

II. System Model

A quasi-static fading multiple access channel with K users is considered. Each user has a single transmit antenna, and the access point has a single receive antenna. The channel between the i-th user and the access point is denoted by

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h_i. The vector of channel gains $\mathbf{h} = \begin{bmatrix} h_1 & \cdots & h_K \end{bmatrix}^T$ is assumed to be a zero-mean complex Gaussian random vector with identity covariance, corresponding to independent and identically distributed (i.i.d.) Rayleigh fading. The average received SNR at the access point is denoted by ρ. In this paper, each user is assumed to contribute an equal fraction $(1/K)$ of the average received SNR. Hence, the received signal y at the access point can be written as

$$y = \sqrt{\frac{\rho}{K}} \sum_{i=1}^{K} h_i x_i + n \tag{1}$$

where x_i is the signal from the i-th user and n is the AWGN with $E[|n|^2] = 1$. The inputs x_i are chosen to be i.i.d. Gaussian with $E[|x_i|^2] = 1$. Let R_i denote the target spectral efficiency (in bps/Hz) of the i-th user and $\mathbf{R} = \begin{bmatrix} R_1 & \cdots & R_K \end{bmatrix}^T$. With these definitions, the capacity region of the multiple access channel (conditioned on $\mathbf{h}$) with perfect channel estimation is given by

$$\mathcal{C}_{\text{MAC}} = \left\{ \mathbf{R} : \sum_{i \in \mathcal{S}} R_i \leq I(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \mathbf{h}), \forall \mathcal{S} \subseteq \{1, \ldots, K\} \right\} \tag{2}$$

where $\mathcal{S}^c$ is the complement of $\mathcal{S}$, $\mathbf{x}_{\mathcal{S}}$ and $\mathbf{x}_{\mathcal{S}^c}$ are vectors of transmitted symbols from the users in $\mathcal{S}$ and $\mathcal{S}^c$, respectively, and

$$I(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \mathbf{h}) = \log_2 \left(1 + \frac{\rho}{K} \sum_{i \in \mathcal{S}} |h_i|^2 \right) \tag{3}$$

is the relevant mutual information conditioned on the channel realization $\mathbf{h}$.

With channel estimation error, the channel for the i-th user is written as $h_i = \hat{h}_i + \tilde{h}_i$, where $\hat{h}_i$ and $\tilde{h}_i$ are the channel estimate and estimation error, respectively, with $E[|\hat{h}_i|^2] = \mathcal{E}_{\hat{h}}$ and $E[|\tilde{h}_i|^2] = \mathcal{E}_{\tilde{h}}$. Hence, the received signal in the presence of channel estimation errors can be written as

$$y = \sqrt{\frac{\rho}{K}} \sum_{i=1}^{K} \hat{h}_i x_i + \sqrt{\frac{\rho}{K}} \sum_{i=1}^{K} \tilde{h}_i x_i + n. \tag{4}$$

Inner and outer bounds on the achievable rate region are obtained using bounds on the mutual information conditioned on the channel estimates $\hat{\mathbf{h}} = \begin{bmatrix} \hat{h}_1 & \cdots & \hat{h}_K \end{bmatrix}^T$. As discussed in [5], an inner bound is obtained by considering the second term in (4) as additional i.i.d. Gaussian noise. By contrast, an outer bound is obtained by regarding the second term as contributing to additional signal power. Hence, inner and outer bounds on the feasible rate region are given by

$$\mathcal{C}_{\text{MAC,inner}} = \Big\{ \mathbf{R} : \sum_{i \in \mathcal{S}} R_i \leq I_L(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}), \forall \mathcal{S} \subseteq \{1, \ldots, K\} \Big\} \tag{5}$$

$$\mathcal{C}_{\text{MAC,outer}} = \Big\{ \mathbf{R} : \sum_{i \in \mathcal{S}} R_i \leq I_U(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}), \forall \mathcal{S} \subseteq \{1, \ldots, K\} \Big\} \tag{6}$$

where

$$I_L(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}) = \log_2 \left(1 + \frac{\frac{\rho}{K} \sum_{i \in \mathcal{S}} |\hat{h}_i|^2}{1 + \frac{\rho}{K} \sum_{i=1}^{K} \mathcal{E}_{\tilde{h}}} \right) \tag{7}$$

$$I_U(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}) = \log_2 \left(1 + \frac{\rho}{K} \sum_{i \in \mathcal{S}} (|\hat{h}_i|^2 + \mathcal{E}_{\tilde{h}}) \right). \tag{8}$$

Note that $G_S = \sum_{i \in \mathcal{S}} (|\hat{h}_i|^2 / \mathcal{E}_{\hat{h}})$ is a gamma random variable with parameters $(|\mathcal{S}|, 1)$, where $|\mathcal{S}|$ is the cardinality of $\mathcal{S}$. Hence, (7) and (8) can be rewritten as follows:

$$I_L(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}) = \log_2 \left(1 + \frac{\rho \mathcal{E}_{\hat{h}} G_{\mathcal{S}}}{K(1 + \rho \mathcal{E}_{\tilde{h}})} \right) \tag{9}$$

$$I_U(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}) = \log_2 \left(1 + \frac{\rho \mathcal{E}_{\hat{h}}}{K} G_{\mathcal{S}} + \frac{\rho \mathcal{E}_{\tilde{h}}}{K} |\mathcal{S}| \right). \tag{10}$$

Note that from (10), $I_U(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}})$ can have a large value even if $G_{\mathcal{S}}$ is near zero. In other words, by assuming the channel estimation error power contributes to signal power, the outer bound predicts a large achievable rate region even if the channel experiences a deep fade. In this sense, the outer bound is a loose bound on the true rate region with channel estimation error. An interesting area for future research is to investigate tighter outer bounds on the feasible rate region with channel estimation errors.

The common outage probability of the multiple access channel is defined as the probability that the target rate vector $\mathbf{R}$ lies outside the achievable rate region. Bounds on the outage probability can be computed using the inner and outer bounds of the rate region. For this computation, outage events $\mathcal{O}_{\mathcal{S},L}$ and $\mathcal{O}_{\mathcal{S},U}$ are defined as follows:

$$\mathcal{O}_{\mathcal{S},L} = \left\{ \hat{\mathbf{h}} \in \mathbb{C}^K : I_L(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}) < \sum_{i \in \mathcal{S}} R_i \right\} \tag{11}$$

$$\mathcal{O}_{\mathcal{S},U} = \left\{ \hat{\mathbf{h}} \in \mathbb{C}^K : I_U(\mathbf{x}_{\mathcal{S}}; y | \mathbf{x}_{\mathcal{S}^c}, \hat{\mathbf{h}}) < \sum_{i \in \mathcal{S}} R_i \right\} \tag{12}$$

where $\mathbb{C}^K$ is the K-dimensional complex vector space. Now, the common outage probability satisfies the following inequalities:

$$P_{\text{out}}^U \leq P_{\text{out}} \leq P_{\text{out}}^L \tag{13}$$

where

$$P_{\text{out}}^L = P\left(\bigcup_{\mathcal{S} \subseteq \{1, \ldots, K\}} \mathcal{O}_{\mathcal{S},L} \right) \tag{14}$$

$$P_{\text{out}}^U = P\left(\bigcup_{\mathcal{S} \subseteq \{1, \ldots, K\}} \mathcal{O}_{\mathcal{S},U} \right). \tag{15}$$

The common outage probability is used in diversity-multiplexing tradeoff analysis as follows. Multiplexing and diversity gains are traditionally defined in the limit as the SNR tends to infinity. For the high-SNR diversity-multiplexing tradeoff computed in [2], the target spectral efficiencies are $R_i \sim r_{i,\text{asymptotic}} \log_2 \rho$, where $r_{i,\text{asymptotic}}$ is the asymptotic multiplexing gain, $i = 1, \ldots, K$. For these target rates, the common diversity gain is $d_{\text{asymptotic}} =$

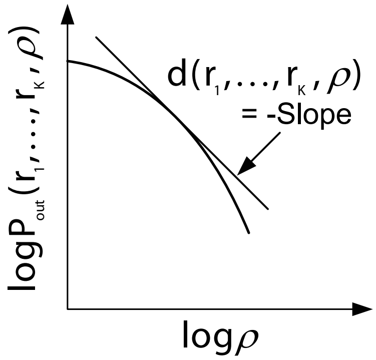

Fig. 1. Illustration of finite-SNR diversity gain.

$-\lim_{\rho\to\infty}(\log_2 P_{\mathrm{out}}/\log_2\rho)$. In the asymptotic regime, the diversity-multiplexing tradeoff for a single-antenna multiple access channel with perfect channel estimates can be computed using [2] and is given by:

$$d_{\mathrm{asymptotic}}(r_{1,\mathrm{asymptotic}},\ldots,r_{K,\mathrm{asymptotic}}) = \min_{\mathcal{S}\subseteq\{1,\ldots,K\}}\left[|\mathcal{S}|\left(1-\sum_{i\in\mathcal{S}} r_{i,\mathrm{asymptotic}}\right)^{+}\right] \tag{16}$$

where $(t)^+ = \max(0,t)$.

In this paper, the finite-SNR definitions given in [4] are used to determine the impact of channel estimation errors in multiple access channels. The finite-SNR multiplexing gain for the i-th user is defined by the ratio of R_i to the capacity of an AWGN channel at SNR ρ:

$$r_i = \frac{R_i}{\log_2(1+\rho)}\ ,\ i=1,\ldots,K. \tag{17}$$

The multiplexing gain r_i indicates the sensitivity to SNR of the i-th user's rate adaptation strategy. From (13) and (17), the common outage probability of the multiple access channel is a function of the user multiplexing gains and SNR, i.e., $P_{\mathrm{out}} = P_{\mathrm{out}}(r_1,\ldots,r_K,\rho)$. The common diversity gain as a function of SNR is defined by

$$d(r_1,\ldots,r_K,\rho) = -\frac{\partial \ln P_{\mathrm{out}}}{\partial\ln\rho} = -\frac{\rho}{P_{\mathrm{out}}}\cdot\frac{\partial P_{\mathrm{out}}}{\partial\rho}. \tag{18}$$

As illustrated in Fig. 1, the finite-SNR diversity gain is the slope of the outage probability curve at a particular SNR for a given set of multiplexing gains. Hence, this diversity gain can be used to estimate the change in SNR necessary to change the outage probability by a specified amount.

The finite-SNR framework discussed above and the inner and outer bounds of the achievable rate region are used in the next section to analyze the impact of channel estimation errors on the diversity-multiplexing tradeoff in multiple access channels.

III. Outage and Diversity Computations with Channel Estimation Errors

A. Outage Probability

The exact computations of P^L_{out} and P^U_{out} given in (14) and (15), respectively, are difficult in general. Hence, upper and lower bounds on these outage probabilities are derived as follows. For the upper bound, the union bound is used:

$$P^L_{\mathrm{out}} \le \sum_{\mathcal{S}\subseteq\{1,\ldots,K\}} P(\mathcal{O}_{\mathcal{S},L}) \tag{19}$$

$$P^U_{\mathrm{out}} \le \sum_{\mathcal{S}\subseteq\{1,\ldots,K\}} P(\mathcal{O}_{\mathcal{S},U}). \tag{20}$$

A lower bound is computed by determining the outage event with highest probability:

$$P^L_{\mathrm{out}} \ge P(\mathcal{O}_{\mathcal{S}_{m,L}}) \tag{21}$$

$$P^U_{\mathrm{out}} \ge P(\mathcal{O}_{\mathcal{S}_{m,U}}) \tag{22}$$

where $\mathcal{S}_{m,L} = \mathrm{argmax}_{\mathcal{S}\subseteq\{1,\ldots,K\}} P(\mathcal{O}_{\mathcal{S},L})$ and $\mathcal{S}_{m,U} = \mathrm{argmax}_{\mathcal{S}\subseteq\{1,\ldots,K\}} P(\mathcal{O}_{\mathcal{S},U})$. Note that from (13), the right hand sides of (19) and (22) are *bounds* on the outage probability P_{out} while (20) and (21) are outage probability *estimates*.

In order to study finite-SNR diversity-multiplexing tradeoffs, let the total multiplexing gain of all users in subset $\mathcal{S}$ be $r_{\mathcal{S}} = \sum_{i\in\mathcal{S}} r_i$. Now, consider the inner bound on the rate region. The probability of outage event $\mathcal{O}_{\mathcal{S},L}$ is given by

$$P(\mathcal{O}_{\mathcal{S},L}; r_1,\ldots,r_K,\rho) = P\left[\log_2\left(1+\frac{\rho\mathcal{E}_{\hat{h}}G_{\mathcal{S}}}{K(1+\rho\mathcal{E}_{\tilde{h}})}\right) < r_{\mathcal{S}}\log_2(1+\rho)\right] = \frac{\gamma(|\mathcal{S}|, a_{\mathcal{S},\rho})}{\Gamma(|\mathcal{S}|)} \tag{23}$$

where $\gamma(m,x) = \int_0^x t^{m-1}e^{-t}\,dt$ is the incomplete gamma function and

$$a_{\mathcal{S},\rho} = \frac{K(1+\rho\mathcal{E}_{\tilde{h}})[(1+\rho)^{r_{\mathcal{S}}}-1]}{\rho\mathcal{E}_{\hat{h}}}. \tag{24}$$

Similarly, one can show for the outer bound that

$$P(\mathcal{O}_{\mathcal{S},U}; r_1,\ldots,r_K,\rho) = \frac{\gamma(|\mathcal{S}|, b_{\mathcal{S},\rho})}{\Gamma(|\mathcal{S}|)} \tag{25}$$

where

$$b_{\mathcal{S},\rho} = \max\left[0, \frac{K}{\rho\mathcal{E}_{\hat{h}}}\left((1+\rho)^{r_{\mathcal{S}}} - 1 - \frac{\rho\mathcal{E}_{\tilde{h}}|\mathcal{S}|}{K}\right)\right]. \tag{26}$$

Note that the maximization in (26) ensures that the argument of the incomplete gamma function is nonnegative for small $r_{\mathcal{S}}$ in the presence of channel estimation errors. Upper and lower bounds on P^L_{out} and P^U_{out} are obtained by substituting (23) and (25) into (19), (20), (21) and (22).

B. Diversity Gain

Four estimates of the finite-SNR diversity gain can be obtained by substituting the right hand sides of (19), (20), (21) and (22) into (18). For instance, the diversity gain estimates corresponding to the inner bound on the rate region are given by:

$$\hat{d}_1^L(r_1,\ldots,r_K,\rho) = \frac{\sum_{\mathcal{S}\subseteq\{1,\ldots,K\}} \frac{a_{\mathcal{S},\rho}^{|\mathcal{S}|-1} e^{-a_{\mathcal{S},\rho}}}{\Gamma(|\mathcal{S}|)} \left(-\rho \frac{\partial a_{\mathcal{S},\rho}}{\partial \rho}\right)}{\sum_{\mathcal{S}\subseteq\{1,\ldots,K\}} \frac{\gamma(|\mathcal{S}|, a_{\mathcal{S},\rho})}{\Gamma(|\mathcal{S}|)}} \tag{27}$$

$$\hat{d}_2^L(r_1,\ldots,r_K,\rho) = \frac{a_{\mathcal{S}_{m,L},\rho}^{|\mathcal{S}_{m,L}|-1} e^{-a_{\mathcal{S}_{m,L},\rho}}}{\gamma(|\mathcal{S}_{m,L}|, a_{\mathcal{S}_{m,L},\rho})} \left(-\rho \frac{\partial a_{\mathcal{S}_{m,L},\rho}}{\partial \rho}\right). \tag{28}$$

Similar expressions for $\hat{d}_1^U(r_1,\ldots,r_K,\rho)$ and $\hat{d}_2^U(r_1,\ldots,r_K,\rho)$ are obtained by replacing $a_{\mathcal{S},\rho}$ with $b_{\mathcal{S},\rho}$ and $\mathcal{S}_{m,L}$ with $\mathcal{S}_{m,U}$ in (27) and (28), respectively. For the case of a symmetric multiplexing gain, $r_i = r,\ i = 1,\ldots,K$ and $r_{\mathcal{S}} = |\mathcal{S}|r$. Note that although the expressions (27) and (28) are more complicated than the corresponding asymptotic diversity gain, the analytical dependence on channel estimation error (discussed in Section III-C) provides insight beyond that available from Monte Carlo simulations.

C. Asymptotic and Limiting Behavior

It is insightful to study the asymptotic and limiting behavior of the diversity gain estimates derived in Section III-B. For this purpose, assumptions on $\mathcal{E}_{\hat{h}}$ and $\mathcal{E}_{\tilde{h}}$ are needed. For the remainder of the paper, linear minimum mean-square error (LMMSE) estimation is assumed with orthogonal training symbols to obtain channel estimates [6], [7]. Under this assumption, $\mathcal{E}_{\hat{h}} = 1 - \mathcal{E}_{\tilde{h}}$. In addition, let $\mathcal{E}'_{\tilde{h}} = \partial \mathcal{E}_{\tilde{h}}/\partial\rho$. With these definitions, the relevant quantities to compute the diversity gain estimates become

$$a_{\mathcal{S},\rho} = \frac{K(1+\rho\mathcal{E}_{\tilde{h}})[(1+\rho)^{rs}-1]}{\rho(1-\mathcal{E}_{\tilde{h}})} \tag{29}$$

$$\begin{aligned}-\rho\frac{\partial a_{\mathcal{S},\rho}}{\partial\rho} &= a_{\mathcal{S},\rho} - \frac{K}{1-\mathcal{E}_{\tilde{h}}}\Big[r_{\mathcal{S}}(1+\rho)^{rs-1}(1+\rho\mathcal{E}_{\tilde{h}}) \\ &\quad + (\mathcal{E}_{\tilde{h}} + \rho\mathcal{E}'_{\tilde{h}})[(1+\rho)^{rs}-1]\Big] \\ &\quad - \frac{K\mathcal{E}'_{\tilde{h}}(1+\rho\mathcal{E}_{\tilde{h}})[(1+\rho)^{rs}-1]}{(1-\mathcal{E}_{\tilde{h}})^2}\end{aligned} \tag{30}$$

$$b_{\mathcal{S},\rho} = \max\left[0, \frac{K}{\rho(1-\mathcal{E}_{\tilde{h}})}\left((1+\rho)^{rs} - 1 - \frac{\rho\mathcal{E}_{\tilde{h}}|\mathcal{S}|}{K}\right)\right] \tag{31}$$

$$\begin{aligned}-\rho\frac{\partial b_{\mathcal{S},\rho}}{\partial\rho} &= \frac{K}{\rho(1-\mathcal{E}_{\tilde{h}})}\left[(1+\rho)^{rs} - 1 - r_{\mathcal{S}}\rho(1+\rho)^{rs-1}\right] \\ &\quad - \frac{K\mathcal{E}'_{\tilde{h}}[(1+\rho)^{rs}-1]}{(1-\mathcal{E}_{\tilde{h}})^2} + \frac{\rho|\mathcal{S}|\mathcal{E}'_{\tilde{h}}}{(1-\mathcal{E}_{\tilde{h}})^2}, \text{if } b_{\mathcal{S},\rho} > 0.\end{aligned} \tag{32}$$

From LMMSE channel estimation, the following form of $\mathcal{E}_{\tilde{h}}$ versus SNR is used [6], [7]:

$$\mathcal{E}_{\tilde{h}} = \frac{1}{1+\alpha\rho} \tag{33}$$

where α depends on the training time and the training SNR. The high-SNR diversity gain estimates can be computed by taking the limits of (27), (28), and similar expressions for $\hat{d}_1^U$ and $\hat{d}_2^U$ as $\rho \to \infty$. For instance, it can be seen that terms with $r_S \geq 1$ tend to zero. For $0 < r_{\mathcal{S}} < 1$ and $\rho \to \infty$, $a_{\mathcal{S},\rho} \approx K(1+1/\alpha)/\rho^{1-r_S}$, $\gamma(|\mathcal{S}|, a_{\mathcal{S},\rho}) \approx \{K(1+1/\alpha)/\rho^{1-r_S}\}^{|\mathcal{S}|}/|\mathcal{S}|$, and $-\rho(\partial a_{\mathcal{S},\rho}/\partial\rho) \approx K(1+1/\alpha)(1-r_{\mathcal{S}})/\rho^{1-r_S}$. For the same asymptotic conditions, $\mathcal{S}_{m,L} \to \mathrm{argmin}_{\mathcal{S}\subseteq\{1,\ldots,K\}}[|\mathcal{S}|(1-r_{\mathcal{S}})^+]$. Thus,

$$\begin{aligned}&\lim_{\rho\to\infty} \hat{d}_2^L(r_1,\ldots,r_K,\rho) \\ &= \lim_{\rho\to\infty} \frac{|\mathcal{S}_{m,L}|}{\left(\frac{K(1+1/\alpha)}{\rho^{1-r_{\mathcal{S}_{m,L}}}}\right)} \cdot \left(\frac{K(1+1/\alpha)}{\rho^{1-r_{\mathcal{S}_{m,L}}}}\right)(1-r_{\mathcal{S}_{m,L}}) \\ &= |\mathcal{S}_{m,L}|(1-r_{\mathcal{S}_{m,L}}),\ 0 < r_{\mathcal{S}_{m,L}} < 1 \\ &= \min_{\mathcal{S}\subseteq\{1,\ldots,K\}}[|\mathcal{S}|(1-r_{\mathcal{S}})^+].\end{aligned} \tag{34}$$

From similar analysis, it can be shown that the same result is obtained for the asymptotic diversity gain estimates $\lim_{\rho\to\infty}\hat{d}_1^L(r_1,\ldots,r_K,\rho)$, $\lim_{\rho\to\infty}\hat{d}_1^U(r_1,\ldots,r_K,\rho)$, and $\lim_{\rho\to\infty}\hat{d}_2^U(r_1,\ldots,r_K,\rho)$. Note that the asymptotic diversity gain estimate (34) is exactly equal to (16). Since $\mathcal{E}_{\tilde{h}}$ does not appear in the high-SNR asymptotic region, finite-SNR analysis is needed to account for channel estimation errors.

Another limit of interest is the diversity gain as the symmetric multiplexing gain r tends to zero. A multiplexing gain of zero often, but not always (e.g., for finite SNR with line-of-sight components [8]), corresponds to the maximum diversity gain. For $\hat{d}_1^L$ and $\hat{d}_2^L$, as $r \to 0$, $a_{S,\rho} \approx K|\mathcal{S}|r\ln(1+\rho)[1+(1+\alpha)\rho]/(\alpha\rho^2)$ and $-\rho(\partial a_{S,\rho}/\partial\rho) \approx K|\mathcal{S}|r[(1+\rho)(2+(1+\alpha)\rho)\ln(1+\rho) - \rho(1+(1+\alpha)\rho)]/[\alpha\rho^2(1+\rho)]$. Furthermore, one can show that $|\mathcal{S}_{m,L}| \to 1$. Hence,

$$\lim_{r\to 0}\hat{d}_2^L(r,\ldots,r,\rho) = \frac{2+(1+\alpha)\rho}{1+(1+\alpha)\rho} - \frac{\rho}{(1+\rho)\ln(1+\rho)}. \tag{35}$$

The same result is obtained for $\lim_{r\to 0}\hat{d}_1^L(r,\ldots,r,\rho)$.

Note that for the case of perfect channel estimation at any SNR, we have $\alpha \to \infty$. Under this condition, (35) agrees with previous results that assume no channel estimation errors [9], [3], [4]. For the outer bound on the feasible rate region, note that a limiting diversity gain as $r \to 0$ is not meaningful since the channel estimation error variance contributes to nonzero mutual information irrespective of the fading realization. This observation highlights once again the fact that the outer bound (6) can be quite loose.

IV. Numerical Results

In this section, numerical results are given to illustrate the impact of channel estimation errors on the outage and diversity performance in multiple access channels. The results in this section correspond to a multiple access channel with $K = 2$ users. Since the inner bound (5) is a tighter approximation to the feasible rate region with channel estimation errors than the outer bound (6), results are given for the outage probability P_{out}^L. In Fig. 2, the outage probability P_{out}^L associated with the

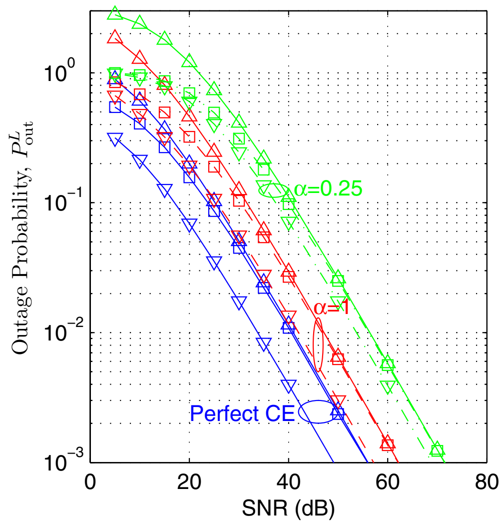

Fig. 2. Outage probability P^L_{out} of inner bound on rate region for symmetric multiplexing gain $r = 0.33$ and different channel estimation errors (parametrized by α). "Perfect CE" indicates perfect channel estimation. Legend: $\triangle \rightarrow$ upper bound (19), $\nabla \rightarrow$ lower bound (21), $\square \rightarrow$ exact simulation results.

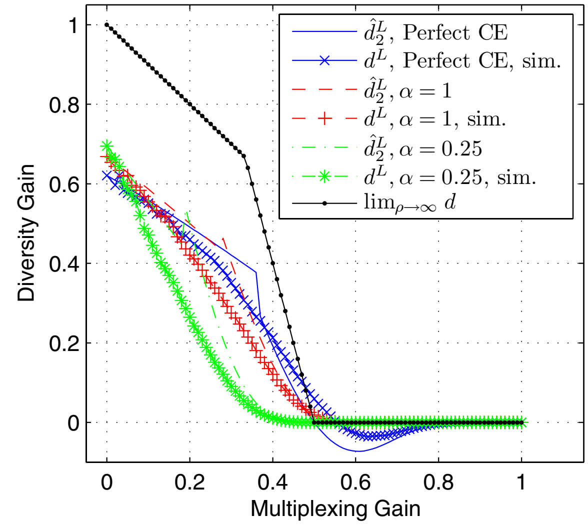

Fig. 3. Finite-SNR diversity-multiplexing tradeoff curves corresponding to inner bound on rate region, SNR = 10 dB, and different channel estimation errors. The actual diversity gains from Monte Carlo simulations are also plotted.

inner bound of the rate region is plotted for symmetric multiplexing gain $r = 0.33$. In Fig. 2, the upper and lower bounds (19) and (21) of P^L_{out} are compared to exact Monte Carlo simulations. The figure indicates significant loss in outage performance due to imperfect channel estimates. Furthermore, for low to moderate SNR with channel estimation errors, the union bound (19) can be greater than unity, while the lower bound (21) is closer to the actual outage probability.

Since the lower bound (21) is closer to the actual outage probability in the presence of channel estimation errors at low to moderate SNR, the diversity gain (for symmetric multiplexing gain) is estimated using $\hat{d}^L_2(r, r, \rho)$. This estimate is used to plot finite-SNR diversity-multiplexing tradeoff curves in Fig. 3 for SNR = 10 dB. Tradeoff curves obtained from Monte Carlo simulations are also plotted.

The jumps observed in $\hat{d}^L_2(r, r, \rho)$ for various channel estimation errors are due to the transition between the lightly and heavily loaded regimes [2]. The simulated diversity gains have smoother transitions at finite SNR. The nonzero diversity gain observed for $r \geq 1/2$ is due to slight changes in the outage probability as the SNR increases. The finite-SNR tradeoff curves are significantly lower than the asymptotic (high-SNR) tradeoff for $r < 0.4$. In addition, channel estimation errors decrease the achievable diversity even further for nonzero multiplexing gain.

V. Conclusion

A finite-SNR analysis of the diversity-multiplexing tradeoff is presented to evaluate the impact of imperfect channel estimation in rate-adaptive fading multiple access channels. Inner and outer bounds on the feasible rate region with channel estimation errors are used to derive estimates of the common outage probability. These bounds are used to estimate the diversity gain as a function of the user multiplexing gains and SNR. At realistic SNR and channel estimation quality, the diversity-multiplexing tradeoff is significantly lower than asymptotic results. These finite-SNR results can be used to predict multiple access performance achievable in practical systems with channel estimation errors.

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