ESTIMATION OF THE NONSTATIONARY MEAN SIGNAL IN WIRELESS SYSTEMS USING WAVELETS

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Abstract

A new technique is described for estimating the nonstationary mean signal received at a mobile station in a Rayleigh fading environment. The estimate is based on samples taken at the midpoints between the local minima of the received envelope. The continuous wavelet transform is used to estimate the local minima. An estimate of the mean signal is obtained using a fixed number of local minima. This technique requires neither an estimate of the mobile speed nor an adaptive temporal averaging window, in contrast to other estimators. Simulations show that the mean signal is estimated well in a nonstationary environment with variable mobile speed.

1 Introduction

Accurate estimates of the local mean of the received signal are required to increase the realized system capacity by improving the performance of handoff, power control and channel assignment algorithms. In many environments, a direct path is not present between the base station and the mobile station. The received signal consists of a sum of waves that have been reflected by objects such as mountains, trees, and buildings. The sum of many waves at the receiver gives rise to small-scale spatial variation of the received envelope (on the order of a wavelength). The received signal is nonstationary for distances on the order of building sizes since the mean of the small-scale variation changes considerably. This large-scale variation of the mean is known as shadowing. The mean of the shadowing also decreases as the distance between the base station and the mobile station increases.

The mean signal is the local mean of the small-scale variation (up to a constant) and represents the distance-dependent trend and shadowing. The most widely used estimate of the mean signal is the average of samples of the received envelope (or logarithm of the envelope) taken at a constant temporal interval. An adaptive method to estimate the mean signal is proposed in [1] where the squared deviations of the logarithm of the envelope are used to estimate the maximum Doppler frequency. The maximum Doppler frequency is then used to adapt the number of signal samples that are averaged to estimate the mean signal. The literature has considered only the problem of a constant mobile speed. For variable mobile speed, the number of temporal signal samples used in the estimate of the mean signal must be constantly adapted, and the rate of adaptation is critical to the performance of the estimator. In particular, errors in the estimate of the maximum Doppler frequency as described in [1] could magnify errors in the estimate of the mean signal due to suboptimal temporal averaging windows.

A new method of estimating the mean signal using wavelets is described in this paper. The wavelet transform at different scales provides a variety of window lengths and, hence, eliminates the necessity of adapting the duration of a single temporal averaging window. The method presented here utilizes the fact that the small-scale spatial variation of the received envelope is dominated by the positions of the mobile and base stations. This spatial variation has a characteristic scale that is on the order of a carrier wavelength. By averaging over a fixed number of samples taken at the midpoints between the local minima of the received envelope, one obtains an estimate of the mean signal over a spatial scale that is selected based on the variation of the distance-dependent trend and the shadowing.

The paper is organized as follows. In Section 2, wireless propagation and noise models are presented. Section 3 presents a method to estimate the local minima of the received envelope using the continuous wavelet transform (CWT). Signal samples taken at the midpoints between the local minima are then used to estimate the mean signal. Section 4 addresses the selection of parameters used in the estimate of the mean signal and compares the performance with the adaptive averaging method of [1] for constant and variable mobile speed. Conclusions are given in Section 5.

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2 Wireless Propagation and Noise Models

The propagation model discussed here takes into account three effects which are present in many wireless environments: correlated Rayleigh fading, correlated log-normal shadowing, and a distance-dependent trend [2]. The received bandpass signal at a mobile consists of a sum of contributions from several paths. We denote $p(\mathbf{x},\mathbf{x_B})$ to be the received power at the mobile station averaged over a small neighborhood of $\mathbf{x}$ due to a base station located at $\mathbf{x_B}$. Let λ represent the carrier wavelength and $r_c(\mathbf{x})$ denote the complex received envelope. A model for $r_c(\mathbf{x})$ is constructed using a weighted sum of plane waves arriving at different angles.

The average received power $p(\mathbf{x},\mathbf{x_B})$ contains the distance-dependent trend and the log-normal shadowing [3]. Let α represent the exponent of the distance-dependent trend. Furthermore, let $10^{L(\mathbf{x},\mathbf{x_B})/10}$ denote the log-normal shadowing between the mobile position $\mathbf{x}$ and the base station position $\mathbf{x_B}$. The received power averaged over a neighborhood of $\mathbf{x}$ due to the base station located at $\mathbf{x_B}$ can then be expressed as

$$p(\mathbf{x},\mathbf{x_B}) = P_0 \,||\mathbf{x}-\mathbf{x_B}||^{-\alpha}\, 10^{L(\mathbf{x},\mathbf{x_B})/10} \tag{1}$$

where P_0 accounts for antenna parameters, transmitted power, and other relevant system parameters. The process $L(\mathbf{x},\mathbf{x_B})$ is modelled as a zero-mean, exponentially correlated Gaussian random process that is wide-sense stationary in the variable $\mathbf{x}$. The received envelope $|r_c(\mathbf{x})|$ is Rayleigh distributed in the absence of noise.

The noise model developed here is based on a receiver model for a digital wireless system in which the envelope of the signal is recovered as follows. White Gaussian noise $n(t)$ is added to the channel output. The result is passed through an ideal bandpass filter (BPF) centered at carrier frequency f_c with bandwidth $2/T$, where T is the symbol duration of the digitally modulated waveform. The signal is then passed through a square-law device followed by a unit-gain, square-root raised cosine lowpass filter (LPF) with bandwidth $2f_m(1+\gamma)/(1-\gamma)$. The maximum Doppler frequency is f_m, and the excess bandwidth factor is γ $(0<\gamma<1)$. The square root of the resulting signal is the envelope $r(t)$.

Let the noise $n_{BPF}(t)$ at the output of the bandpass filter be represented by in-phase and quadrature components: $n_{BPF}(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$. The components $n_I(t)$ and $n_Q(t)$ each have variance σ_n^2. The curve Γ traversed by the mobile station is parametrized by the scalar position variable $x(t)$. Suppose that the mobile travels along Γ with a speed $v(t)$ at time t. Under a narrowband assumption, the square of the received envelope, $r^2(t)$, as a function of t is then

$$r^2(t) \approx |r_c(\Gamma[x(t)])|^2 + LPF\{n_I^2(t)+n_Q^2(t)\} \tag{2}$$

where LPF denotes lowpass filtering with the square-root raised cosine filter and $x(t) = \int_0^t v(t')\,dt'$. The time origin is chosen such that $x(0)=0$. We define $b_0 \equiv p(\mathbf{x},\mathbf{x_B})$ and an input signal-to-noise ratio (SNR) as $SNR_i \equiv b_0/\sigma_n^2$. The logarithm of $r(t)$ is $f(t) = 20\log_{10} r(t)$. The model presented here is used in Section 3 to estimate the mean signal.

3 Estimation of Mean Signal Using CWT

The estimation technique presented here utilizes the characteristic spatial scale of correlated Rayleigh fading. In the following, we consider the case of two-dimensional isotropic scattering such that the incoming power is uniformly distributed in azimuth angle [2]. Fig. 1 plots a typical (noiseless) signal trace as a function of mobile position (measured in wavelengths). This plot shows that the local minima of the signal occur with a separation of a fraction of a wavelength. The mean separation in distance between the local minima of the noiseless envelope is calculated in [4] to be approximately 0.662λ for the model stated above.

Let r_i^{mid} and $r_i^{\mathrm{mid,nl}}$ denote signal samples taken at the midpoints between the local minima of the noisy and noiseless envelopes, respectively. Let the mean of $r_i^{\mathrm{mid,nl}}$ be represented by $E\{r_i^{\mathrm{mid,nl}}\} = \mu\sqrt{b_0}$ where μ is a constant determined in Section 4 and $b_0 = b_0(t)$. An estimate of the mean signal is then

$$\widehat{\sqrt{b_0(t)}} = \frac{1}{\mu}\left(\text{local average}\{r_i^{\mathrm{mid}}\}\right) \tag{3}$$

where the local average is taken in a neighborhood of time t. A method to estimate r_i^{mid} using the CWT is described in the following.

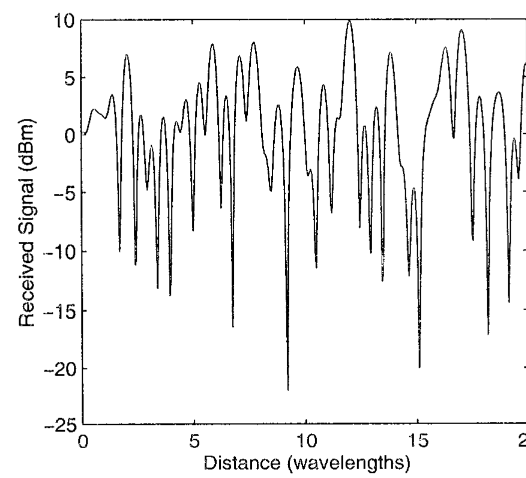

Figure 1: Typical signal trace as a function of distance.

The CWT of a function $h(t) \in L_2(\mathcal{R})$ is given by

$$CWT_h(a,b) \equiv \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi\left(\frac{t-b}{a}\right) h(t)\, dt \qquad (4)$$

where $a \in \mathcal{R}^+$ denotes "scale" and $b \in \mathcal{R}$ denotes "shift". The "mother wavelet" $\psi(t) \in L_2(\mathcal{R})$ is a zero-mean, real function. The CWT has the important property of characterizing singularities of the signal [5, 6, 7]. Since many of the local minima of $f(t)$ correspond to points of discontinuity in the first derivative, the CWT is used to detect the local minima[1]. Detecting the local minima using the CWT permits the tracking of a variable mobile speed profile without requiring an adaptive window. In contrast, other function minimization methods using a three-point bracketing scheme require adaptation of the bracketing time since the time between local minima varies as the speed varies. As discussed in detail in Section 4, the CWT of $f(t)$ is taken over a suitable temporal window of observation to obtain mean signal estimates within an acceptable delay for real-time implementation. The CWT is taken at a discrete set of scales $a = 2^{l+m/M}$, $l = l_{\min}, l_{\min}+1, \ldots, l_{\max}$, $m = 0, \ldots, M-1$ where $l_{\min}$, $l_{\max}$ and M are integers. In order to compare the CWT with a significance threshold which is independent of mobile speed, the CWT of $f(t)$ at each scale a is normalized by $1/\sqrt{a}$ before further processing.

One method to detect the points of discontinuity in the first derivative of $f(t)$ is to identify the wavelet transform modulus maxima (WTMM) [6]. The number of WTMM associated with a singularity of $f(t)$ depends upon the number of local extrema of the analyzing wavelet $\psi(t)$. The number of local extrema of the wavelet is at least one plus the number of vanishing moments of the wavelet. Furthermore, the analyzing wavelet must have at least two vanishing moments in order to characterize points of discontinuous derivative of $f(t)$.

The singularities of interest for mean signal estimation are the points with discontinuous derivative that are also local minima of $f(t)$. This fact can be used to reduce the number of extraneous WTMM by using a modified version of the WTMM detection method as described in the following. For wavelets $\psi(t)$ which satisfy $\sup_t \psi(t) \geq -\inf_t \psi(t)$, the *negative local minima* at each scale of the normalized CWT are identified; otherwise, the *positive local maxima* at each scale are identified. The values identified as above are referred to as the *signed local extrema* of the normalized CWT. The benefit of using the signed local extrema instead of the modulus maxima is that for a suitable analyzing wavelet $\psi(t)$, the local minima of $f(t)$ with discontinuous derivative do not produce extraneous signed local extrema. In contrast, each singularity of $f(t)$ gives rise to extraneous modulus maxima using the WTMM method. Therefore, use of the signed local extrema removes the need to isolate the modulus maxima from the extraneous ones. The signed local extrema with absolute value less than a significance threshold τ are discarded. The scale which has the highest number of significant signed local extrema is identified as the scale of interest and is denoted by a_τ.

Since most of the local minima of $f(t)$ correspond to points of discontinuity in the first derivative, the locations in time of the signed local extrema of the normalized CWT correspond to most of the locations of the local minima of $f(t)$ (and, hence, the local minima of $r(t) = 10^{f(t)/20}$). To compensate for the small fraction of local minima of $f(t)$ with continuous derivative, the significance threshold τ is determined empirically such that the number of local minima of $f(t)$, which are detected by the CWT, is equal to the total number of local minima of $f(t)$. The selection of τ is discussed in more detail in Section 4.

We now consider $(N+1)$ significant signed local extrema occurring at times $t_0, t_1, \ldots, t_N$. The signal samples taken at the midpoints between the local minima of $r(t)$ are estimated by the signal samples taken at the midpoints between the signed local extrema of the CWT, *i.e.*, $\hat{r}_i^{\text{mid}} = r(\tilde{t}_i), i = 0, 1, \ldots, N-1$, where $\tilde{t}_i = (t_i + t_{i+1})/2$. The estimate (3) for $\sqrt{b_0(t)}$ at time $(\tilde{t}_0 + \tilde{t}_{N-1})/2$ then becomes

$$\widehat{\sqrt{b_0((\tilde{t}_0 + \tilde{t}_{N-1})/2)}} = \frac{1}{N\mu} \sum_{i=0}^{N-1} \hat{r}_i^{\text{mid}}. \qquad (5)$$

The next estimate of the mean signal is $\widehat{\sqrt{b_0((\tilde{t}_1 + \tilde{t}_N)/2)}}$, which is obtained by considering a time interval containing a new signed local extremum and the N most recent signed local extrema from the previous time interval. The estimates obtained in this manner are smoothed by a moving average. The following section describes the selection of parameters together with simulations that apply the estimation technique.

4 Parameter Selection and Results

This section determines the mean of the signal samples taken at midpoints between the local minima of the received envelope and selects the significance threshold (τ) and the number (N) of midpoint values used. The choice of the maximum scale ($a_{\max}$), the duration (T_{obs}) of the observation window over which the CWT is taken, and associated boundary effects are also described. The observation window is required to obtain estimates within an acceptable delay for real-time implementation. The performance of the

[1] The local minima of the envelope $r(t)$ correspond to the local minima of the logarithm of the envelope $f(t)$. The noiseless version of $f(t)$ is related to the signal of Fig. 1 through the mobile speed $v(t)$.

mean signal estimator is then compared to the adaptive averaging method described in [1] for constant and variable mobile speed.

The mean of the signal samples taken at the midpoints between the local minima is determined by averaging over 90 independent realizations of the Rayleigh fading process with constant b_0 and with around 1000 local minima per realization. The resulting value is $E\{\hat{r}_i^{\max,\mathrm{nl}}\} = \mu\sqrt{b_0} \approx 1.66\sqrt{b_0}$. The effect of noise on this mean value was also investigated. Using an excess bandwidth of $\gamma = 0.5$ for the lowpass filter, it is found that for $SNR_i > 15$ dB, the mean value converges to the noiseless value of $1.66\sqrt{b_0}$.

The threshold τ is selected such that the number of signed local extrema that are detected at scale a_τ is equal to the number of positive-slope zero crossings of the derivative $f'(t)$ in the absence of noise. The threshold τ depends on the analyzing wavelet $\psi(t)$. The wavelet chosen is a "coiflet" of order 2 (coif1) [5]. This wavelet has two vanishing moments and yields the best results among the five different wavelets considered in [4]. The optimum threshold of $\tau = 0.474$ for the coif1 wavelet was determined by averaging over 90 independent realizations of the Rayleigh fading process.

The normalized bias of the mean signal estimate, $E\{\widehat{\sqrt{b_0}}/\sqrt{b_0} - 1\}$, as a function of the number (N) of midpoint samples is studied for various values of SNR_i. The mean signal power b_0 and noise variance σ_n^2 are kept constant for each value of SNR_i to obtain results under a controlled environment. The bias is found to be negligible for $N > 4$ and $SNR_i > 15$ dB. The normalized mean square error (MSE) of the mean signal estimate, $E\{[\widehat{\sqrt{b_0}}/\sqrt{b_0} - 1]^2\}$, decreases as $1/N$ for large N.

The CWT is applied to blocks of the signal collected over time periods of duration T_{obs} in order to limit the delay in obtaining mean signal estimates for real-time implementation. The determination of T_{obs} is described in the following. Let the lowest speed of interest be denoted by $v_{\min}$, *i.e.*, speeds less than $v_{\min}$ will be regarded as zero. If L_ψ denotes the nominal duration of support of the analyzing wavelet $\psi(t)$, the maximum scale for the CWT is $a_{\max} = 0.662\lambda/(L_\psi v_{\min})$. The duration of the observation window is then the mean time between the local minima of the signal for a mobile speed of $v_{\min}$, *i.e.*, $T_{\mathrm{obs}} = 0.662\lambda/v_{\min}$. The finite observation window results in boundary effects in the detection of the signed local extrema of the CWT. In order to remove these effects, the observation windows are enlarged by $T_{\mathrm{obs}}/2$ for each boundary (left and right endpoints). The enlarged windows slide by an amount T_{obs} as time progresses. For the initial left boundary and the final right boundary, the signal $f(t)$ is reflected about the boundaries such that $f(t)$ and the first derivative $f'(t)$ are continuous at the boundaries. This technique of overlapping windows results in a factor of two increase in computation for the CWT and a $T_{\mathrm{obs}}/2$ increase in delay of the mean signal estimate.

The mean signal estimator presented here is compared with the adaptive averaging method described in [1]. The adaptive averaging technique uses the squared deviations of $f(t)$ to estimate the maximum Doppler frequency f_m. The estimate of f_m is used to determine the number of samples of $f(t)$ that are averaged to estimate $10\log_{10} b_0(t)$. Fig. 2 compares the normalized bias of the two methods for a constant mobile speed. The spatial averaging distance for both methods is 20λ, in accordance with the results given in [8]. The spatial averaging distance of 20λ implies that the number of midpoint samples used in (5) is $N = 20/0.662 \approx 30$. For the adaptive averaging method, the number of samples of $f(t)$ corresponding to 20λ depends on the current estimate of f_m. Fig. 3 plots the normalized MSE of the two estimation methods. It can be seen that the mean signal estimator using wavelets performs better over a large range of useable SNR_i.

The performance improvement of the wavelet method over the adaptive averaging method is vivid for a variable mobile speed profile. Fig. 4 plots the error (in decibel) between the estimate and the true mean signal power for the two estimation methods using a variable speed profile. The duration of the moving average for smoothing of the estimates obtained in (5) is chosen to be T_{obs}. The relevant parameters for this example are: $v_{\min} = 1.8$ km/h, $N = 30$, $M = 6$, carrier wavelength $\lambda = 1/3$ m, correlation length of the log-normal shadowing $d_0(\mathbf{x_B}) = 50$ m, standard deviation of the log-normal shadowing $\sigma_L = 10$ dB, and exponent of distance dependence $\alpha = 4$.

The estimation error in both methods at the left and right

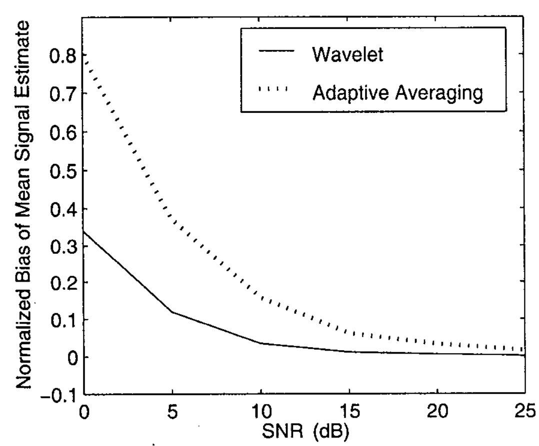

Figure 2: Normalized bias versus SNR_i for wavelet and adaptive averaging methods.

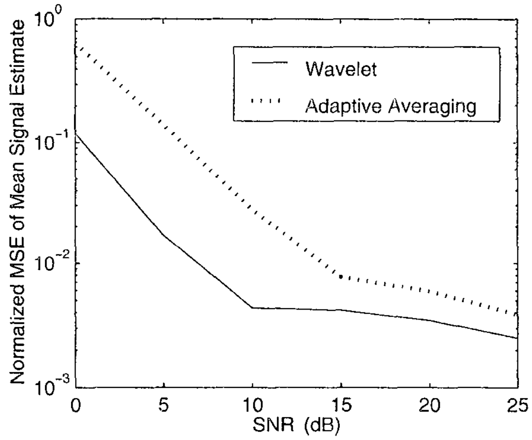

Figure 3: Normalized MSE versus SNR_i for wavelet and adaptive averaging methods.

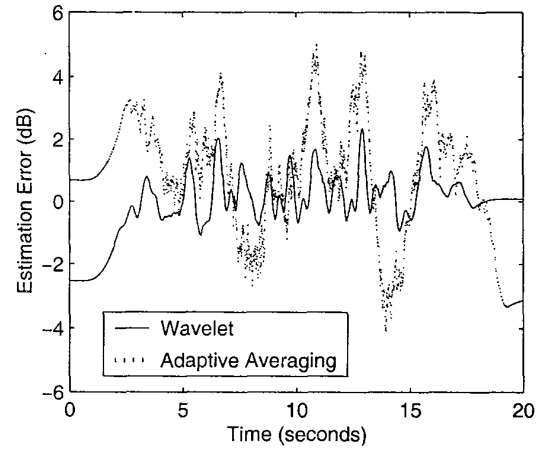

Figure 4: Estimation error for wavelet and adaptive averaging methods using variable speed profile.

boundaries of Fig. 4 is due to a mobile speed of zero at those locations. At very low mobile speeds, any method to estimate the mean signal will have significant errors since a sufficient number of samples of the Rayleigh fading process is not available within a given time interval to obtain accurate estimates. An advantage of the wavelet method is that the method can detect the presence of local minima in the signal. If no minima are detected, one concludes that the mobile speed is very low (less than $v_{\min}$), and hence no system action (*e.g.*, handoff or channel assignment) need be taken. Fig. 4 shows that the mean signal estimator using wavelets performs significantly better than the adaptive averaging method for variable mobile speed.

5 Conclusion

The paper presents a new technique for estimating the mean signal in a Rayleigh fading environment. A wireless propagation model is used which accounts for correlated Rayleigh fading, correlated log-normal shadowing, and a distance-dependent trend. A noise model is also described which takes into consideration the signal processing required to obtain the received envelope. The mean signal is estimated by computing a local average of the signal samples taken at the midpoints between adjacent local minima of the received envelope. With an empirically determined significance threshold, the significant signed local extrema of the CWT are used to detect the local minima. This estimation technique is robust under variable mobile speeds and performs better than an adaptive averaging method described in the literature.

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