3D Physics-Based Fabric Rendering

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1 Introduction to Fabric Modeling

The question of how to model cloth has as many useful applications as there are ways to answer that question. From rendering with more realism in video games and movies, to streamlining the garment industry, to the invention of entirely new woven materials, a computer-generated and physics-based method of modeling cloth can prove indispensable. With a great variety of practical uses, it is no wonder that the problem of modeling fabric with physics-based solutions has been about long before there were computers to run a 3-D rendering. The problem of formulating a mathematical model of this physical phenomenon is very complex, and the difficulty of solving the problem can be seen in the wide variety of solutions that are used to solve it. Analysis of fabric mechanics begins with taking measurements of fabric properties. Models are then built in an attempt to recreate those properties.

In the physical world, fabric is a continuous surface, without macroscopically discernible segments. However, in the realm of computation, calculating the infinite number of points that exist in any continuous dimension is impossible. Well, it's possible to try, but as finite beings we cannot wait for an infinite loop to complete. In order to model our continuous world, approximations must be made. The fabric model must be broken into a series of discernible points at which the modeling calculations are to be performed. Each of these points, hence forth “nodes”, needs to contain the following information about the fabric: location in space, vector of the forces acting upon it, and its physical characteristics for the space it represents (mass, friction coefficient, and stretchiness).

Location in space is where the point lies in three dimensions. Much like in our view of reality, for the fabric there exists the dimensions of width, height, and depth— often referred to in mathematics as the x, y, and z axis. Forces acting upon a node consist of forces from the outside world and forces from other fabric nodes. The different forces are summed together to create a net force which changes the location of the node over time. The physical characteristics of a node influence the strength of interacting forces. The mass of the node affects the force of gravity, the stretchiness influences the inter-fabric interactions, and the friction coefficient affects how readily fabric slides against itself and other objects. In a typical simulation, the fabric will be modeled over time. While the end result may be captured in a single moment of time, the vector forces and position of the fabric nodes need to be calculated over multiple discrete units of time. Note that the location in space and force vector can change over time, but the mass and stretch characteristics of the fabric nodes will remain constant.

2 Interpolation

Many forces calculated do not interact exactly on a fabric node. Gravity is perhaps the easiest force to calculate, as the mass at a node is really the mass of the area of fabric which the node represents. However forces stemming from collisions will usually happen in the planes between nodes. In this case, interpolation is used to calculate the forces at each of the corners of the plane. Linear interpolation, while simple, adequately calculates the force at any point on the fabric [1]. This is because fabric's properties are uniformly distributed throughout its area. To find a value, say force, between any two points, each with a known force, is as simple as using the relative distance from the unknown force to the two known forces to calculate the relative influence of the known forces on the unknown. To find the value in two dimensions, repeat the one-dimensional procedure for the additional second dimension. This technique of repeating interpolation extends quite well to an arbitrary number of dimensions [2].

$$d_x = d_i + s * (d_{i+1} - d_i)$$
$$d_{x,y} = d_{x,j} + t * (d_{x,j+1} - d_{x,j})$$

Figure 1: Bilinear interpolation [Interactive Data Visualization]. A series of linear interpolations is performed to calculate the final value at a given point ($d_{x,y}$) surrounded by points of known value ($d_{i,j}$), ($d_{i+1,j}$), ($d_{i,j+1}$), and ($d_{i+1,j+1}$). In this case, the value in the x dimension (d_x) is calculated first by scaling (to get "s") the x values at $d_{i+1,}$ and d_i by the percentage of the way d_x is between the two. The second equation shows this process repeated in the y dimension. The x value has already been calculated, so only the y value is set to change.

Fabric, although existing in three dimensions, relies primarily on two-dimensional interpolation. Most fabric modeling techniques make the approximation of fabric as a two-dimensional object because of the plate-shell theory[1]. In the case that the fabric planes are represented by three points (as triangles), linear interpolation is the sum of the three corner values scaled by the distance to each.

$$d_{x,y} = (d_{corner\ 1} * \Delta_1) + (d_{corner\ 2} * \Delta_2) + (d_{corner\ 3} * \Delta_3)$$

Figure 2: Linear interpolation [2] of a point inside a triangle. Where $d_{x,y}$ is the location of the value to be found, $d_{corner\ 1,2,3}$ is the value at each corner, and $\Delta_{1,2,3}$ is the scalar distance from that corner to $d_{x,y}$.

2 The Forces at Work

When rendering fabric there are essentially six types of forces at work. Inter-fabric interactions, inter-fabric collision reactions, inter-fabric friction, external forces, external collision reactions, and external friction. Inter-fabric interactions are those discussed in the *Rod Elements and Bending Springs* section (spring force model, plate-shell theory, etc.). Inter-fabric and external collision reactions are the forces that result when fabric collides with other fabric and with other objects, respectively. External forces refers to the interactions with other forces in the scene, such as gravity (a single-directional force vector), or any wind field that may be present. External and fabric-friction are differentiated to provide a compartmentalized friction coefficient for the different materials that may be present.

1Plate theory is used to model 3-dimensional objects with a small thickness compared to their other two dimensions. Because of the relative difference in thickness, the plate theory mechanics reduce the plates into two-dimensional planes and decrease computational complexity[9].

2.1 Inter-fabric Collisions

Inter-fabric collisions are the most problematic forces to deal with. To realistically represent the complex shapes and folds of real-life fabric behavior, fabric "nodes" need to be present in the order of hundreds of thousands, or better yet millions. The nature of the nodes is to be in close proximity with one another, which gives the fabric the tendency to intersect and "pass through" itself. These intersections must be prevented with collision detection and reactions in order to persevere the illusion of a solid form. This leads to a very complex shape that is computationally intense to calculate, as multiple iterations of calculations must be performed upon the detection of a collision [1].

The variations of detecting collisions stem from that need to re-calculate the fabric positions after the collision behavior has been accounted for. The most simple to understand collision detection method is brute force. For every node in the fabric, check to ensure that every other node in the scene does not collide with it for every interval of time (using a simple geometry method such as point-plane intersection). This leads to calculations that are far too costly for rendering fabric of a reasonable level of detail: a sheet of fabric with 100 nodes would require 10,000 checks of collision detection. This number does not include the additional processing that comes into effect when a collision is detected.

The brute force method may work for simple world models with few defined boundaries, but is impractical when dealing with the number of nodes needed for fabric. "A single time step may have thousands of collisions and contacts […] characteristic of highly deformable bodies like cloth"[4]. These thousands of collisions make processing one collision point at a time far too slow for any practical application. A method proposed by Birdson, et al. is to first apply "repulsion forces" to cloth to prevent most collisions from happening. Repulsion forces come into effect when points of fabric are within an approximate "width" of the fabric (eg, 1mm).

To determine the proximity of fabric nodes, Birdson et al. checks both the proximity of fabric "triangles" (composed of three adjacent nodes), and "edge pairs" (two edges of two nodes each). Two types of proximities are needed to calculate two types of interaction forces. The first proximity (triangle) determines the presence of an inelastic collision force: the two triangles colliding represent surfaces that hit each other. The inelastic force not only models a realistic interaction of two surfaces bouncing off each other, but also saves the computational load of re-iterating needed by a spring force.

$$F_s = -k_s(d_o - d_f)$$

Figure 3: Hooke's Law of Elasticity. F_s is the calculated spring force , d_o is the rest length, and d_f is the current length, and K_s is the "spring constant", a relative term reflecting the elasticity of the material (must be calculated for different materials).

The second proximity (edge pairs) determines the presence of a "spring-based force". This spring-based force models the compression across the diameter of the fabric, coming into effect only when the fabric particles overlap more than the "width" of the fabric. As is characteristic of springs, the force is proportional to the change from rest width (ie, the standard fabric thickness). Note that this force needs to be capped at a maximum when the objects touch [4].

2.2 External Collisions

While still complex, external collisions (fabric node and world object) are simpler than fabric collisions. If the world object is immobile, it is a simple matter of reflecting the incoming fabric force back at the node of fabric. If the world object can move, the weight of the fabric may cause it to slide (absorbing some of the force). However, both of these cases lack the more complex "springy" interactions that inter-fabric collisions must calculate.

External forces in general are all forces that act upon the fabric that are not directly from other fabric nodes. Excluding the more specific external collisions and external friction, external forces act upon fabric without the fabric acting upon them. The two best examples of such forced are wind and gravity. While technically the fabric "pushes" the air and floor, no calculations need to be done with regards to changing the wind or the location of the floor from the fabric's counteracting force. The floor is not going to move, and the wind field has no visible form to be moved. This makes the calculation very easy. In the case of gravity it is a simple addition of a standard force (-9.8 m/s^2 here on earth) in the downwards direction to every node on the fabric. And wind is the force of a wind field on each fabric node. Unlike gravity, wind magnitude and direction change with location, but the calculation of the force can be done with an interpolation.

2.3 Friction

Friction must be calculated whenever a force is applied to a fabric in contact with any surface [5]. Therefore, the force of friction must be applied when collisions are detected. Friction has been studied extensively, and there are multiple models to consider. Classical friction models are based around the idea that "friction opposes motion and that its magnitude is independent of velocity and contact area" [5]. One such model is Coulomb friction (which does not include friction at zero velocity) often combined with "stiction" (static friction, or friction at rest as defined by Olsson et al.) to account for the forces at work when the velocity is zero. More recently proposed friction models attempt to better address the zero-velocity problem and have more accurate dynamic friction properties. These models include the Karnopp Model, Armstrong's Model, the Dahl Model, the Britsle Model (and it's more computationally feasible related Rest Integrator Model), and models by Bliman and Sorine [5]. As velocity is already precisely known for most fabric nodes, Coulomb's model with accounting for "sticktion" will suffice for most purposes (although experimentation with other friction models is highly encouraged in the interests of exploring scientific progress).

$$m * x'' + f \sin(x') + k*x = 0$$

$$x(t) = [x_0 - (2n+1)\Delta_X] \cos \omega t + (-1)^n \Delta_X$$

$$F = \begin{cases} F_e & \text{if } v = 0 \text{ and } |F_e| < F_S \\ F_S \sin(F_e) & \text{if } v = 0 \text{ and } |F_e| \geq F_S \end{cases}$$

Figure 4: Coulomb's friction model .

Top- the classic Coulomb friction model, where m is the mass, f is the kinetic friction force, k is the spring constant, x'' is the second derivative, x' is the first derivative, and x is the calculation of position [6].

Middle- where x_0 is the initial position, Δ_X is the change in position, ω is the angular frequency, t is time, and n is an integer specifying the number of half-cycle turning points from t = 0. [6].

Bottom- Friction force (F) broken up into two cases of zero velocity (v). F_e represents the external forces, and F_S is static friction[5].

3 Completing the Calculations

Once all the forces at work in the fabric have been calculated and summed together, the fabric may advance to its new position in time. Using the velocity at each node, the fabric moves a distance relative to the unit of time in the direction of the force. After the move, velocities at all nodes must be recalculated for the next move through time [1]. Depending on a node's new position the forces acting upon it will likely change. Most of the aforementioned forces can change with respect to position; new collisions may occur, friction may cease (node is lifted away from other surfaces) or change (fabric moves onto a new surface), or the wind may have a different force at the new location.

4 Inter-fabric Modeling Techniques

While by no means an entirely comprehensive list of all techniques, the following are a selection of the modeling techniques that can be used to calculate the inter-fabric interactions in the progression of calculating the overall forces at work over time. Often, mathematical models that were developed for different real-world phenomena can be slightly repurposed to represent fabric. These models include modeling fabrics as thin plates with Euler's column-buckling formulation analysis, as well as small-deformation linear-plate shell theory. Other models include using load-deformation curves for shearing, numerical integration of a series of equations of fabric properties (moment-curvature equations, moment and force equilibrium equations, and orientation-coordinate equations) to describe the fabric by obtaining coordinates in space, using rod elements and bending springs, and different variations on a finite-element model [7].

5 Rod Elements and Bending Springs

In this solution to the fabric-modeling problem, the model is a geometric-based representation where the fabric is a series of points in a grid connected with “springs”. While springs are certainly their own real world phenomena, they are applied in a manner almost akin to weaving threads to create a fabric. In the more complex models, each spring is not necessarily connected in a grid-like fashion, but instead multiple springs stem from the same point, connecting other points of varying distances and overlapping with other springs.

The spring force is calculated for every point on the fabric, starting with the initial point of force applied, taking into account the forces of the surrounding grid points as well as the effects of other outside forces (such as gravity). Spring force is calculated with Hooke's Law of Elasticity. Compression and expansion of the spring (connections between fabric “points”) result in either negative or positive exertion of force by the spring.

Self-collision detection and resolution also must be applied to the points. Collisions must be detected because fabric particles cannot freely pass between each other. Example resolutions add an additional force to the “bottom” layer of fabric from the layer that has come to rest on top of the bottom layer.

Spring dampers, proportional to the velocity of the force applied to the spring, can be used to further control the behavior of fabric. Fabric behaves differently depending on how much tension is applied to it. Outside forces acting on the fabric points are the weight of the other fabric particles (force of gravity), forces of inter-particle interactions (the stiffness of surrounding particles inhibiting a curve of the fabric), and any outside objects acting on the fabric (eg, the surface over which it is draped, or air resistance) [8].

6 Implementation

Concessions upon the aforementioned methodology were made in the implementation. Currently, the project implements only elastic collisions with world objects where the force reflected is the projection of the incoming velocity onto the normal of the surface impacted multiplied by a “bounce” constant. The force not reflected upwards is directed out orthogonal to the upward bounce. The collision detection method is merely brute force, and the resolution of the output fabric has been reduced accordingly. The inter-fabric collisions have been implemented, but have some unresolved issues with the floor, and as such are turned off in the final program.

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