Israel Journal of Ecology & Evolution, 2013
http://dx.doi.org/10.1080/15659801.2013.815435

Invasion biology and the success of social collaboration networks, with application to Wikipedia

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(*Received 5 May 2013; accepted 10 June 2013*)

We adapt methods from the stochastic theory of invasions – for which a key question is whether a propagule will grow to an established population or fail – to show how monitoring early participation in a social collaboration network allows prediction of success. Social collaboration networks have become ubiquitous and can now be found in widely diverse situations. However, there are currently no methods to predict whether a social collaboration network will succeed or not, where success is defined as growing to a specified number of active participants before falling to zero active participants. We illustrate a suitable methodology with Wikipedia. In general, wikis are web-based software that allows collaborative efforts in which all viewers of a page can edit its contents online, thus encouraging cooperative efforts on text and hypertext. The English language Wikipedia is one of the most spectacular successes, but not all wikis succeed and there have been some major failures. Using these new methods, we derive detailed predictions for the English language Wikipedia and in summary for more than 250 other language Wikipedias. We thus show how ideas from population biology can inform aspects of technology in new and insightful ways.

Keywords: invasion biology; stochastic population theory; social collaboration networks; Wikipedia

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Introduction

It is difficult to overestimate the effect that Charles Elton's book *The Ecology of Invasions by Animals and Plants* (1958) had on population biology. It remained the most highly cited source on the subject 50 years after its publication (Richardson & Pysek 2008). Although invasion biology covers a wide intellectual swath, one of the fundamental questions concerning invasions is whether a founding population of a certain size will establish itself or fail – that is, will the descendants of the initial propagule(s) reach a level considered to be self-maintaining or fall back to 0? Stochastic population theory is used to answer this question (MacArthur & Wilson 1967; Mangel 2006).

Social collaboration networks have become ubiquitous (Newman 2010; Easley & Klineberg 2011) and when such a network is started, one can envision it as a propagule of users 'invading' the worldwide web. Understanding whether a social collaboration network will succeed or fail is a problem with wide interest, with applications that range from investment in social media, to monitoring terrorist networks, to science education (e.g. Micklos et al. 2011), but we currently lack methods to make predictions of success. In this paper, we use methods from stochastic population theory (Mangel & Ludwig 1977; Mangel 1994; Mangel 2006) to show how such a prediction can be made. Population biologists will find our approach familiar, but with a new application and connection to the world of technology. Technologists interested in social networks will find novel features that include: (1) characterizing the stochastic growth of a social collaboration network; (2) a clear definition of success in terms

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of reaching a specified number of active users before falling to 0 [or any other low number] of active users; (3) derivation and solution of the differential equation characterizing the probability of success; (4) application of Maximum Likelihood Estimation (MLE) and the Akaike Information Criterion (AIC) to determine parameters and then weight possible models of success given data on participation; and (5) application of Markov Chain Monte Carlo (MCMC) methods to determine the distribution of parameters for each potential model. We mainly focus on the English language Wikipedia, but consider the remaining 250+ languages in the supplemental information (SI) (supplemental material is available online). In summary, we show a novel and important interdisciplinary approach that is broadly applicable.

About wikis

A wiki is a website in which multiple, dispersed participants create and edit interlinked web pages (Eberscbach et al. 2008; Mader 2008). One of the most famous wikis is Wikipedia, an online encyclopedia in more than 250 different languages, which has had remarkable success. Indeed, when viewed backwards across the time scale of a decade, the success of the English language Wikipedia (henceforth Wikipedia), measured by the number of individuals actively creating and editing pages (henceforth editors), seems to have been inevitable (Figure 1a). However, early on, the growth of Wikipedia shows fluctuations (Figure 1b) and the monthly change in the number of editors was sometimes very negative (Figure 1c).

Not all wikis succeed, and some of them – including those created by the *Los Angeles Times,* the Penguin publishing company and Amazon.com – were very short-lived (Mader 2008). Because wikis and other social collaboration networks are important in many fields, it is vital to ask how we can predict the success of a social collaboration network given early data, such as that shown in Figures 1(b) and 1(c). Although wikis can be of many types, we use Wikipedia as a case study.

In light of the fluctuations in the number of editors, the predicted success of a wiki must be a probabilistic quantity. Thus, our goal becomes computing the probability $u(w)$ that a wiki currently with w editors (the w indicating that individuals are writing and editing web pages) reaches a specified upper threshold w_u in which it is considered to be established before it falls to 0 editors.

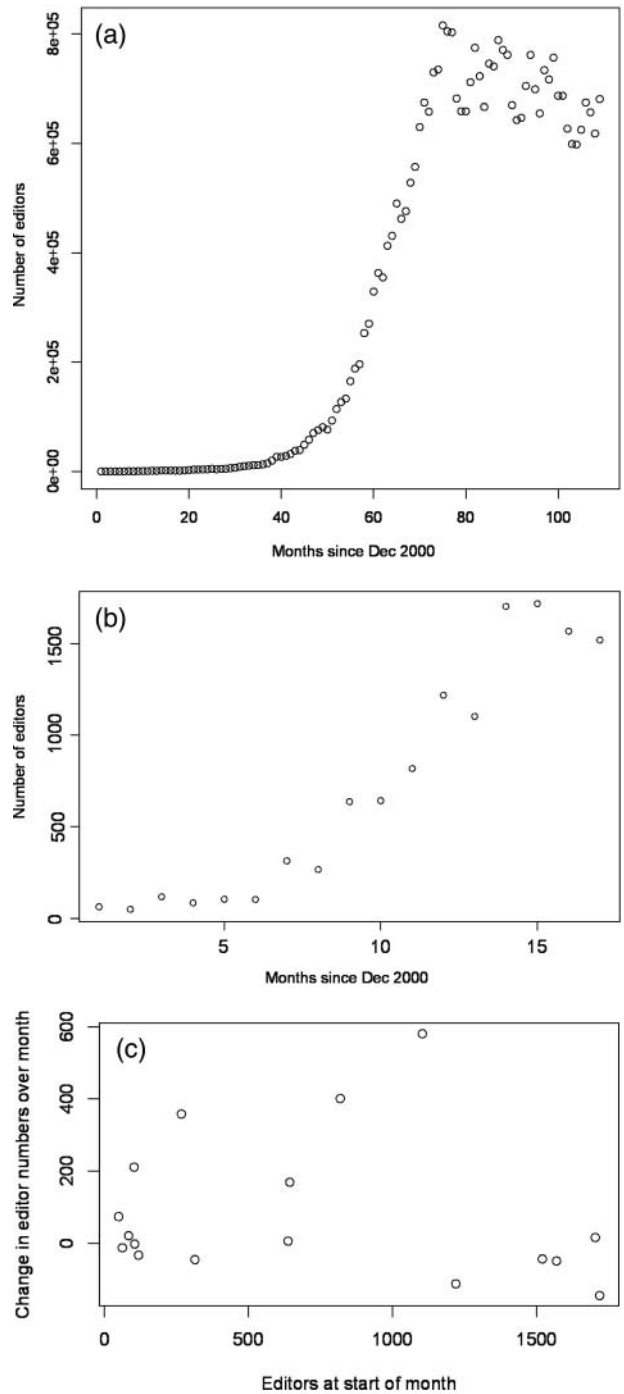


Figure 1. (a) When viewed backwards across about a decade, the growth of the English language Wikipedia seems inevitable, rising exponentially then saturating. (b) Early on, however, the number of active editors shows growth but considerable fluctuations. (c) When those data are converted to monthly change as a function of the current number of editors, large fluctuations can occur even when the number of editors is considerable.

Methods

In order to keep technical details to a minimum, we have put most of them in the Appendix, but there are some details that cannot be skimmed over. To capture the dynamics shown in Figure 1 requires a model that is both nonlinear (because of the sigmoidal shape) and stochastic (because of the fluctuations). We let $W(t)$ denote number of editors at time t and $dW = W(t+dt) - W(t)$. The entire trajectory in Figure 1(a) could then be described by a stochastic differential equation of the form (Schuss 1980):

$$dW = r(W)dt + \sigma(W)dB \tag{1}$$

where $r(W)$ and $\sigma(W)$ are functions of the number of editors, which have to be determined, and dB is an increment

in Brownian motion: it is normally distributed with mean 0 and variance $dt + o(dt)$, where $o(dt)$ denotes terms that are higher powers of dt.

For the analogy with a biological invasion, we assume that the number of editors is small, so that 'density dependent' effects that lead to the sigmoidal growth can be ignored. We then Taylor-expand the functions on the right-hand side of Equation (1), keeping only the linear terms to obtain

$$dW = (r_0 + r_1 W)dt + (\sigma_0 + \sigma_1 W)dB \qquad (2)$$

where r_0, r_1, σ_0 and σ_1 are constants to be determined. As these constants vary, we have many possible models, depending upon which of these parameters is set equal to 0. In the absence of fluctuations (i.e. the second term on the right-hand side is set equal to 0), when $r_1 > 0$, Equation (2) corresponds to $W = 0$ being an unstable steady state and will lead to exponential-like growth in the number of editors. By setting some of the parameters in Equation (2) equal to 0, we generate various forms of the model. For example, a model with constant deterministic and stochastic components is obtained by setting $r_1 = \sigma_1 = 0$.

The statistical interpretation of Equation (2) is that given $W(t) = w$, dW is normally distributed with mean $(r_0 + r_1 w)dt + o(dt)$ and variance $(\sigma_0 + \sigma_1 w)^2 dt + o(dt)$, where $o(dt)$ denotes higher powers of dt. We used techniques of Maximum Likelihood Estimation (MLE) to determine the most likely values of the parameters using the data shown in Figure 1(c) and Equation (2). We then used the Akaike Information Criterion (AIC) (Burnham & Anderson 2002) to select the most parsimonious models among the variants in which one or more of the parameters equal to zero, and to weight the support that the data give to the various models.

In the Appendix we show that the probability of success $u(w)$ satisfies

$$\frac{(\sigma_0 + \sigma_1 w)^2}{2}\frac{d^2u}{dw^2} + (r_0 + r_1 w)\frac{du}{dw} = 0 \qquad (3)$$

(also see Mangel & Ludwig 1977; Keizer 1987; Stratonovich 1992; Mangel 1994).

Since $u(w)$ is the probability of reaching w_u editors before falling back to 0 given that the current number of editors is w, the associated boundary conditions are $u(0) = 0$ and $u(w_u) = 1$. For the Wikipedia with data shown in Figure 1(c), the solution to Equation (3) is a prediction of the probability success given the data up to month 18.

To reflect model uncertainty, at each value of the number of editors, we computed the mean weighted by AIC weight (Burnham & Anderson 2002) and variance (across the models) of the probability of success. In order to account for parameter uncertainty within each model, rather than between models, we used Markov Chain Monte Carlo (MCMC) methods to obtain distributions for the parameters in each of the models. We used a Bayesian approach to describe the joint posteriors of the model parameters and generate median, central 68% and central 95% credible intervals on $u(w)$. These details are described in the Appendix.

Based only on a single language Wikipedia, we cannot assess model performance in terms of how well predicted success probability matches the frequency of success of replicate wikis. To assess the ability of the AIC-determined best model to predict the relative risk faced by multiple Wikipedias, we downloaded data on monthly counts of "active users" (at least five edits per month) in 268 different language Wikipedias.

For each language, we identified the first month with a non-zero editor count and then extracted the first 18 monthly counts, allowing the estimation of 18 values of the number of editors and 17 values of the monthly change in the number of editors. We then fit the best-supported and most parsimonious model according to our original analysis for the English Wikipedia to each dataset, assessed uncertainty of the parameters for each language, and estimated $u(w)$ its associated uncertainty for a range of values of w, including the actual editor count on the 18th month of existence of Wikipedia in that language. Since some languages have an inherently small user base and would be unlikely to sustain more than 1000 editors even if successful, we used both 50 editors and 1000 editors as the upper value w_u.

Results

In Table 1 we show the best models identified by the AIC analysis and in Table 2 the parameters for the five best models, which have 97% of the AIC weight. In Figure 2(a), we show the weighted mean probability of success with error bars determined by the square root of the variance. From this figure, we see that based on the weighted mean, once the Wikipedia crossed the 250-editor mark there was about a 70% chance of growing to 1000 editors before returning to 0 editors. The relatively wide error bars in Figure 2(a) are due to the considerable variation in the individual solutions, which we show in Figure 2(b).

Table 1. Results of the AIC analysis for the data shown in Figure 1(c) under different assumptions about the stochastic dynamics* (k = number of parameters).

Model	k	AICc	AIC Weight ($AIC_w(i)$)
1) $r = r_0, \sigma = \sigma_0$	2	232.01	0.36
2) $r = r_0, \sigma = \sigma_0 + \sigma_1 w$	3	232.22	0.32
3) $r = r_1 w, \sigma = \sigma_0$	2	234.12	0.12
4) $r = r_0 + r_1 w, \sigma = \sigma_0$	3	234.62	0.098
5) $r = r_0 + r_1 w, \sigma = \sigma_0 + \sigma_1 w$	4	235.57	0.061
6) $r = r_1 w, \sigma = \sigma_0 + \sigma_1 w$	3	238.73	0.012
7) $r = r_0, \sigma = \sigma_1 w$	2	239.00	0.011
8) $r = r_1 w, \sigma = \sigma_1 w$	2	240.95	0.004
9) $r = r_0 + r_1 w, \sigma = \sigma_1 w$	3	241.64	0.0029

*Given $W(t) = w$ and setting $dW = W(t + dt) - W(t)$; $dW \sim N((r_0 + r_1 w)dt + o(dt), (\sigma_0 + \sigma_1 w)^2 dt + o(dt))$; where $N(\mu, \sigma^2)$ is a normal distribution with mean μ and variance.

Table 2. Non-zero MLE values of parameters for the five best-supported models from Table 1.

Model	Parameters
1	$r_0 = 81.99, \sigma_0 = 192.86$
2	$r_0 = 78.79, \sigma_0 = 101.30, \sigma_1 = 0.13$
3	$r_1 = 0.0453, \sigma_0 = 205.20$
4	$r_0 = 115.04, r_1 = -0.0467, \sigma_0 = 190.74$
5	$r_0 = 66.70, r_1 = 0.0361, \sigma_0 = 95.23, \sigma_1 = 0.14$

In Figure 3, we show two-dimensional plots of the joint posteriors of all pairs of parameters for the English language Wikipedia, but in the case of more complicated models these are still integrated over the uncertainty in the remaining, unseen parameters.

In Figure 4, languages of non-English Wikipedias with similar success probabilities predicted by Model 1 (individual results are in the online supplemental information) are grouped together and the actual fraction of success is counted for each group. In general, languages with a higher predicted probability of success were more often successful, but there was not a close correspondence between the predicted success probability and the fraction of languages succeeding. In particular, most of the data fall below the 1:1 line if we use the MLE success probability (Figure 4a) but are on either side of the 1:1 line if we use the 68% credible interval (Harte 2001; Figure 4b). The failure of Model 1 to more accurately predict the probability of success across all the Wikipedias may indicate that each language needs its own best model; in the case of languages with small numbers of readers, nonlinear terms in Equation (1) may become important long before the 50-editor mark is reached.

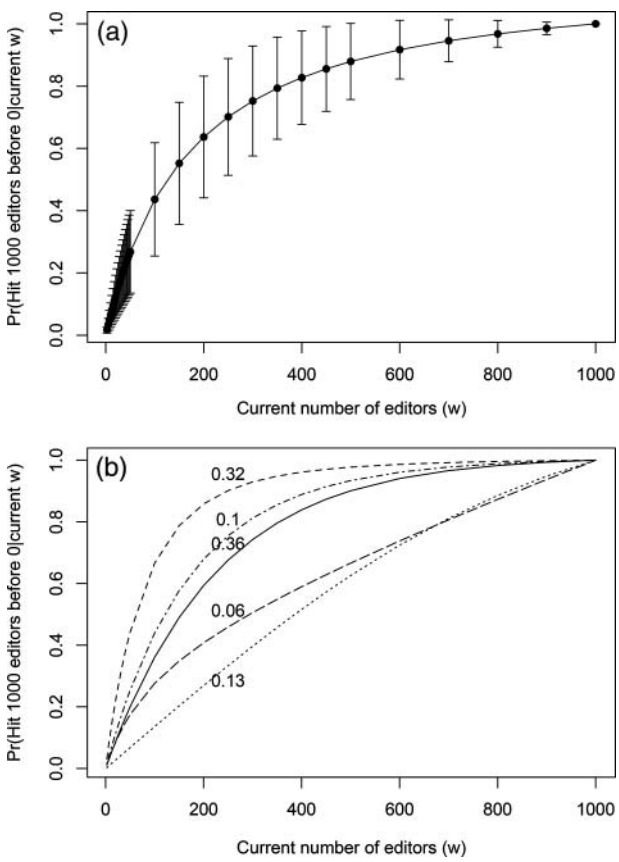


Figure 2. (a) The weighted mean probability of the English Wikipedia hitting 1000 editors before returning to 0 editors given the current number of editors, for the five top models in Table 1, with AIC weights rescaled to sum to 1. The error bars are determined by the square root of the AIC weighted variance (see Methods). (b) The individual solutions, along with their AIC weights.

Discussion

Simberloff (2009) recently reviewed the role of propagule pressure in biological invasions and our results have a clear analogue through the mean rate of growth and its dependence on the current number of editors. The success of a social collaboration network can also be viewed as an "epidemic", characterized (Gladwell 2000) by contagion – the deterministic component Equation (1), which measures the mean rate of growth; and stickiness – the stochastic component of Equation (1), which measures the volatility of editor numbers. Individuals interested in increasing the success probability of a social collaboration network (e.g. Lewis et al. 2012) could use our methods to ask if the probability of success is improved more by increasing the deterministic component or by decreasing the volatility.

The probability that we compute does not characterize the sustainability (large-scale adoption in the sense of Mader (2008)) of a wiki or other social collaboration network, but does identify those that are more likely to succeed than not. Indeed, a wiki could fall to 0 editors and then rebound – the analogue of a local extinction and then re-invasion. Similarly, we have not discussed a mechanism by which the social collaboration network grows in its nonlinear phase. Mechanism becomes much more important if one wishes to understand the full trajectory, including the point of inflection, saturation and fluctuations around the level of saturation. Although that is a subject for a different paper, we can give a few comments here. First, since editors may join a wiki because of other editors ("my friend is doing this, so I will too") or because of articles that inspire one to become an editor, a minimal model for mechanism should likely include articles and editors as states. In addition, we may also need to consider the behavioral ecology of editors and the "fitness" metric involvement in a social network. Second, although the paper is about social networks, we assumed no influence of structure on wiki success. That is, we used the simplifying assumption that all editors are the same (have the same influence in generating new editors). But structure is a key characteristic of networks, and social networks often have hubs, where few individuals connect a large proportion of other less connected individuals. As with invasion biology, in which all invaders are not equal, there are likely key highly connected editors that have more impact than others. We hypothesize that if editors are connected to their articles in a small-world network, then the predicted probability of success of the network would in general be less than the observed probability of success because most editors have little influence on the number

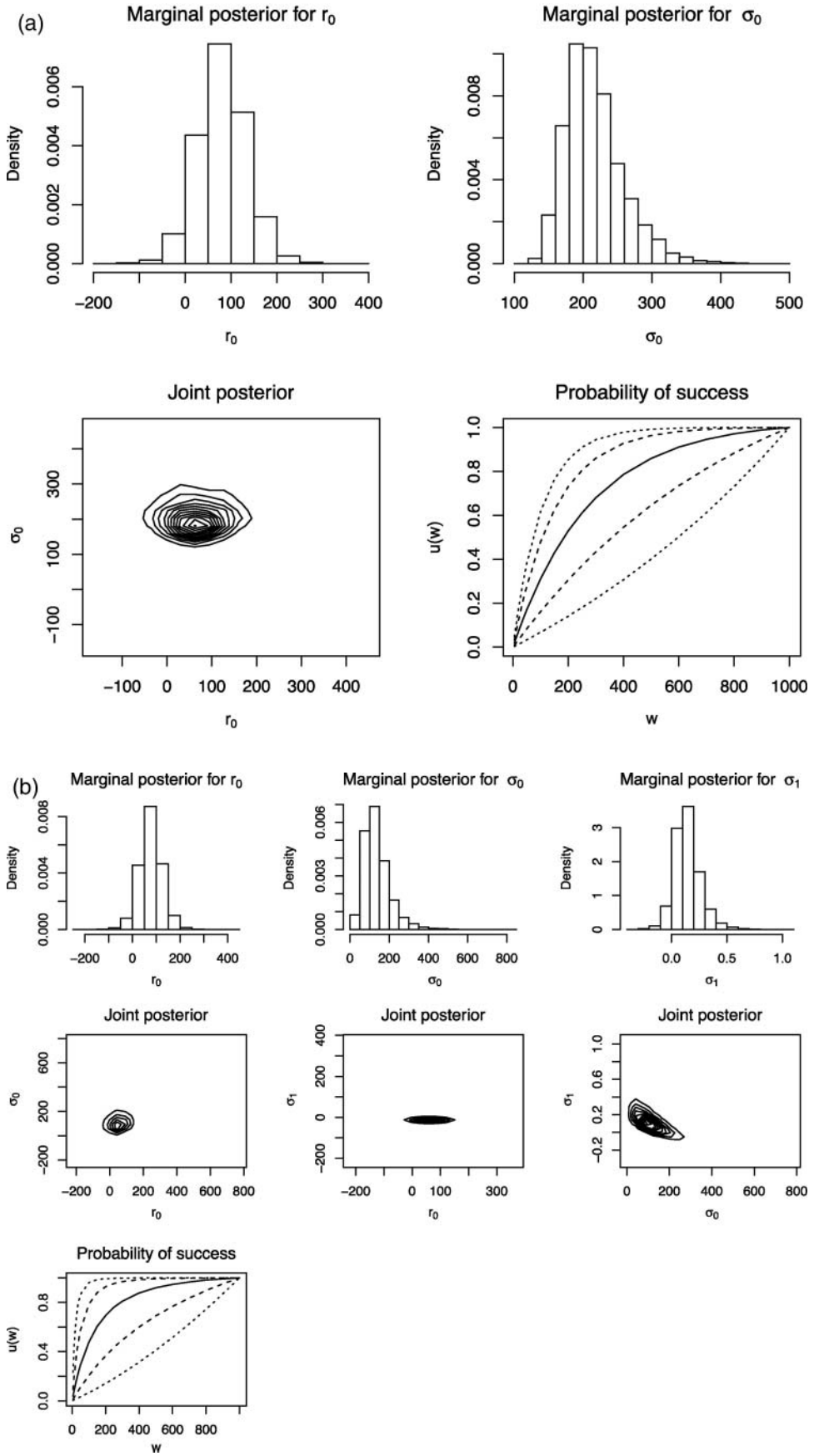


Figure 3. (a–e) Posterior distributions of parameters for the top five models for early dynamics of editors. For the plots of $u(w)$, the solid line is the posterior median, the heavily dashed lines are the 68% credible interval and the lightly dashed lines are the 95% credible interval. Panels a–e correspond to models 1–5.

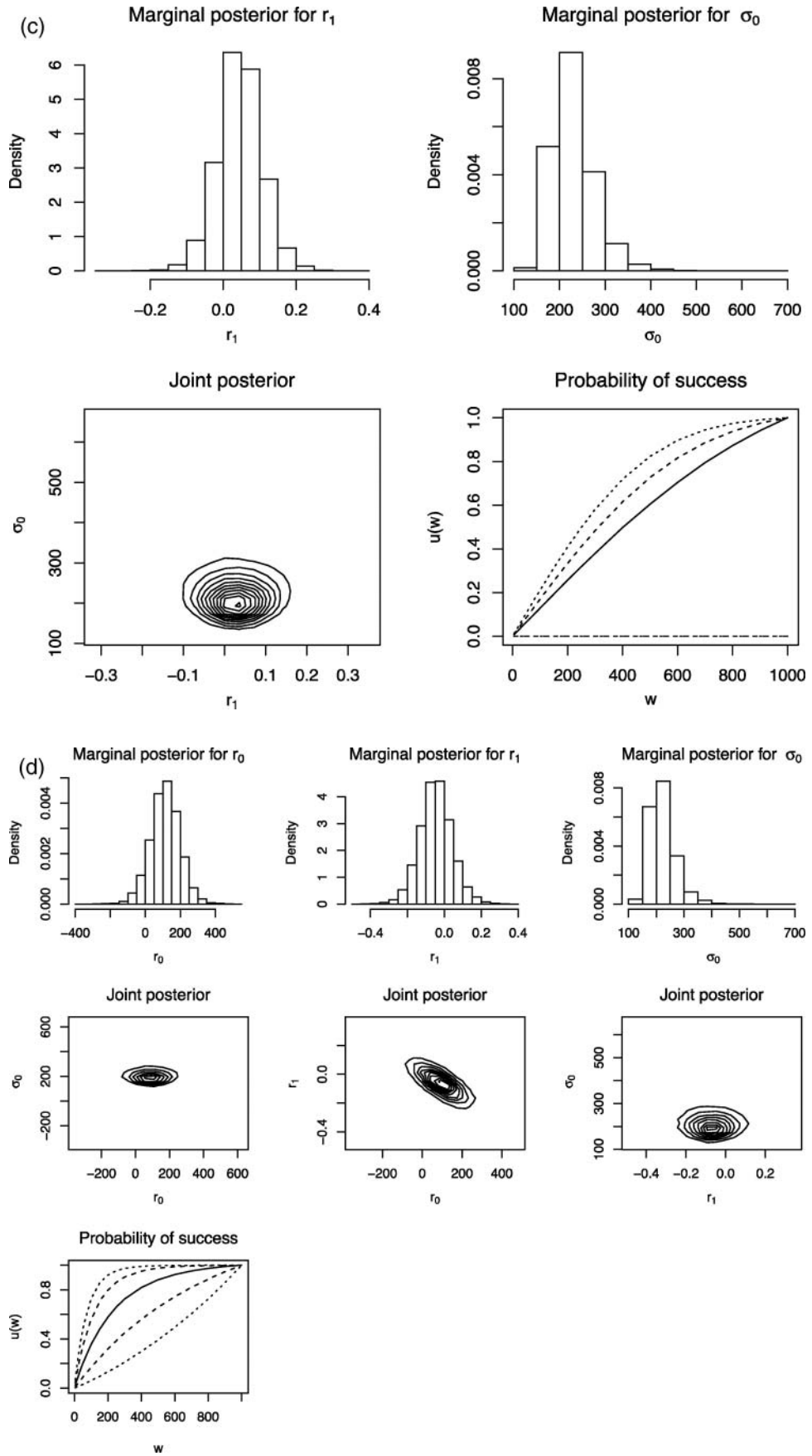


Figure 3. (a–e) Continued.

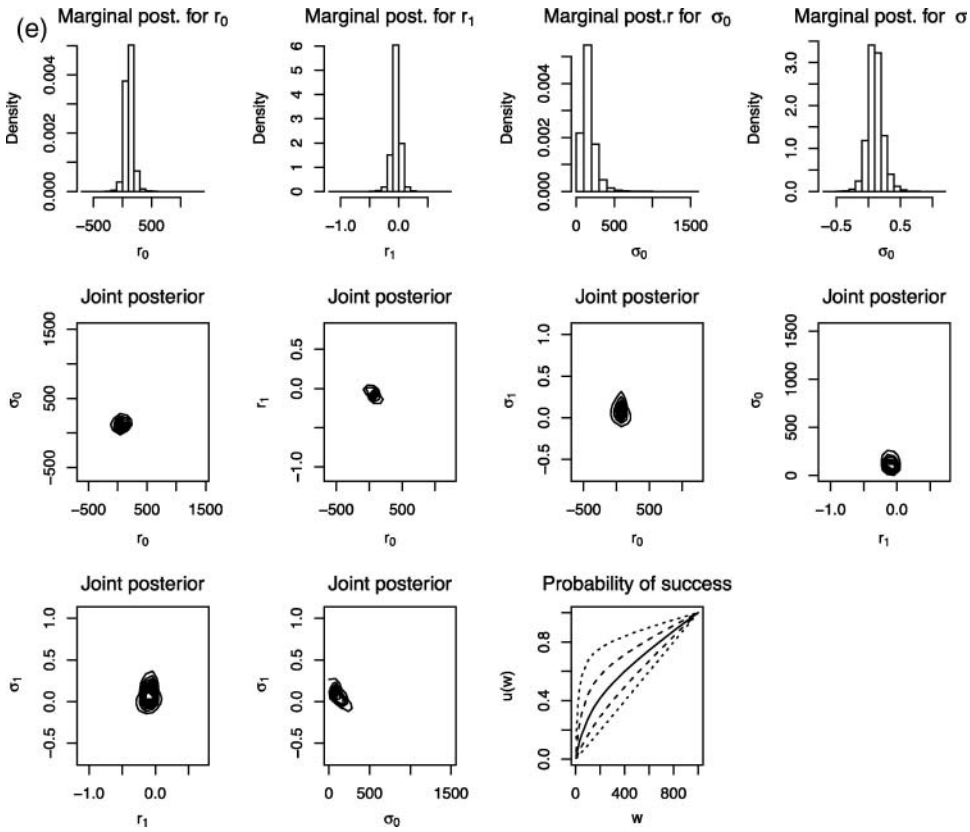


Figure 3. (a–e) Continued.

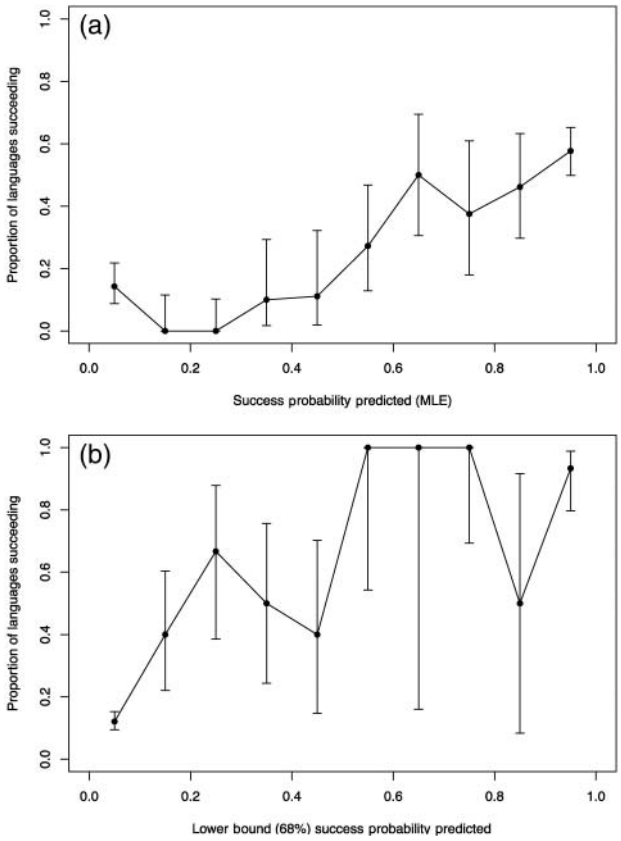


Figure 4. (a–b) Cross-language success in predicting the probability of different language Wikipedias succeeding. We bin languages: 1) by MLE predicted success, and plot the proportion of models with predicted success in each bin that actually succeeded (error bars are 68% binomial confidence intervals (*9*)); or 2) based on the lower bound of the 68% credible interval on success probability.

of articles, which could in part explain the results in Figure 4.

Our computation of the probability of success for different Wikipedias has additional analogies to the field of Population Viability Analysis (PVA) in conservation biology, which has the goal of predicting extinction probabilities for endangered populations (Gilpin & Soule 1986; Beissinger & Westphal 1998). While PVAs have proven useful in ranking relative risks and evaluating alternative management strategies (Brook et al. 2000), confidence intervals on extinction probability ranging nearly from 0 to 1 are common (Ludwig 1990; Satterthwaite et al. 2002) and results are better interpreted as rankings of relative risks rather than absolute predictions of outcome probabilities (Beissinger & Westphal 1998).

The first major step in making a network tool for collaboration in an organization is to conduct a pilot with both a time frame and a long-term vision (Mader 2008) – this is like seeding with a propagule. The methods we have described allow quantitative evaluation of the probability that the collaborative social network will succeed on a larger scale, given the data of the pilot. For that reason, our approach is broadly applicable and is an example of how ideas from population biology inform understanding of technology (Crawford 2001; Wischmann et al. 2012).

Acknowledgements

This work was sponsored by the Defense Advanced Research Projects Agency Defense Sciences Office (DSO) Program:

Thermodynamically Evolving Robust and Adaptive Physical Intelligence (THERA-PI) ARPA Order No. Z191/00, Program Code: 0D10 Issued by DARPA/CMO under Contract No. HR0011-10-C-052; the final stages were supported by NSF Grant EF-0924195 to M.M. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressly or implied, of the Defense Advance Research Projects Agency or the US Government. We appreciate the comments of Colin Clark, Christopher Friedrichs and Donald Ludwig, Robin Snyder and Justin Yeakel on previous versions of this work.

References

Abramowitz M, Stegun IA. 1964. Handbook of mathematical functions with formulas, graphs, and mathematical tables. Washington (DC): National Bureau of Standards.

Beissinger S, Westphal MI. 1998. On the use of demographic models of population viability in endangered species management. J Wildlife Manag. 62:821–841.

Brook BW, O'Grady J, Chapman AP, Burgman MA, Akçakaya HR, Frankham R. 2000. Predictive accuracy of population viability analysis in conservation biology. Nature 404:385–387.

Burnham KP, Anderson DR. 2002. Model selection and multimodel inference: a practical–theoretic approach. Springer-Verlag: New York (NY).

Crawford M. 2011. Biologists using social-networking sites to boost collaboration. BioScience 61:736–736.

Easley D, Klineberg J. 2011. Networks, crowds and markets. Reasoning about a highly connected world. Cambridge (UK): Cambridge University Press.

Eberscbach A, Glaser M, Heigl R, Warta A. 2008. Wiki web collaboration. Heidelberg: Springer-Verlag.

Elton C. 1958. The ecology of invasions by animals and plants. London: Methuen.

Gelman A, Carlin JB, Stern HS, Rubin DR. 2004. Bayesian data analysis. Boca Raton (FL): Chapman & Hall/CRC Texts in Statistical Science.

Geweke J. 1992. Evaluating the accuracy of sampling-based approaches to calculating posterior moments. In: Bernardo JM, Bayarri MJ, Berger JO, Dawid AP, Heckerman D, Smith AFM, West M, editors. Bayesian statistics. Oxford: Clarendon Press; p. 169–193.

Gilpin M, Soule ME. 1986. Minimum viable populations: processes of species extinction. In: Soule ME, editor. Conservation biology: the science of scarcity and diversity. Sunderland (MA): Sinauer Associates; p. 19–34.

Gladwell M. 2000. The tipping point. How little things can make a big difference. New York (NY): Black Bay Books, Little, Brown and Company.

Harte D. 2011. Non-asymptotic binomial confidence intervals [Internet]. Wellington: Statistics Research Associates. Available from: www.statsresearch.co.nz/pdf/confint.pdf.

Keizer JE. 1987. Statistical thermodynamics of nonequilibrium processes. New York (NY): Springer-Verlag.

Lewis K, Gonzalez M, Kaufman J. 2012. Social selection and peer influence in an online social network. Proc Nat Acad USA 109:68–72.

Ludwig D. 1990. Is it meaningful to estimate a probability of extinction? Ecology 80:298–310.

MacArthur RH, Wilson EO. 1967. The theory of island biogeography. Princeton (NJ): Princeton University Press.

Mader S. 2008. Wikipatterns: a practical guide to improving productivity and collaboration in your organization. Indianapolis (IN): Wiley.

Mangel M, Ludwig D. 1977. Probability of extinction in a stochastic competition. SIAM J Appl Math. 33:256–266.

Mangel M. 1994. Barrier transitions driven by fluctuations, with applications to ecology and evolution. Theor Pop Biol. 45:16–40.

Mangel M. 2006. The theoretical biologist's toolbox. Quantitative methods for ecology and evolutionary biology. Cambridge (UK): Cambridge University Press.

Micklos D, Lauter S, Nisselle A. 2011. Lessons from a science education portal. Science 334:1657–1658.

Newman M. 2010. Networks. An introduction. New York (NY): Oxford University Press.

Plummer M, Best N, Cowles K Vines K. 2010. Coda: Output analysis and diagnostics for MCMC. R package version 0.13-5 [Internet]. Available from: http://CRAN.R-project.org/package = coda

R Development Core Team. 2011. R: A language and environment for statistical computing [Internet]. Vienna: R Foundation for Statistical Computing. Available from: http://www.R-project.org/

Raftery AE, Lewis SM. 1995. The number of iterations, convergence diagnostics and generic Metropolis algorithms. In: Gilks WR, Richardson S, Spiegelhalter D, editors. Practical Markov Chain Monte Carlo. London: Chapman and Hall; p. 115–130.

Richardson DM, Pysek P. 2008. Fifty years of invasion ecology – the legacy of Charles Elton. Diversit Distrib. 14:161–168.

Satterthwaite WH, Menges ES, Quintana-Ascencio P. 2002. Assessing scrub buckwheat population viability in relation to fire using multiple modeling techniques. Ecol Applics. 12:1672–1687.

Schuss Z. 1980. Theory and applications of stochastic differential equations. New York (NY): John Wiley and Sons.

Simberloff D. 2009. The role of propagule pressure in biological invasions. Annu Rev Ecol Evol Syst. 40:81–102.

Stratonovich RL 1992. Nonlinear nonequilibrium thermodynamics I. Linear and nonlinear fluctuation-dissipation theorems. New York (NY): Springer-Verlag.

Wischmann S, Floreano D, Keller L. 2012. Historical contingency affects signaling strategies and competitive abilities in evolving populations of simulated robots. Proc Nation Acad USA 109:864–868.

Appendix. Further details on the methods

AIC analysis for the parameters

According to Equation (2), given $W(t) = w$,

$$dW \sim N((r_0 + r_1 w)dt + o(dt), (\sigma_0 + \sigma_1 w)^2 dt + o(dt)) \quad (4)$$

where $N(\mu, \sigma^2)$ is a normal distribution with mean μ and variance σ^2, and *o(dt)* denotes terms that are higher powers of *dt*.

We used maximum likelihood techniques, assuming the normal distribution for *dW* specified in Equation (4), to determine the most likely values of the parameters in Equation (2) using the data shown in Figure 1c. We report these in Table 1. We assessed support for each of the different models using AIC analysis.

Derivation and solution of the equation for the probability of success

From the definition of *u(w),* the law of total probability (Mangel 2006) leads to

$$u(w) = E_{dw}\{u(w + dW)\} \quad (5)$$

Taylor-expanding the right-hand side of Equation (5) to second order gives

$$u(w) = E_{dW}\left\{u(w) + \frac{du}{dw}dW + \frac{1}{2}\frac{d^2u}{dw^2}dW^2 + o(dW^2)\right\} \quad (6)$$

In light of the properties of the increment of Brownian motion

$$\begin{aligned} E\{dW|W(t)=w\} &= (r_0+r_1w)dt+o(dt) \\ E\{dW^2|W(t)=w\} &= (\sigma_0+\sigma_1w)^2dt+o(dt) \end{aligned} \tag{7}$$

so that Equation (6) becomes

$$u(w)=u(w)+(r_0+r_1w)\frac{du}{dw}dt+\frac{(\sigma_0+\sigma_1w)^2}{2}\frac{d^2u}{dw^2}dt+o(dt) \tag{8}$$

We divide by dt and let $dt\to 0$ to obtain

$$\frac{(\sigma_0+\sigma_1w)^2}{2}\frac{d^2u}{dw^2}+(r_0+r_1w)\frac{du}{dw}=0 \tag{9}$$

In light of the definition of $u(w)$, the boundary conditions associated with Equation (9) are

$$u(0)=0; u(w_u)=1 \tag{10}$$

The general solution of Equation (9) is

$$u(w)=\frac{\int_0^w \exp\left[-2\int_0^y \frac{r_0+r_1x}{(\sigma_0+\sigma_1x)^2}dx\right]dy}{\int_0^{w_u} \exp\left[-2\int_0^y \frac{r_0+r_1x}{(\sigma_0+\sigma_1x)^2}dx\right]dy} \tag{11}$$

We evaluate this integral exactly for Model 1

$$u(w)=\frac{1-e^{-\frac{2r_0}{\sigma_0^2}w}}{1-e^{-\frac{2r_0}{\sigma_0^2}w_u}} \tag{12}$$

For cases in which the MLE $r_0=0$, Equation (12) must be evaluated using l'Hospital's rule (or equivalently by Taylor-expanding the exponentials as $r_0\to 0$).

We evaluated the solution for Model 2

$$u(w)=\frac{\int_0^w \exp\left[\frac{1}{\sigma_1}\bullet\frac{2r_0}{\sigma_0+\sigma_1y}\right]dy}{\int_0^{w_u} \exp\left[\frac{1}{\sigma_1}\bullet\frac{2r_0}{\sigma_0+\sigma_1y}\right]dy} \tag{13}$$

using numerical integration.

The solution for Model 3

$$u(w)=\frac{\int_0^w \exp\left[-\frac{r_1}{\sigma_0^2}y^2\right]dy}{\int_0^{w_u} \exp\left[-\frac{r_1}{\sigma_0^2}y^2\right]dy}=\frac{\int_0^{w\frac{\sqrt{2r_1}}{\sigma_0}} \exp\left[-\frac{y^2}{2}\right]dy}{\int_0^{w_u\frac{\sqrt{2r_1}}{\sigma_0}} \exp\left[-\frac{y^2}{2}\right]dy} \tag{14}$$

can be evaluated quickly by using the numerical approximation for normal distributions (Abramowitz & Stegun 1964).

We also evaluated the solution for Model 4

$$u(w)=\frac{\int_0^w \exp\left[-\frac{2}{\sigma_0^2}(r_0y+\frac{1}{2}r_1y^2)\right]dy}{\int_0^{w_u} \exp\left[-\frac{2}{\sigma_0^2}(r_0y+\frac{1}{2}r_1y^2)\right]dy} \tag{15}$$

numerically.

Finally, for Model 5, no simplifications are possible for the integrands in the exponentials, so we used Equations 3.3.16–3.1.19 from Abramowitz and Stegun (1964) for evaluation of integrals of the form $\int\frac{mx+n}{ax^2+bx+c}dx$.

Construction of Figure 2(a)

To construct Figure 2(a), we let subscript i denote the result obtained from Equation (11) when model i in Table 1 is used (with weights rescaled to sum to 1.0 for the top five models) and define

$$\begin{aligned} \overline{u}(w) &= \sum_{i=1}^{5} AIC_w(i)u_i(w) \\ \overline{u}^2(w) &= \sum_{i=1}^{5} AIC_w(i)u_i^{2V}(w) \\ Var(u(w)) &= \overline{u}^2(w)-(\overline{u}(w))^2 \end{aligned} \tag{16}$$

to represent the AIC weighted solutions for a mean of Equation (11) and a measure of variability around that mean. For Figure 2(a), we show the weighted mean with error bars determined by $\sqrt{Var(u(w))}$.

Bayesian methods

Our goal is to estimate the joint posterior for the model parameters, which we denote by a vector v whose form will vary based on the model under consideration. The posterior predictive distribution for the success probability *u(w)* can then be determined by applying the formula for *u(w)* to draws of model parameters from their joint posterior distribution.

The maximum likelihood fits described earlier allow us to calculate the probability distribution of dW given the parameters and the current number of editors, which we denote by p($dW|v$, W). To generate the posterior for the model parameters we compute the probability distribution of the parameters, given the current number of editors and the observed change in editors, which we denote by $p(v|W,dW)$.

Using Bayes's theorem and assuming independent priors on W, dW and σ and assuming independent priors on del parameter $p_1(v|W,dW)$ given the current number of editors and the change in the number of editors is

$$p_1(v|W,dW)=\frac{p_0(v)p(dW|W,v)}{\int p_0(v)p(dW|W,v)dv}\propto p_0(v)p(dW|W,v) \tag{17}$$

In this expression, $p_0(v)$ is the prior probability for the parameters (we assume limited knowledge and thus uniform, unbounded priors except that $\sigma_0>0$ since the standard deviation in growth rate must be positive even as W becomes small) and $p(dW,W|v)$ is of the probability of drawing each observed value of dW from a normal distribution with mean and standard deviation calculated according to the current model formulation, given W and the proposed values of v. The denominator in the middle expression in Equation (17) is a normalization constant, so that the posterior density integrates to 1. The final expression follows from this.

We approximated the posterior distribution for v by using Metropolis algorithm Markov Chain Monte Carlo sampling of the equation for the posterior density given above (Gelman et al. 2004). We used custom R code (R Development Core Team) with symmetric, normal proposal distributions manually tuned to give acceptance probabilities of ~ 0.4. Within each iteration of the chain, we updated each element of v one parameter at a time, conditioned on the current value of the other parameters. After (potentially) updating each element of v, we calculated *u(w)* for a series of values of w ranging from 2.5 to 50 in increments of

2.5 and from 50 to 500 in increments of 25 (and then up to 1000 in increments of 100 for English). We recorded each of these estimates of $u(w)$ to generate chains of draws from the posterior predictive distribution.

The priors we used are improper (do not have finite integrals), but our data had a bounded range of support (and sufficient power for likelihoods to dominate priors), resulting in proper posteriors. We also ran the models with a Jeffreys prior (Gelman et al. 2004) for σ_0, $p(\sigma_0) \sim 1/\sigma_0$ for all $\sigma_0 > 0$, with essentially identical results. Since the posterior distributions of individual variables integrate over the full uncertainty in the remaining parameters, their modes do not always coincide exactly with the maximum likelihood estimates.

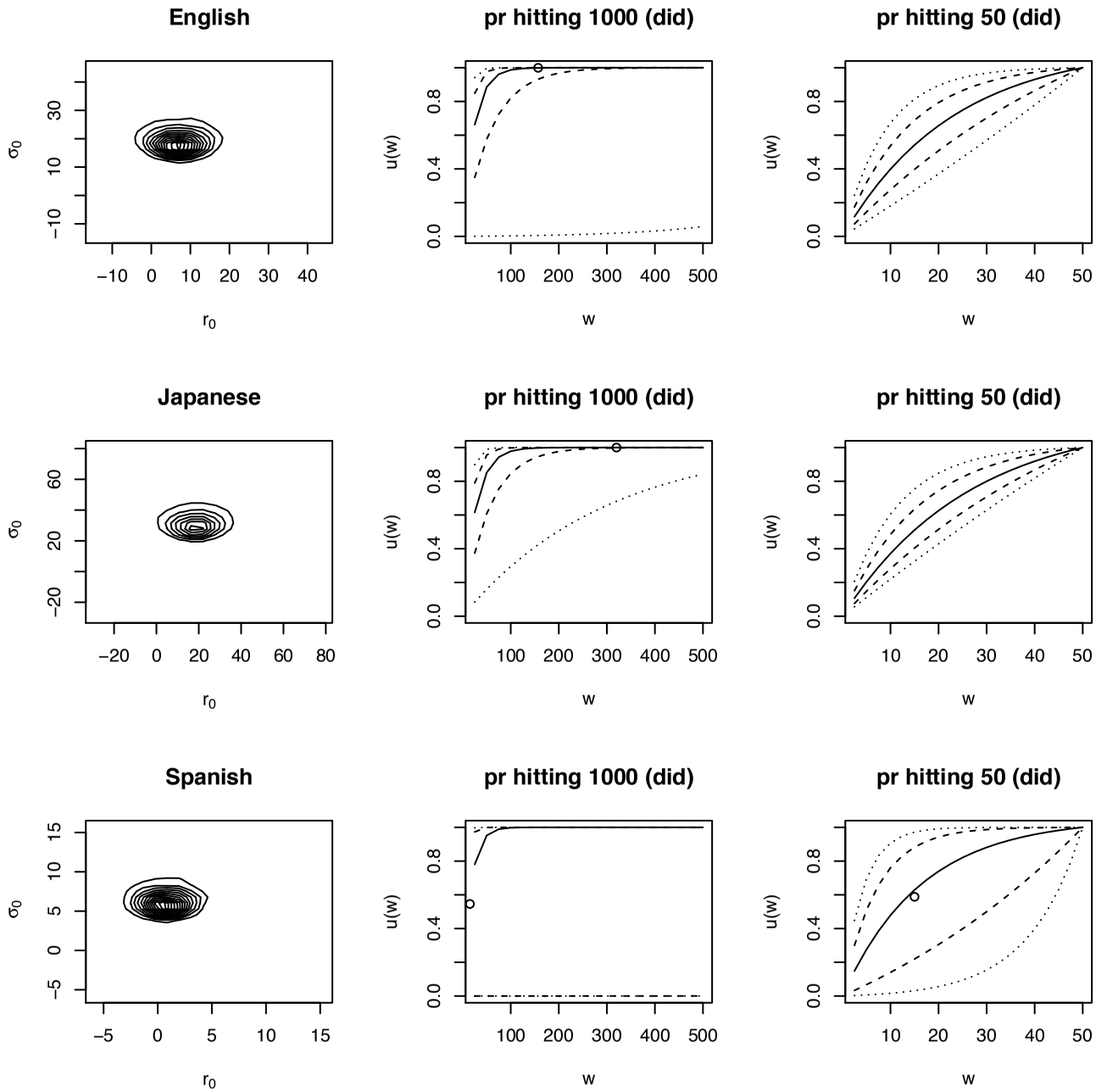
After manual tuning of the proposal distributions, for each model formulation (i.e. Models 1–5) or dataset (we extended this analysis to multiple languages – see below) we ran an initial burn-in of 20,000 iterations, starting from the MLE estimates of v. We then used the diagnostic method of Geweke (1992) as implemented in "gewke.diag" in R library "coda" (Plummer et al. 2010) to compare the mean of the first 10% of the retained chain to the last 50% of the chain. If $|z|$-scores for all elements of v and all estimates of $u(w)$ were < 2, we considered the chains to have converged and burn-in adequate. If not, we ran an additional 20,000 iterations until the last 20,000 iterations yielded passing diagnostics. Finally, we used the Raftery and Lewis (1995) approach implemented via the command "raftery.diag" in library "coda" to calculate the recommended post burn-in length of the posterior chain for each parameter or predictive output required to estimate the .025 quantile with accuracy of 0.005 with 95% probability. We rounded the required chain length up to the nearest 20,000 (if $\leq$100,000 iterations required) or 250,000.

Supporting OnLine Material

In the figures that follow, we plot the joint posterior distributions for r_0 and σ_0 when Model 1 is applied to the first 18 months' data for each language's Wikipedia, along with the predicted probability of reaching 50 or 1000 editors before reaching 0 editors, conditioned on the number of editors at month 18. The solid line is the posterior median predicted success, the heavily dashed lines are the 68% credible interval, and the lightly dotted lines are the 95% credible interval. The open circle denotes the actual editor count on month 18 along with the probability of success associated with that number editors, given the maximum likelihood estimates of r_0 and σ_0.

If there is no open circle visible, the language Wikipedia had exceeded the threshold level of editors before month 18.

Note that with skewed distributions, the MLE may not match the posterior median. The mode for σ_0 was typically smaller than the median, resulting in more optimistic projections based on the MLE.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pr hitting 50 (did)
Japanese
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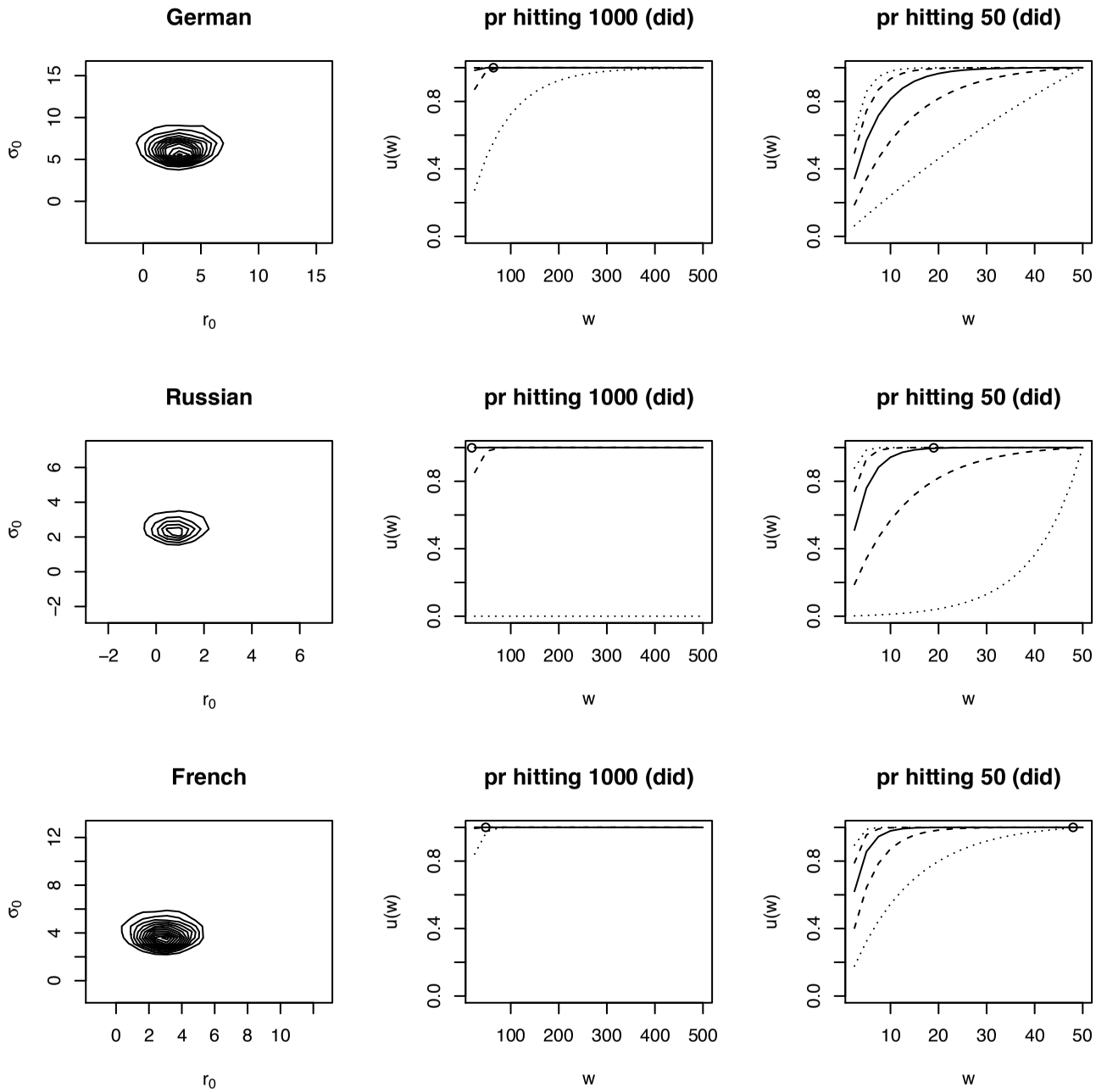
pr hitting 50 (did)
Spanish
pr hitting 1000 (did)
pr hitting 50 (did)
σ0
r0
u(w)
w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pr hitting 1000 (did)
pr hitting 50 (did)
Russian
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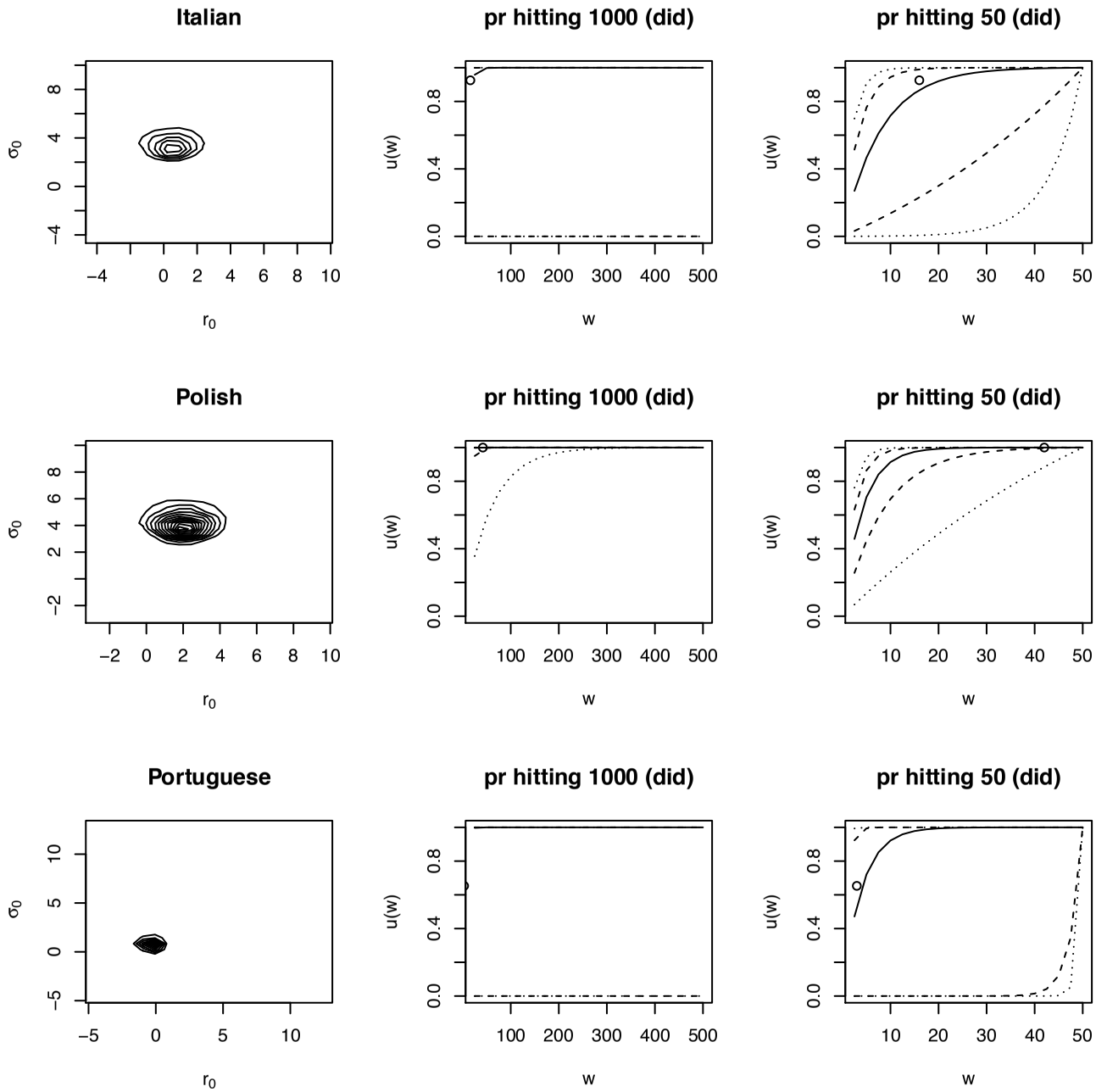
pr hitting 50 (did)
French
pr hitting 1000 (did)
pr hitting 50 (did)
σ0
r0
u(w)
w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pr hitting 1000 (did)
pr hitting 50 (did)
Polish
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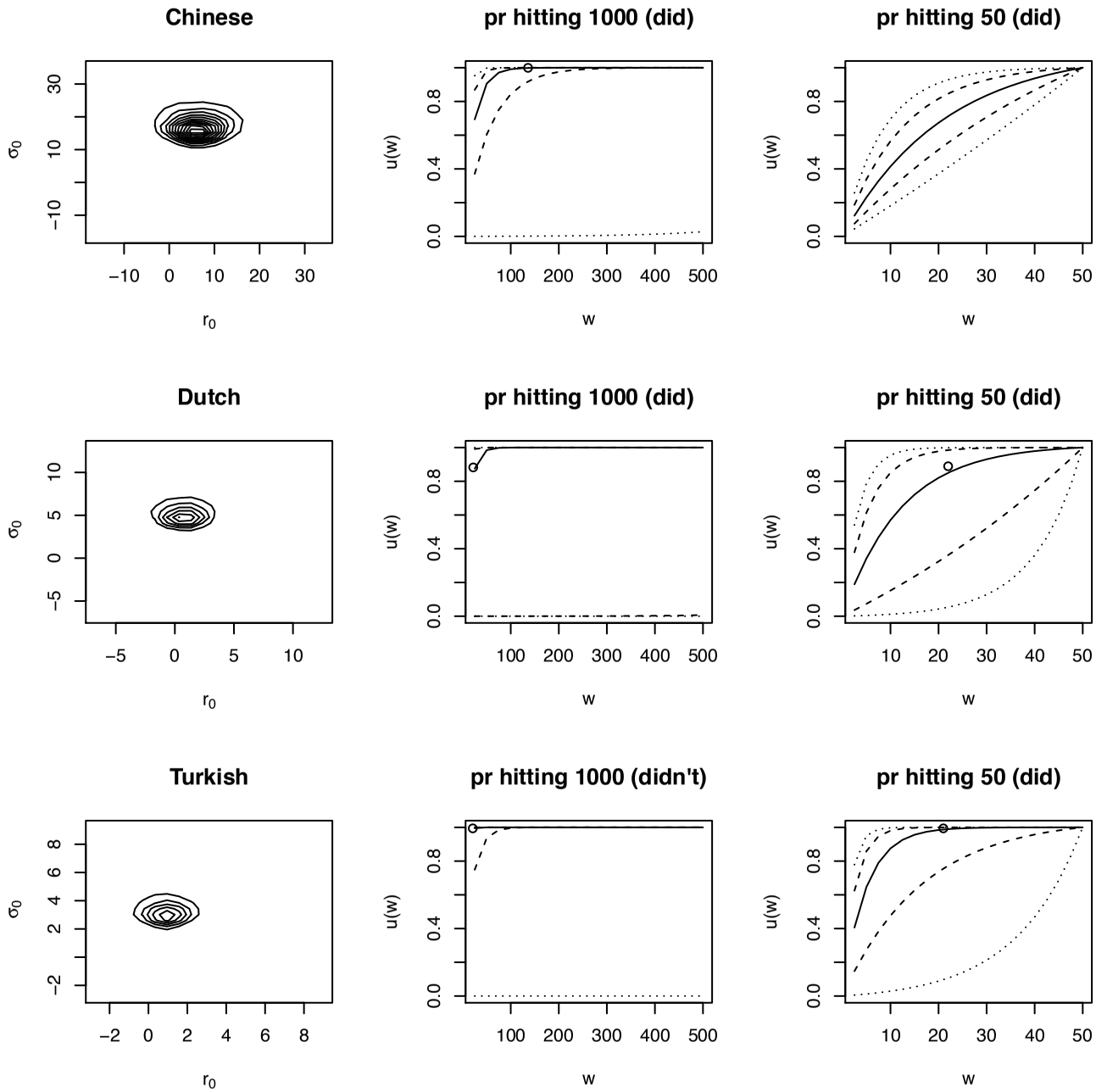
pr hitting 50 (did)
Portuguese
pr hitting 1000 (did)
pr hitting 50 (did)
σ0
r0
u(w)
w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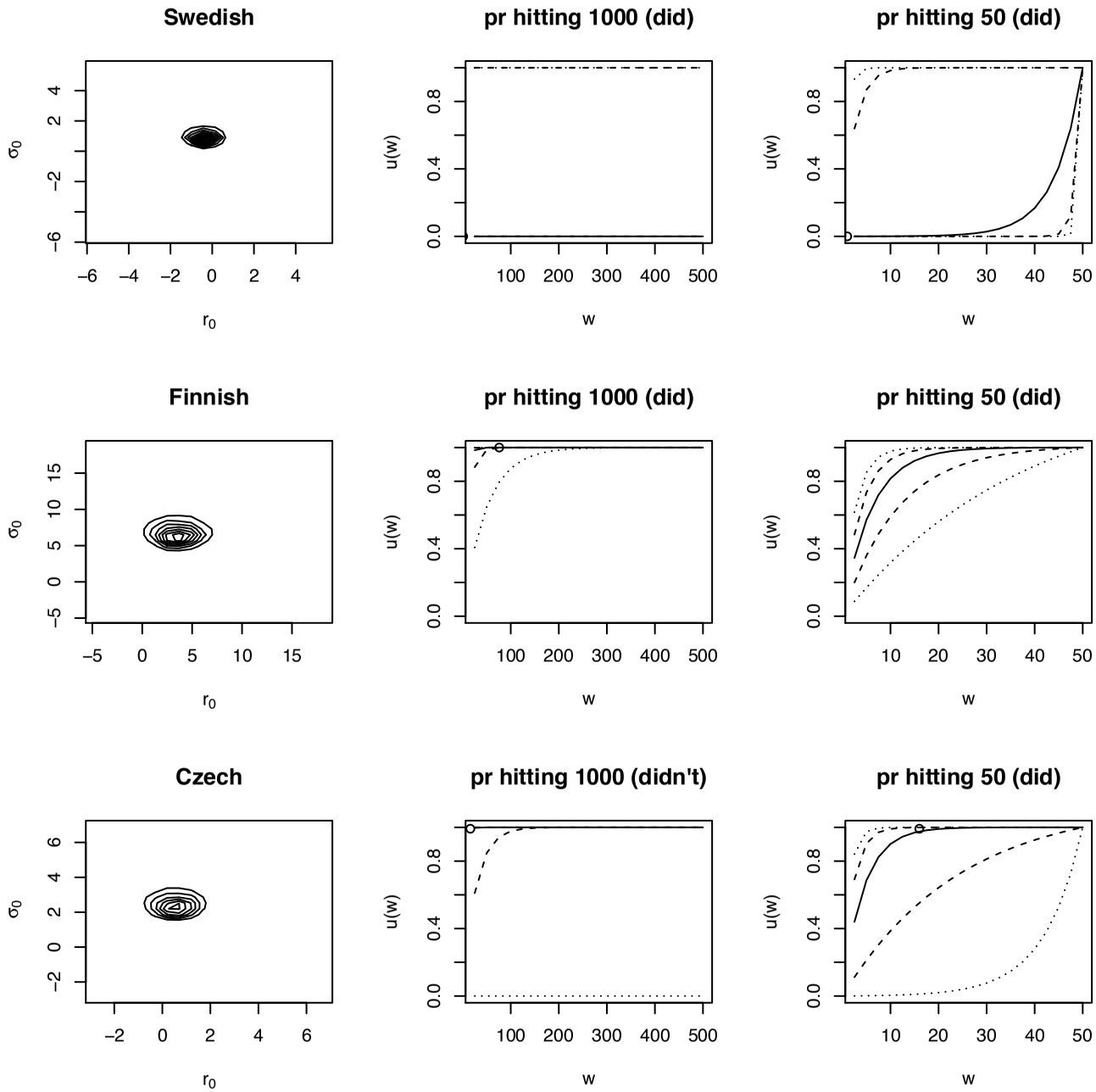
pr hitting 50 (did)
Dutch
pr hitting 1000 (did)
pr hitting 50 (did)
Turkish
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w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pr hitting 1000 (did)
pr hitting 50 (did)
Finnish
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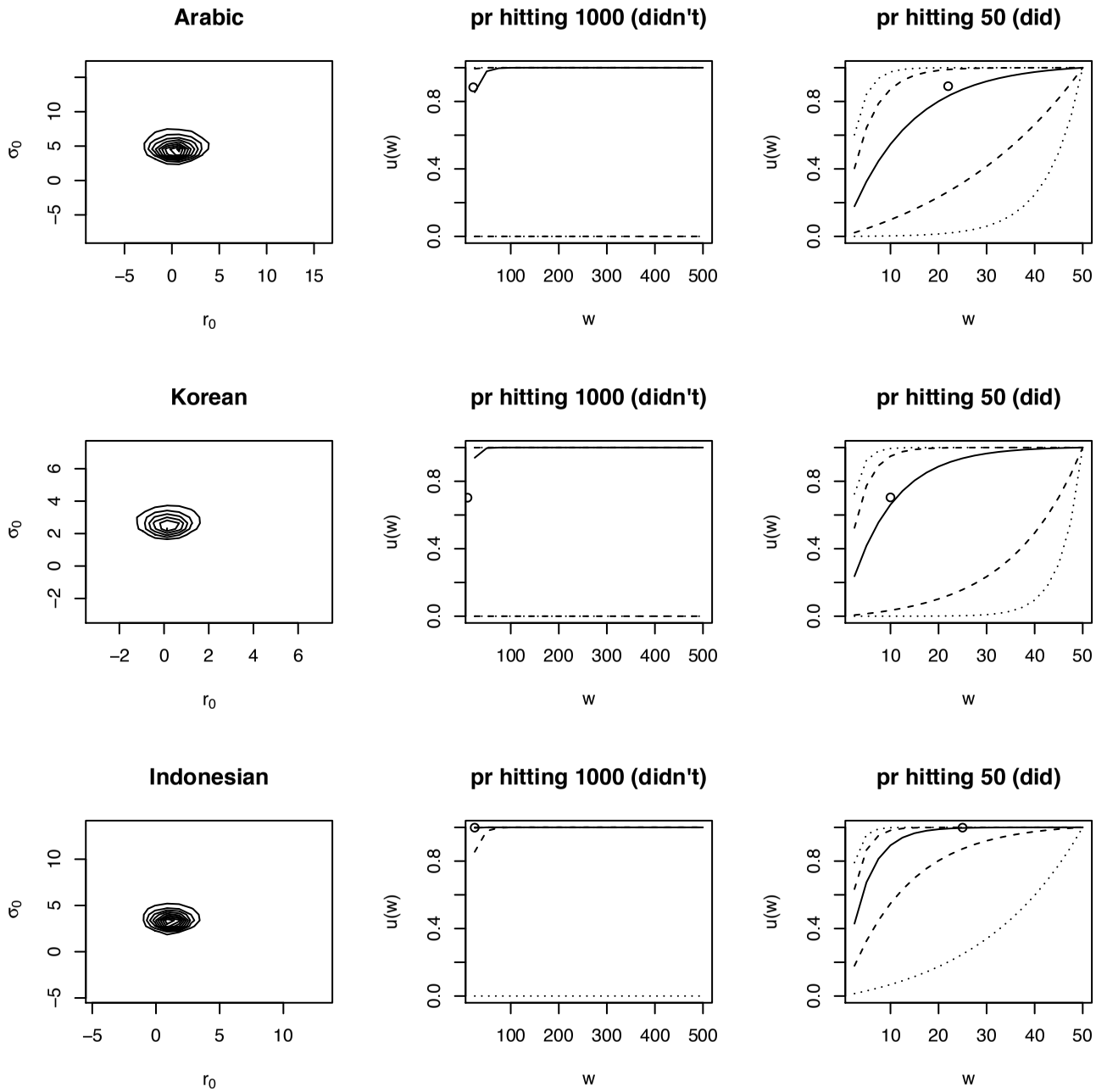
pr hitting 50 (did)
Czech
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w


Arabic
pr hitting 1000 (didn't)
pr hitting 50 (did)
Korean
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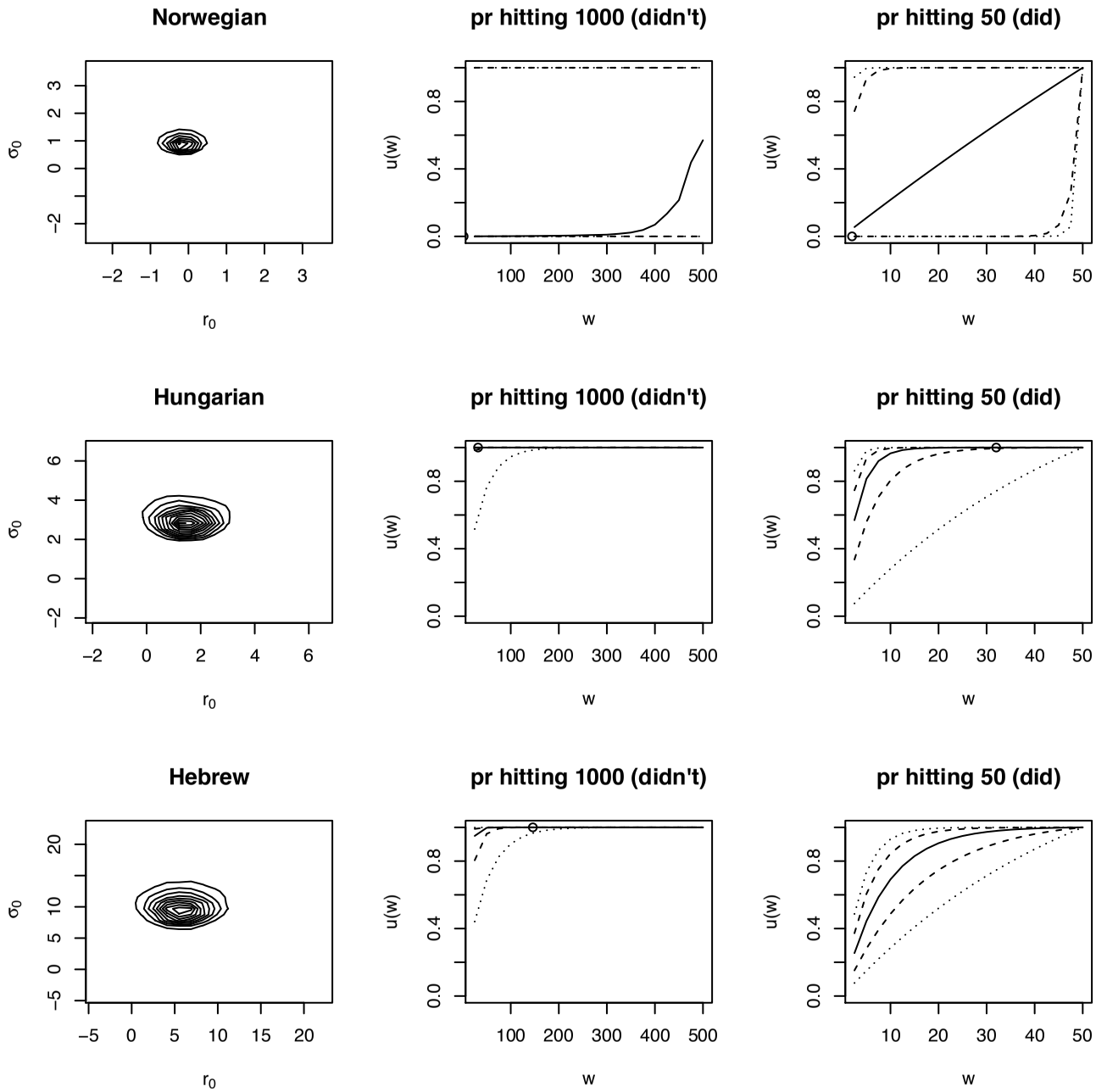
pr hitting 50 (did)
Indonesian
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w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pr hitting 50 (did)
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Hebrew
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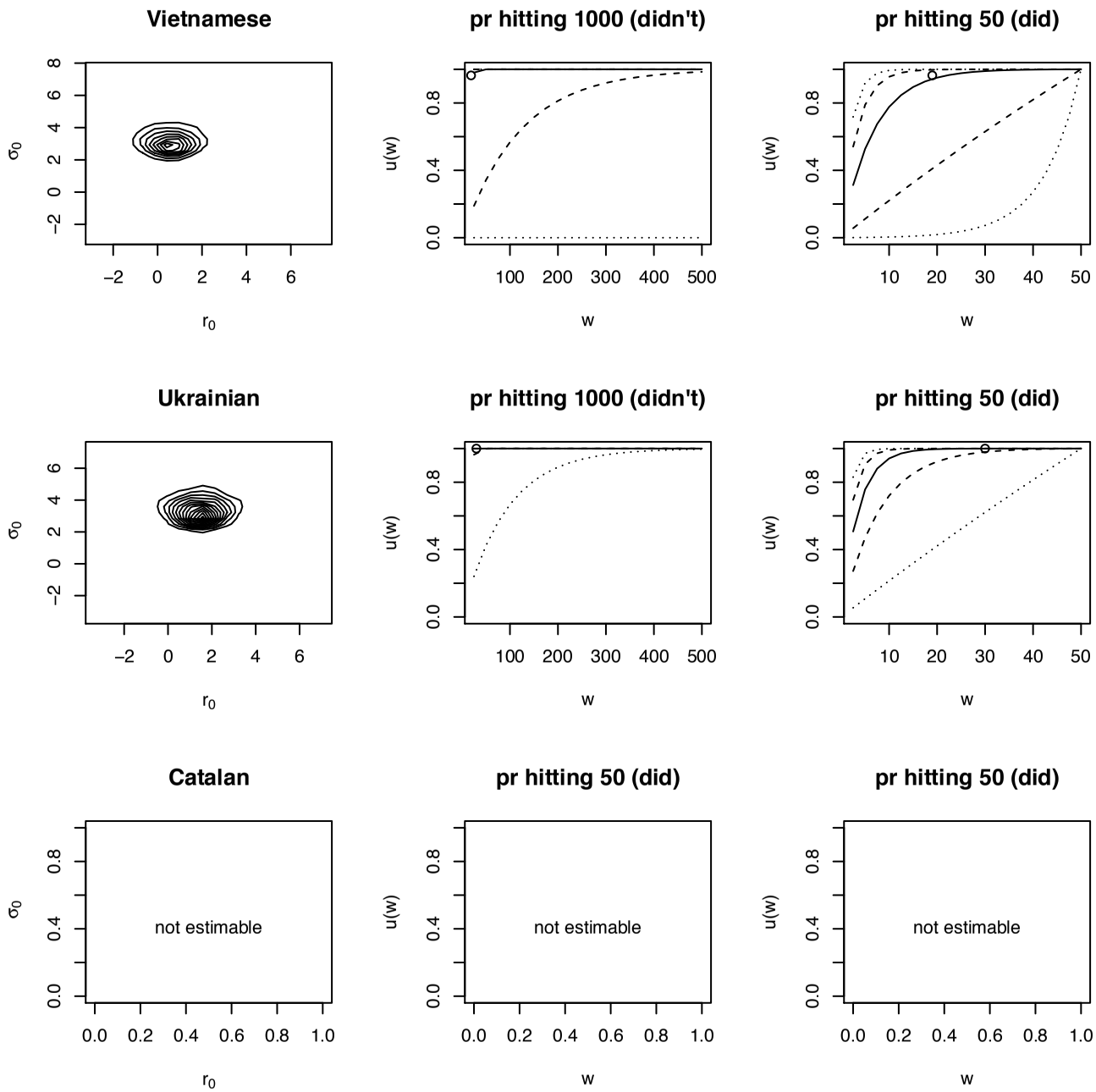
pr hitting 50 (did)
σ0
r0
u(w)
w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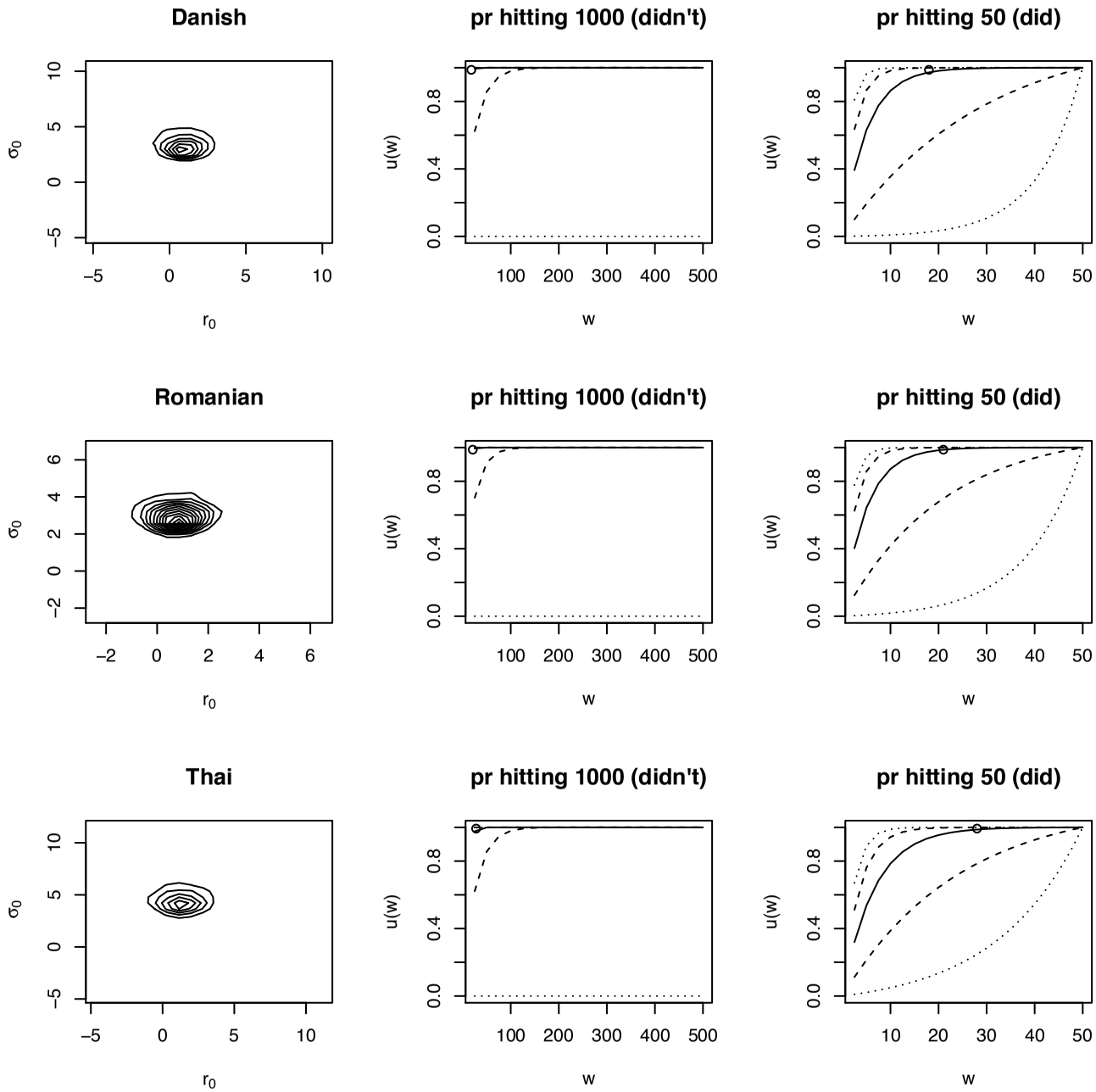
pr hitting 50 (did)
Ukrainian
pr hitting 1000 (didn't)
pr hitting 50 (did)
Catalan
pr hitting 50 (did)
pr hitting 50 (did)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w


Danish
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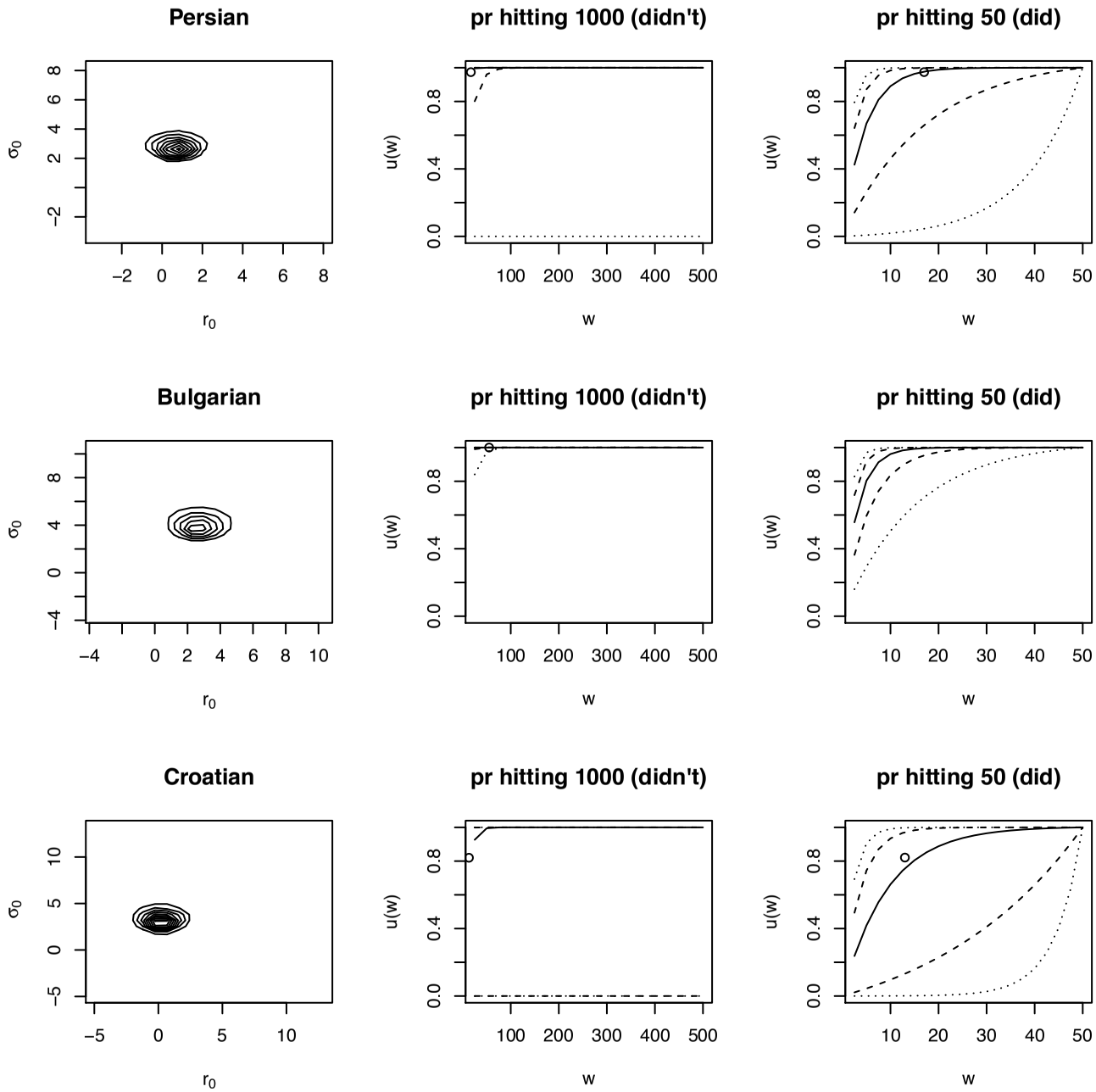
pr hitting 50 (did)
Romanian
pr hitting 1000 (didn't)
pr hitting 50 (did)
Thai
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w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pr hitting 50 (did)
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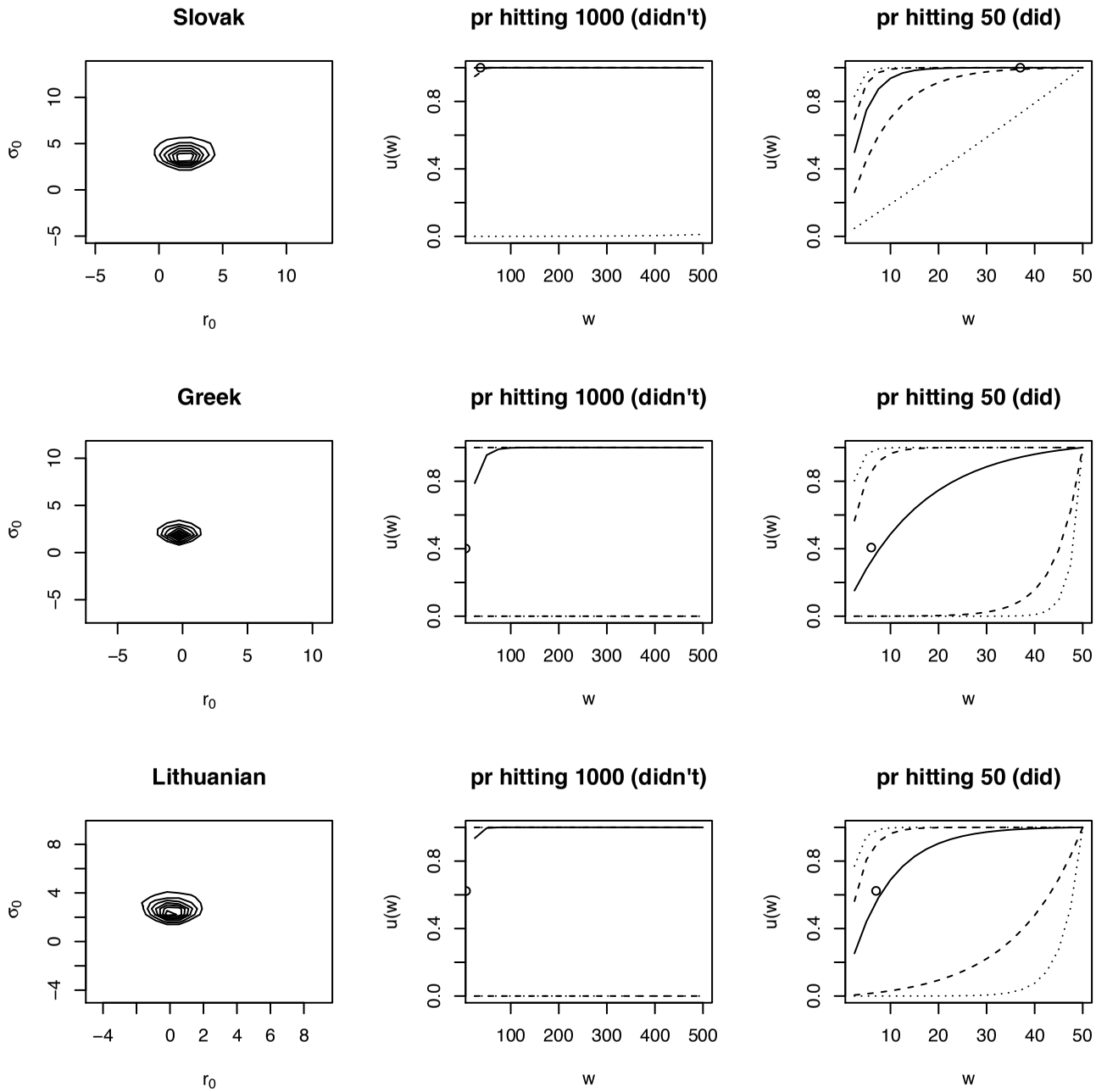
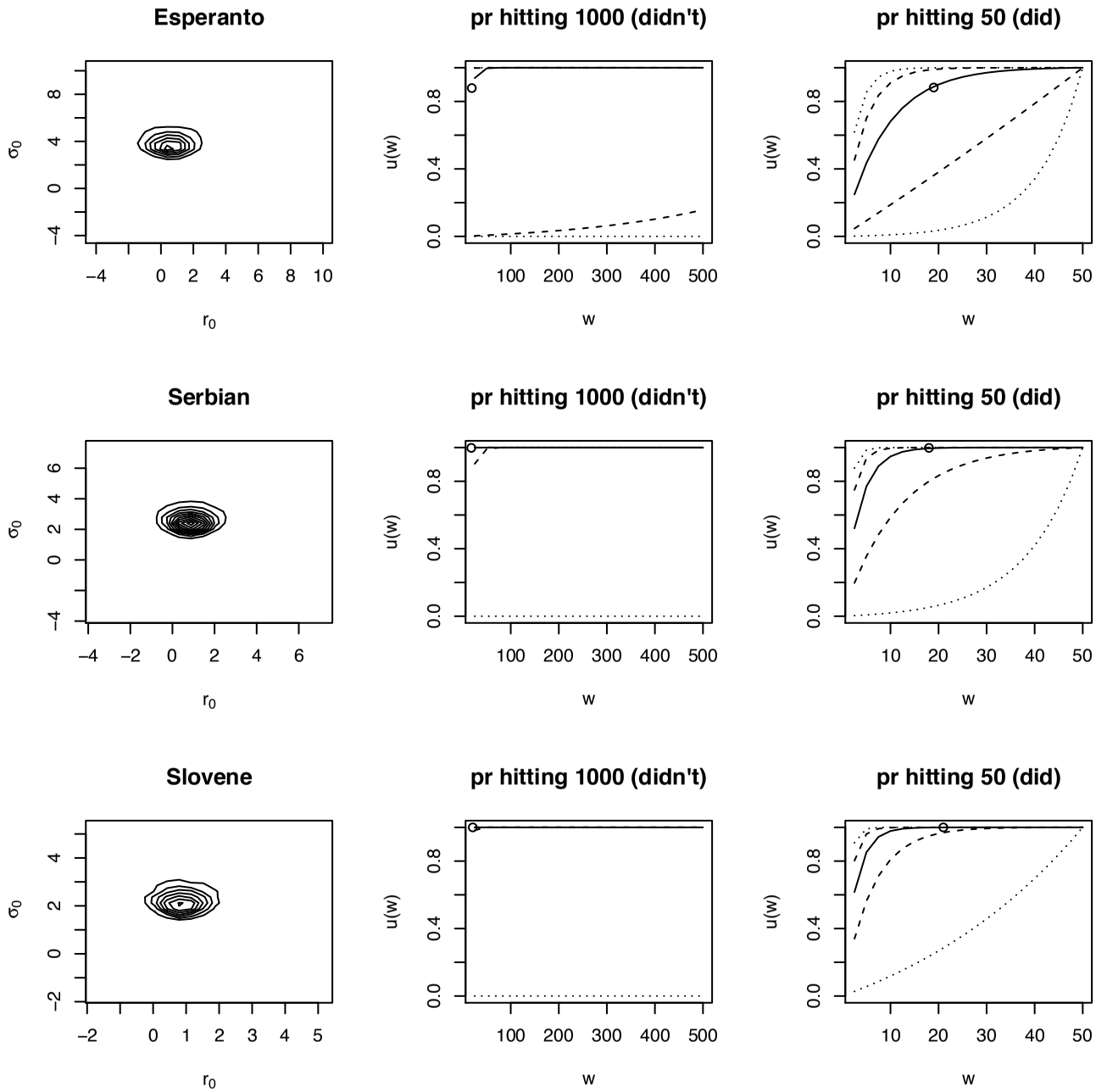
pr hitting 50 (did)
Croatian
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w


Slovak
pr hitting 1000 (didn't)
pr hitting 50 (did)
Greek
pr hitting 1000 (didn't)
pr hitting 50 (did)
Lithuanian
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w


Esperanto
pr hitting 1000 (didn't)
pr hitting 50 (did)
Serbian
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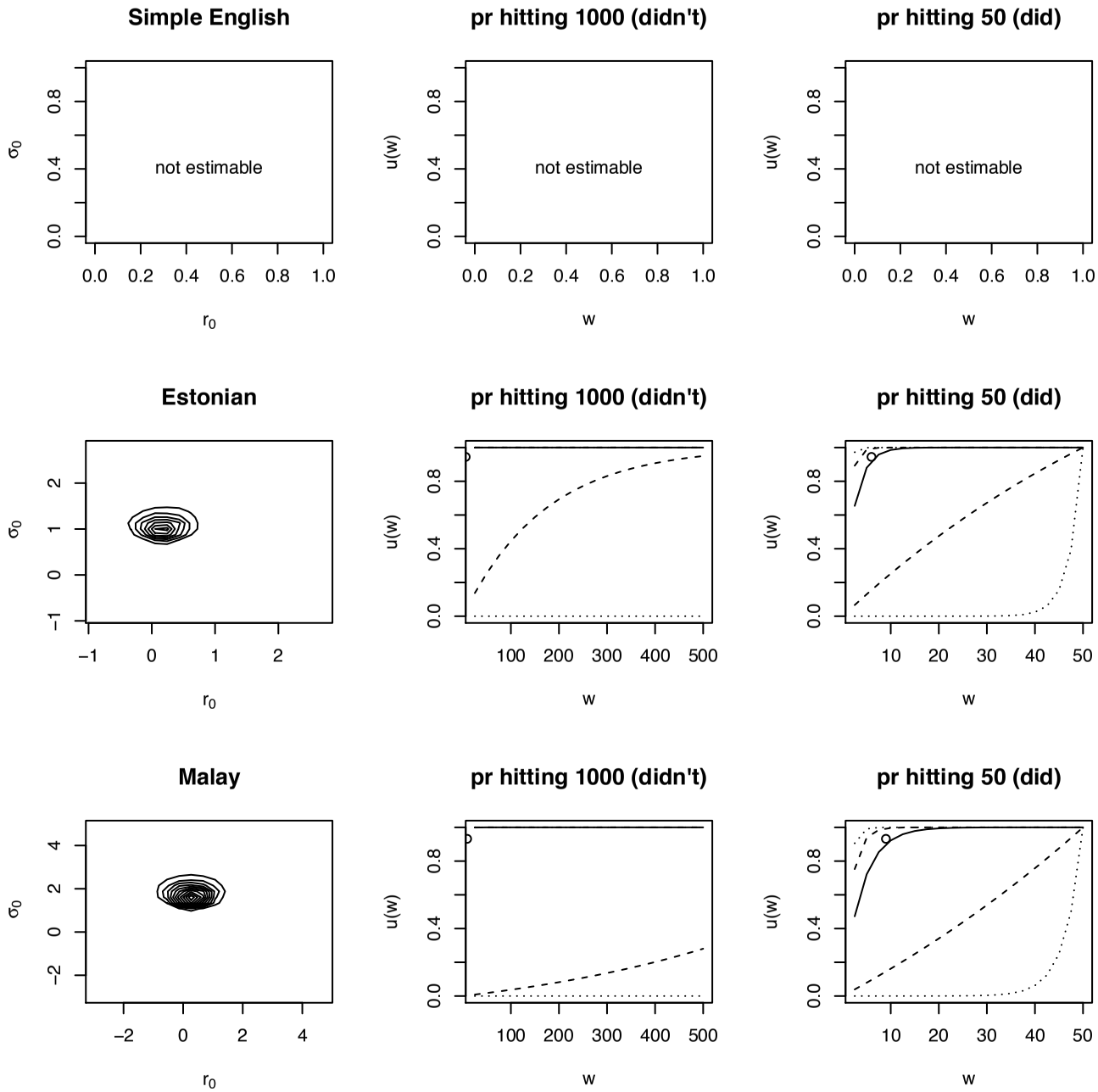
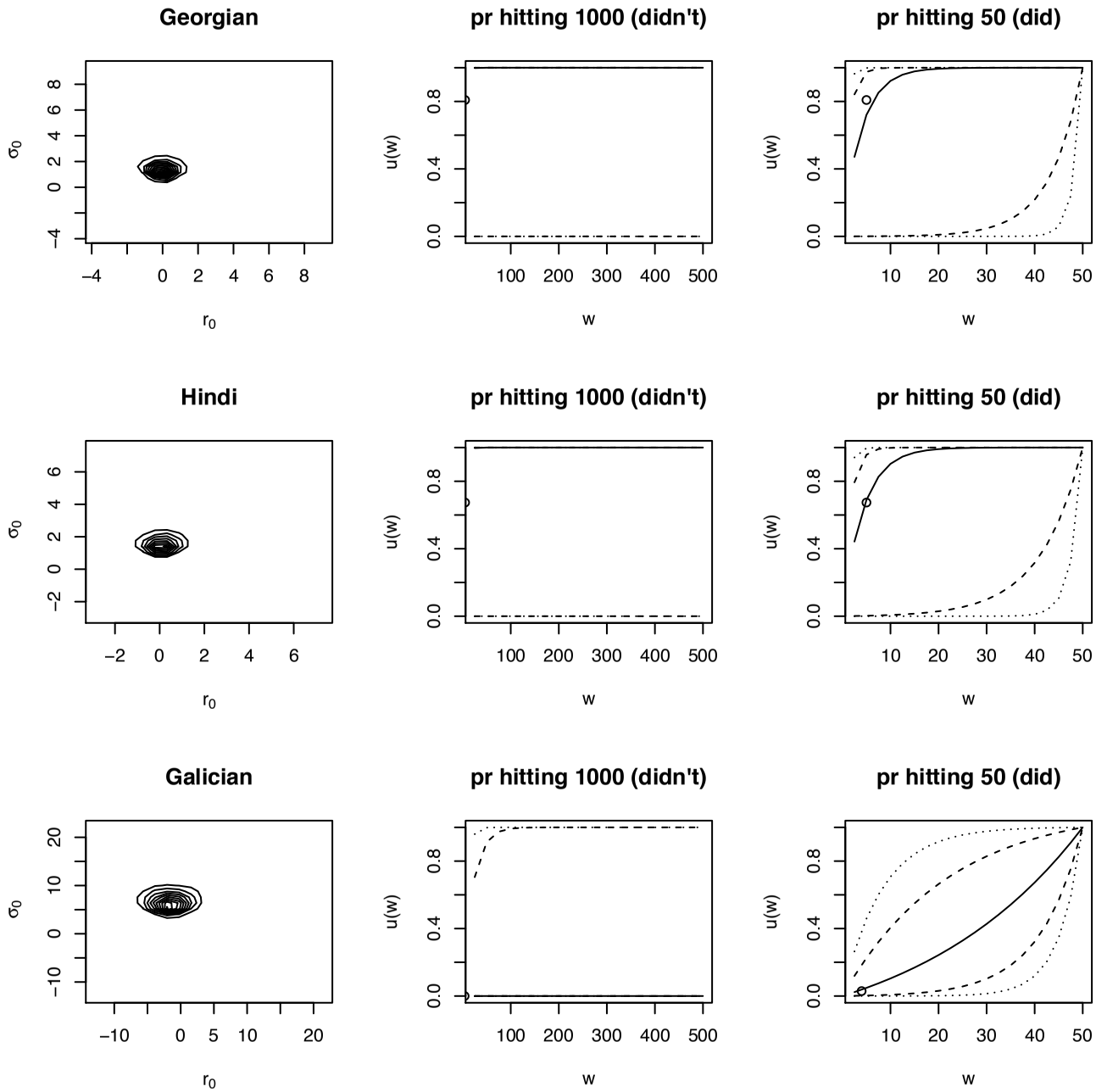
pr hitting 50 (did)
Slovene
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w


Simple English
not estimable
σ_0
r_0
pr hitting 1000 (didn't)
not estimable
u(w)
w
pr hitting 50 (did)
not estimable
u(w)
w
Estonian
σ_0
r_0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (did)
u(w)
w
Malay
σ_0
r_0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (did)
u(w)
w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pr hitting 50 (did)
Hindi
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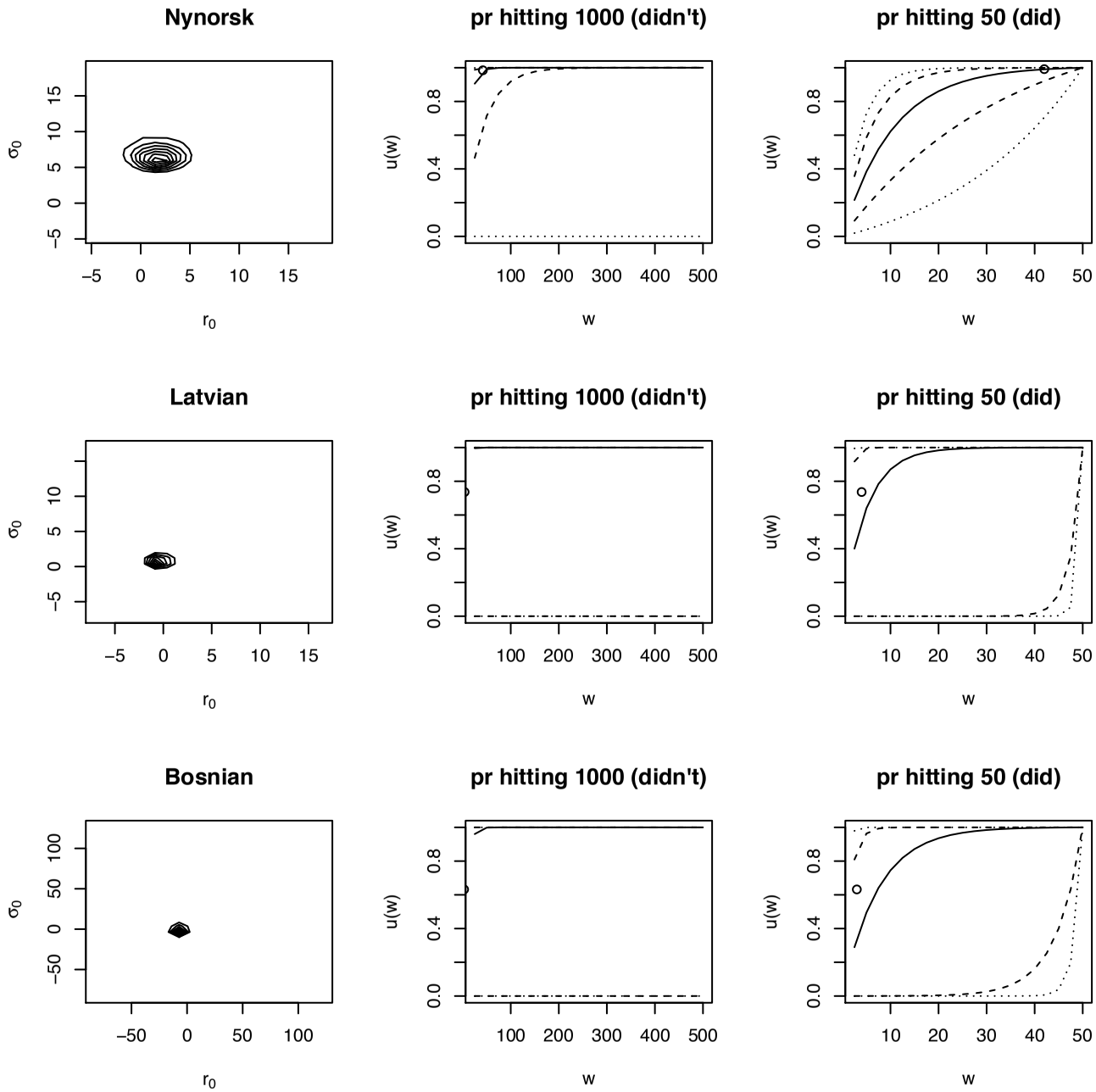
pr hitting 50 (did)
Galician
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w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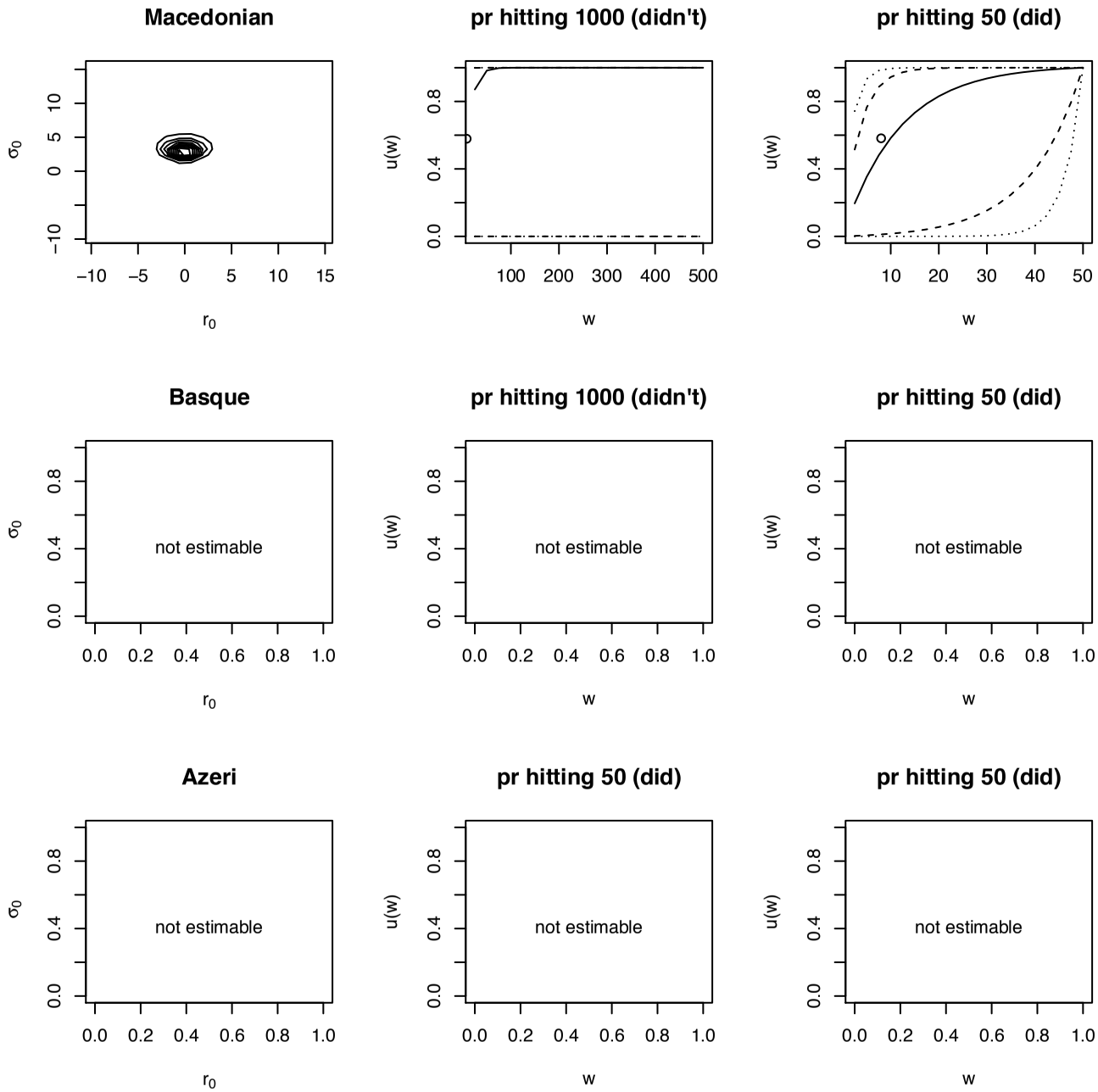
pr hitting 50 (did)
Latvian
pr hitting 1000 (didn't)
pr hitting 50 (did)
Bosnian
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w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pr hitting 50 (did)
Basque
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pr hitting 50 (did)
Azeri
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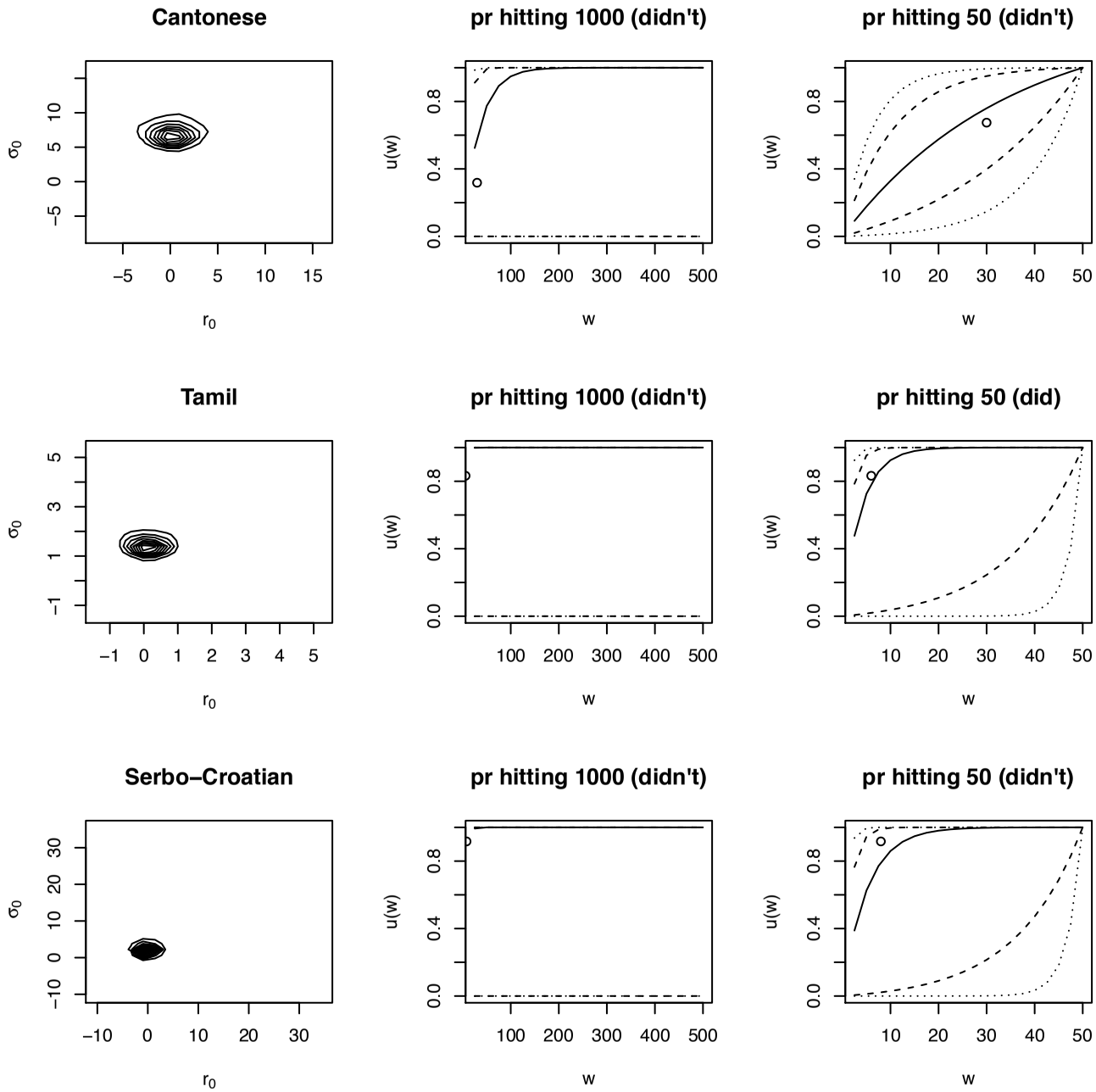
pr hitting 50 (did)
not estimable
not estimable
not estimable
not estimable
not estimable
not estimable
σ0
r0
u(w)
w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pr hitting 50 (didn't)
Tamil
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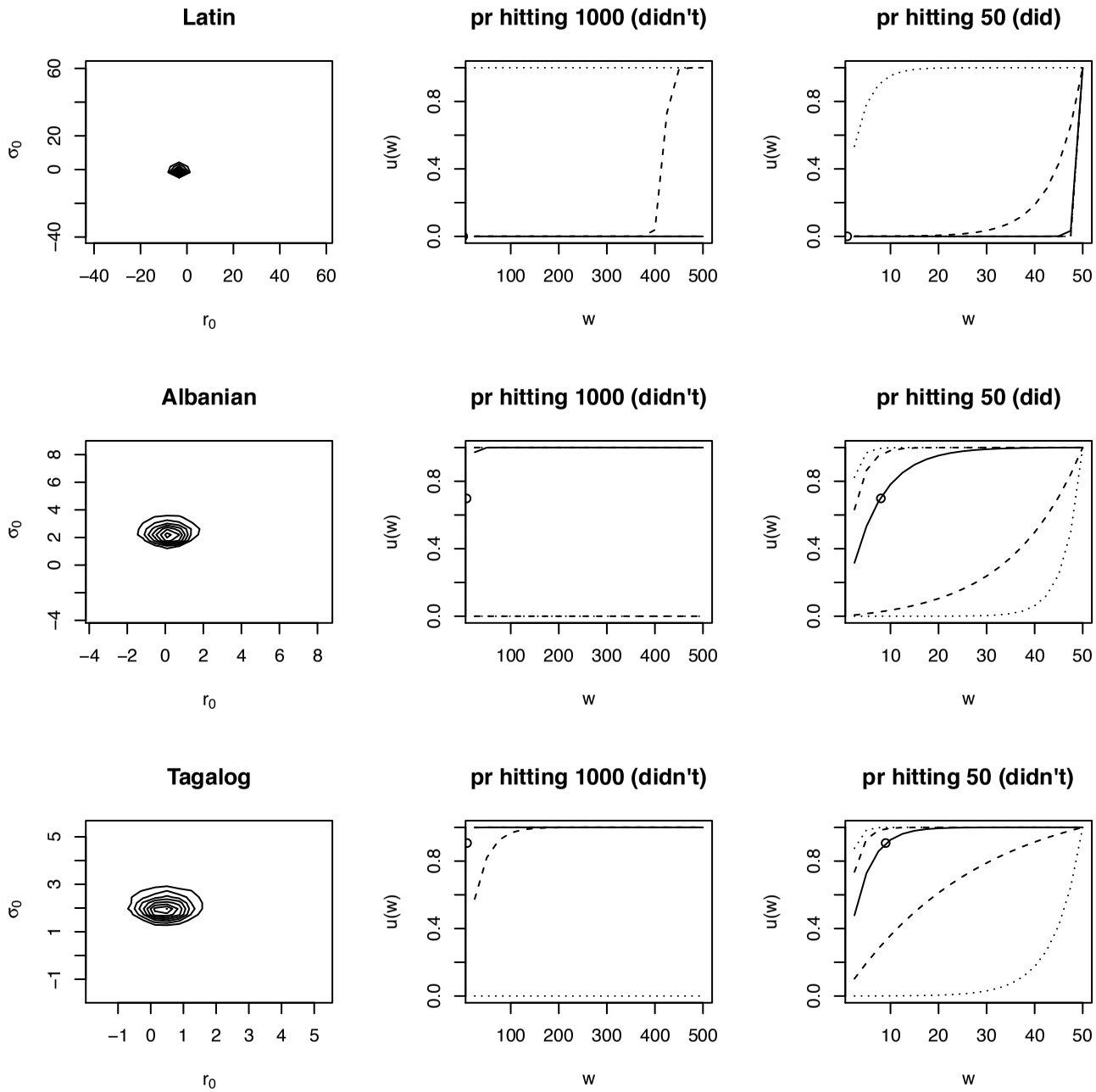
pr hitting 50 (did)
Serbo–Croatian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w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pr hitting 50 (did)
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pr hitting 50 (did)
Tagalog
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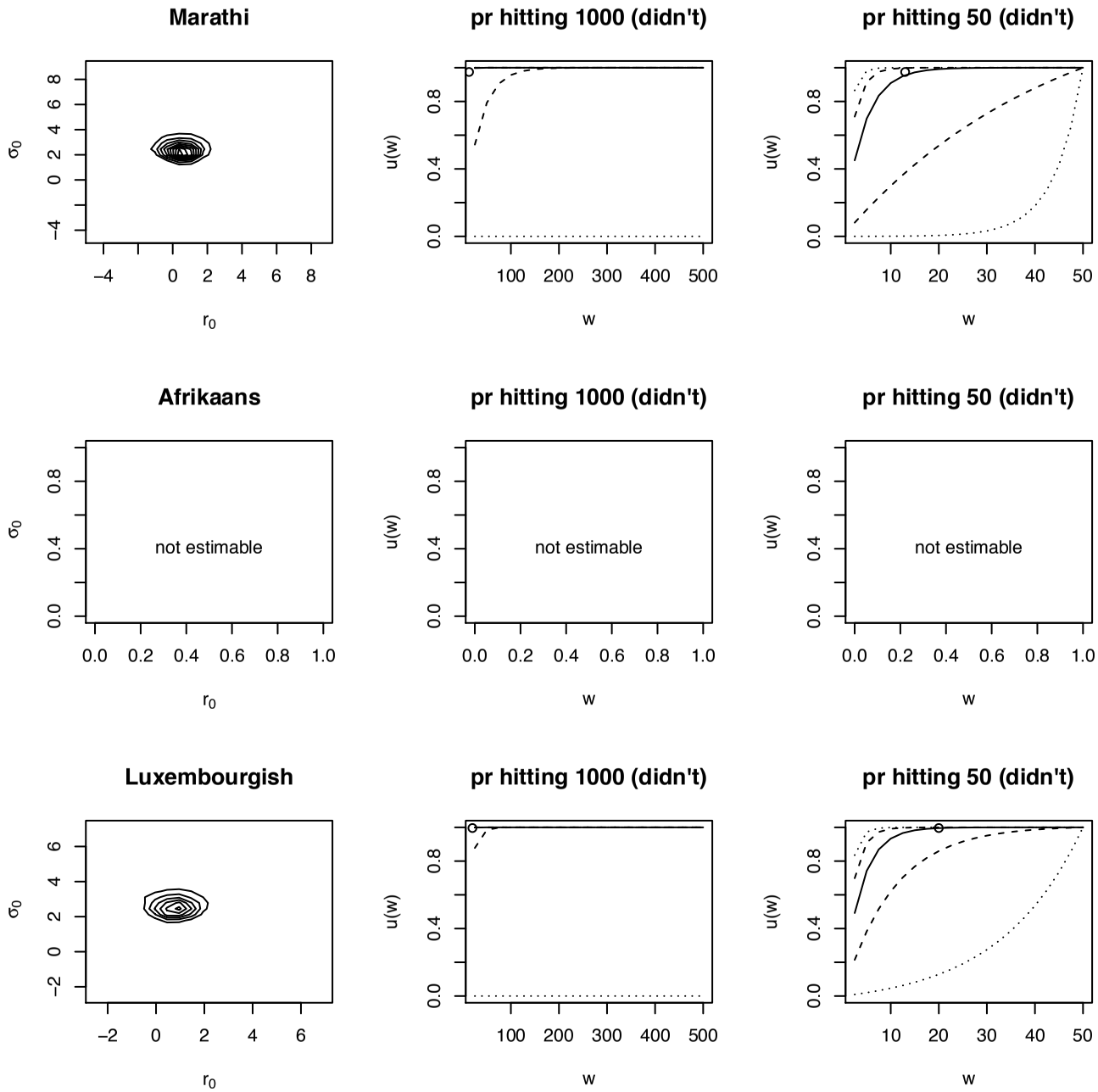
pr hitting 50 (didn't)
σ0
r0
u(w)
w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pr hitting 50 (didn't)
Afrikaans
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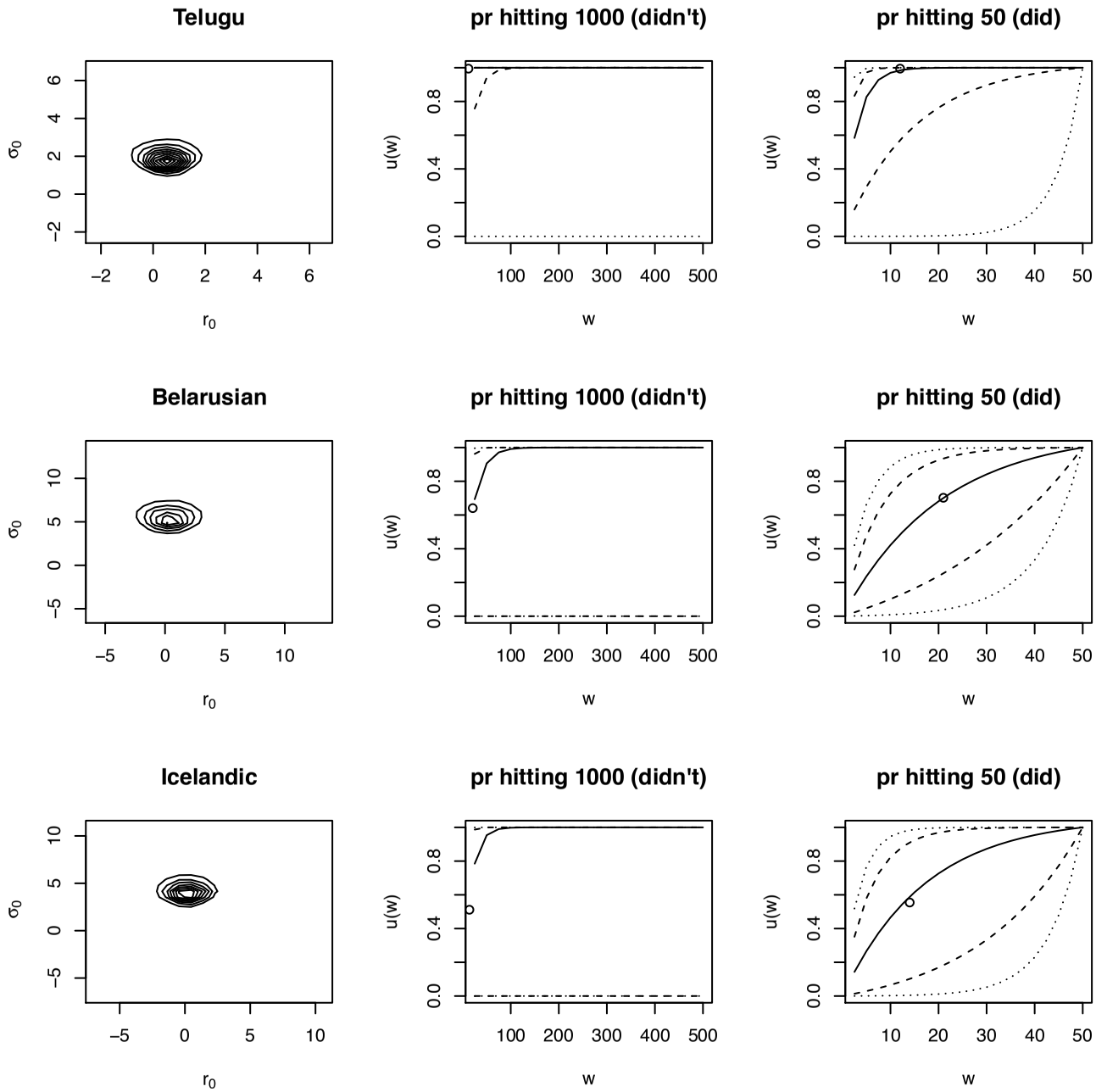
not estimable
not estimable
Luxembourgish
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w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pr hitting 1000 (didn't)
pr hitting 50 (did)
Belarusian
pr hitting 1000 (didn't)
pr hitting 50 (did)
Icelandic
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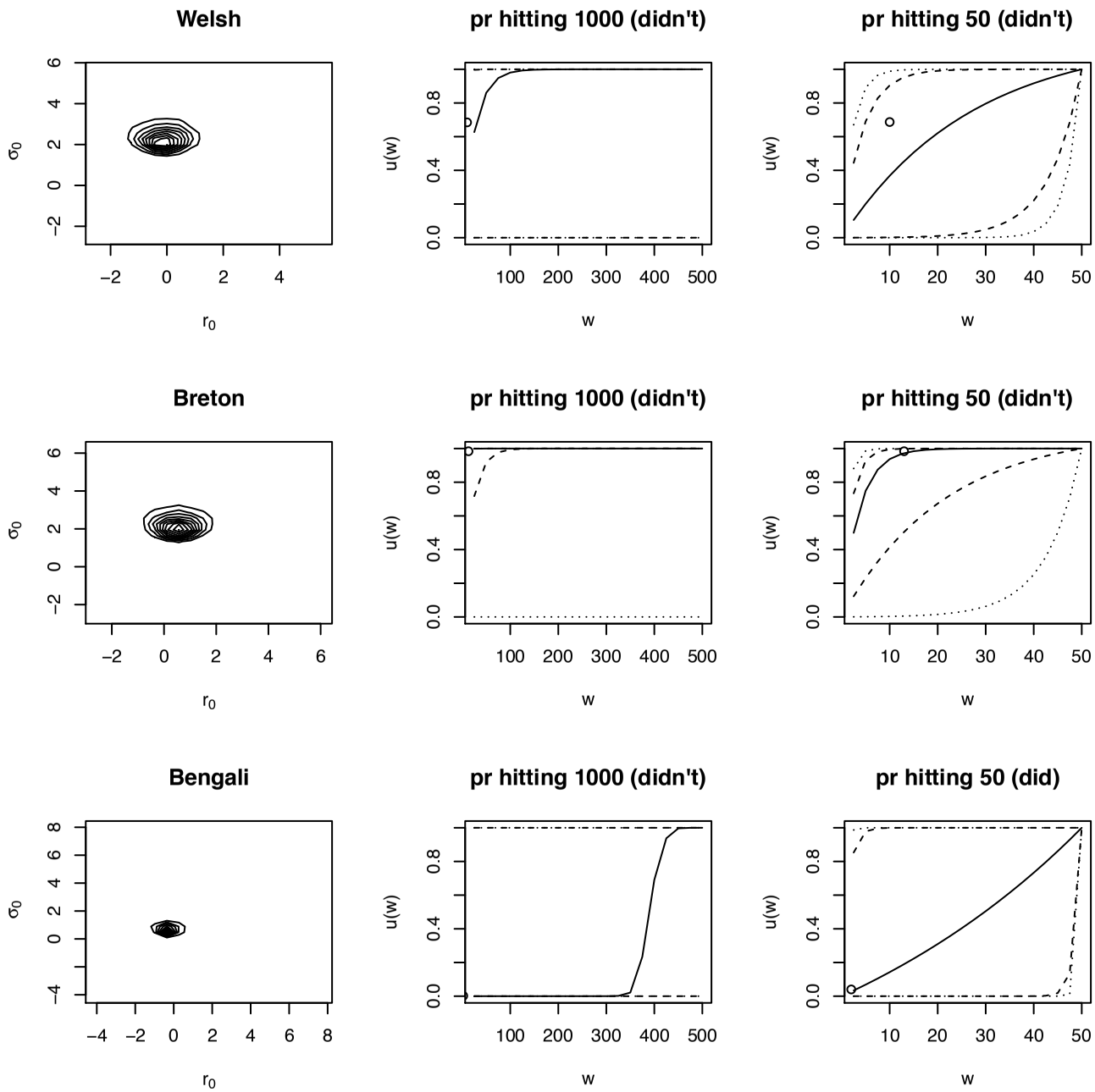
pr hitting 50 (did)
σ0
r0
u(w)
w


Welsh
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Breton
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pr hitting 50 (didn't)
Bengali
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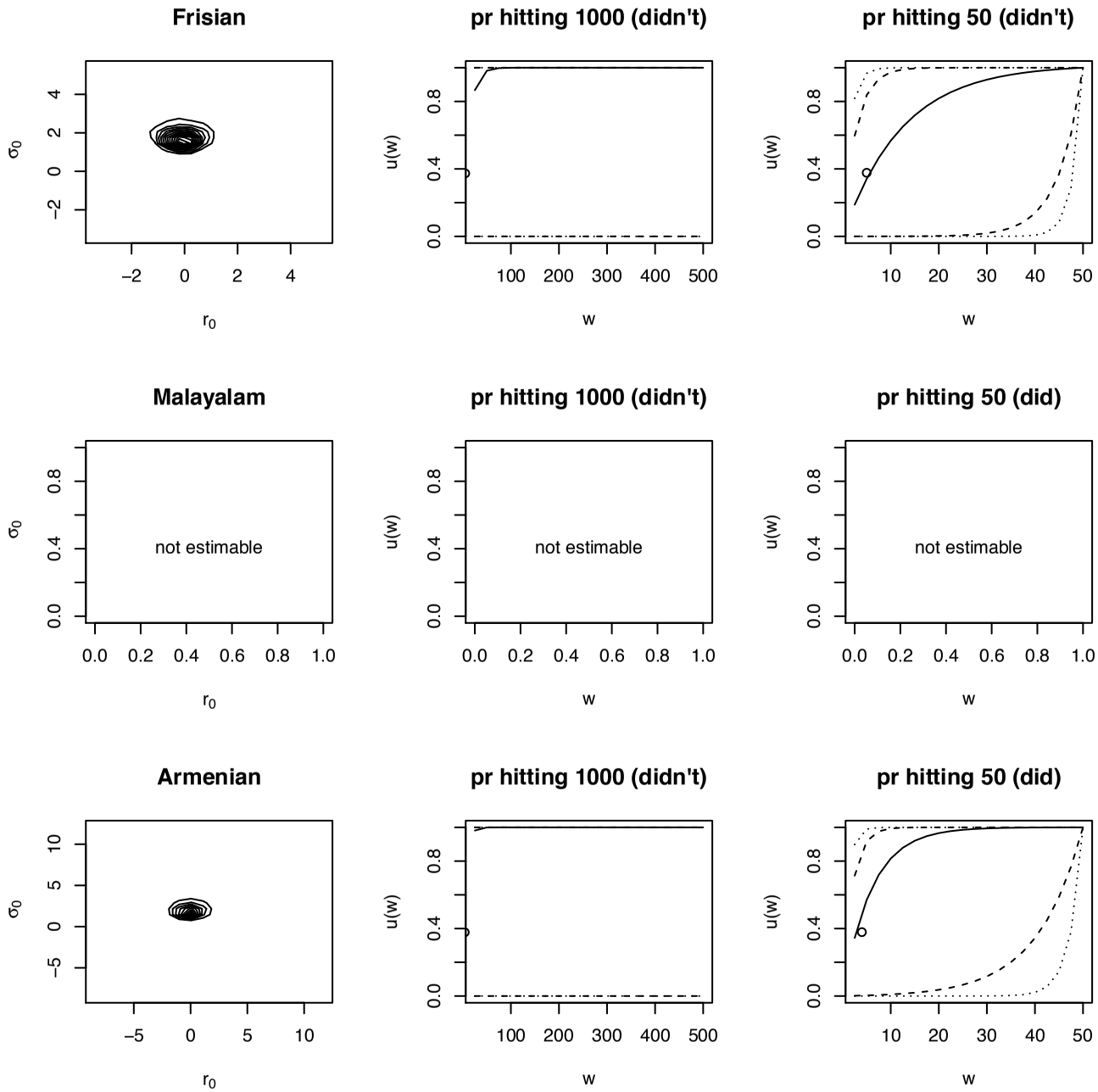
pr hitting 50 (did)
σ0
r0
u(w)
w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Malayalam
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pr hitting 50 (did)
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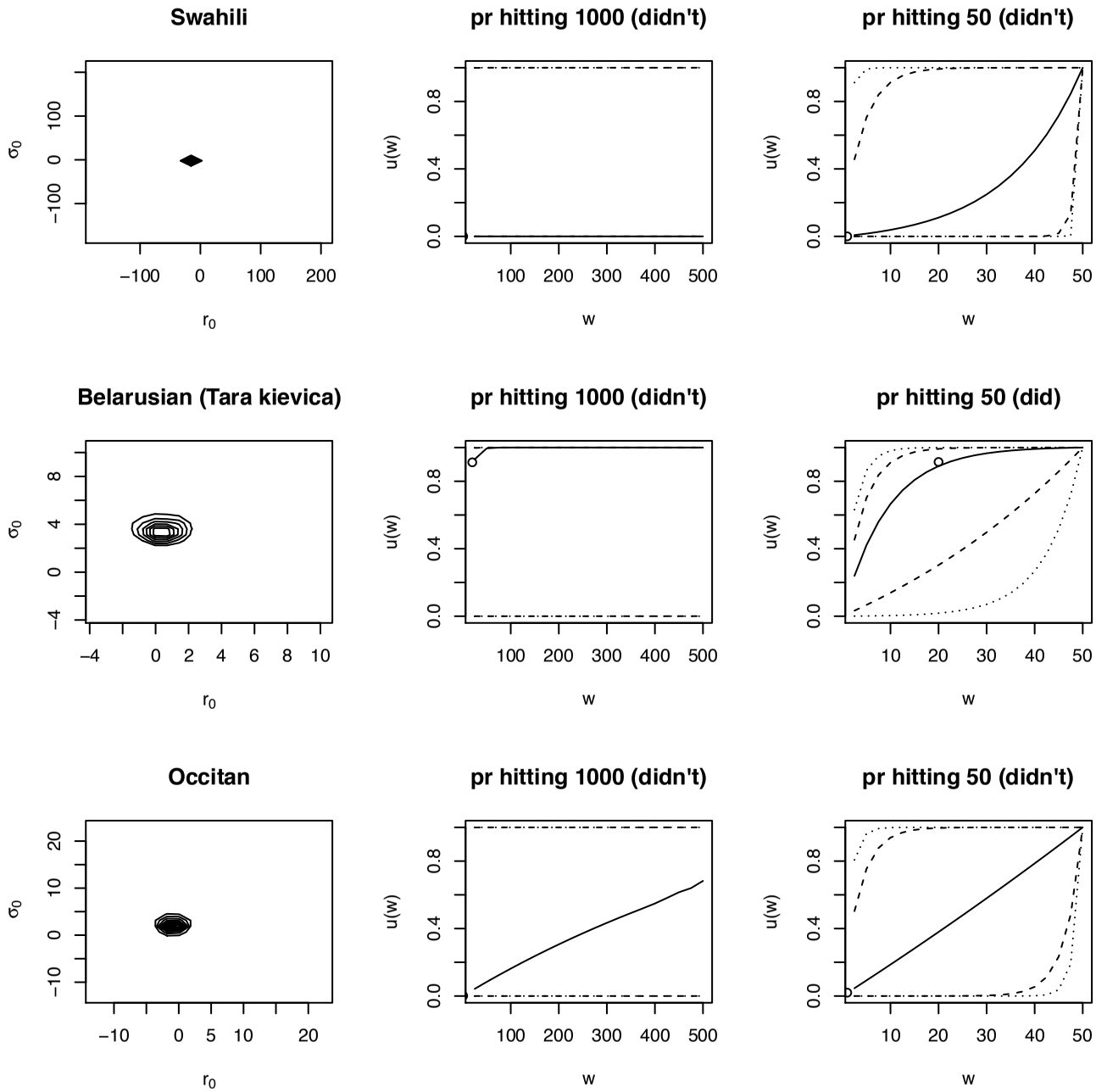
not estimable
not estimable
σ0
r0
u(w)
w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pr hitting 50 (didn't)
Belarusian (Tara kievica)
pr hitting 1000 (didn't)
pr hitting 50 (did)
Occitan
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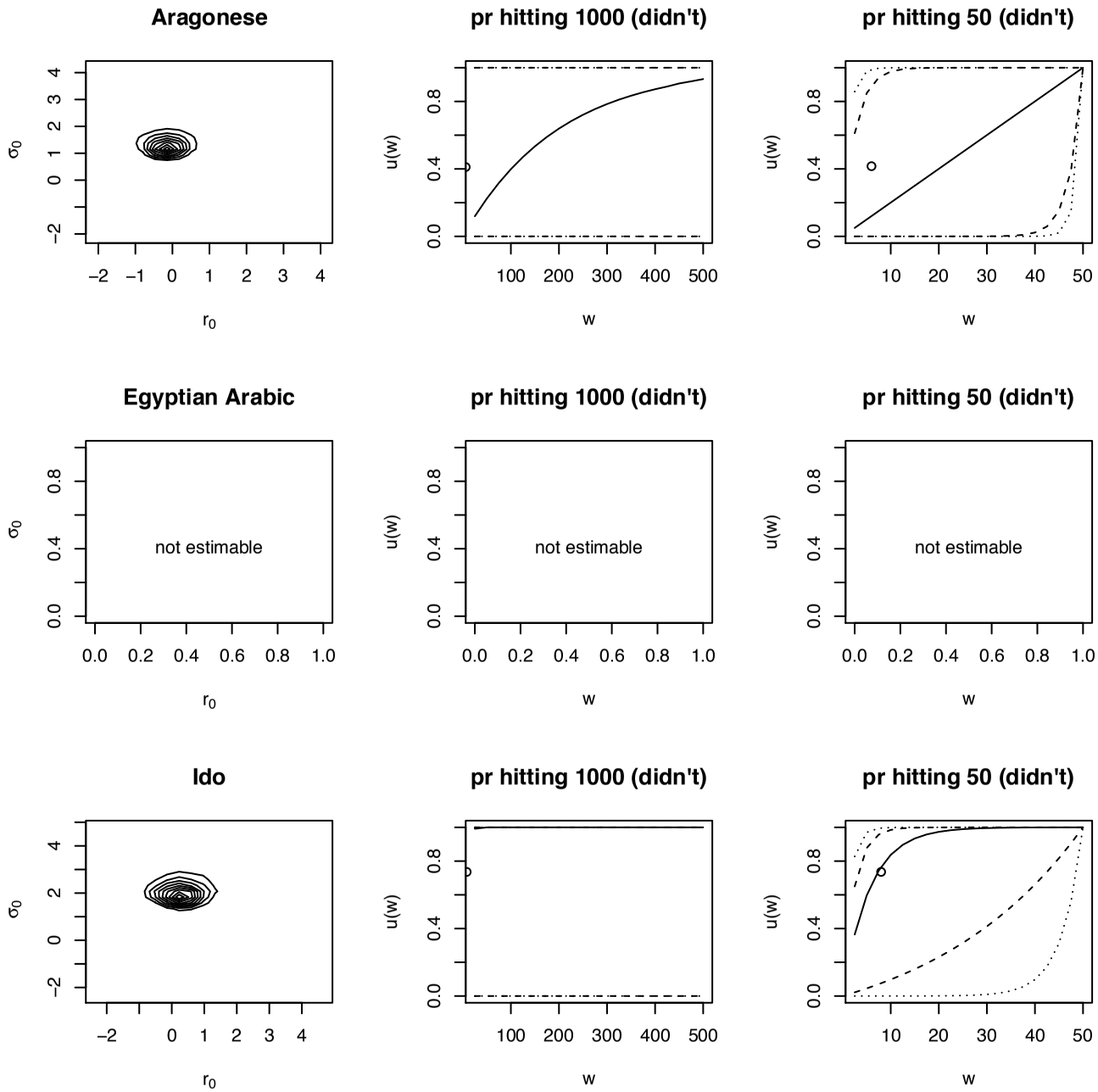
pr hitting 50 (didn't)
σ0
r0
u(w)
w


Aragonese
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pr hitting 50 (didn't)
Egyptian Arabic
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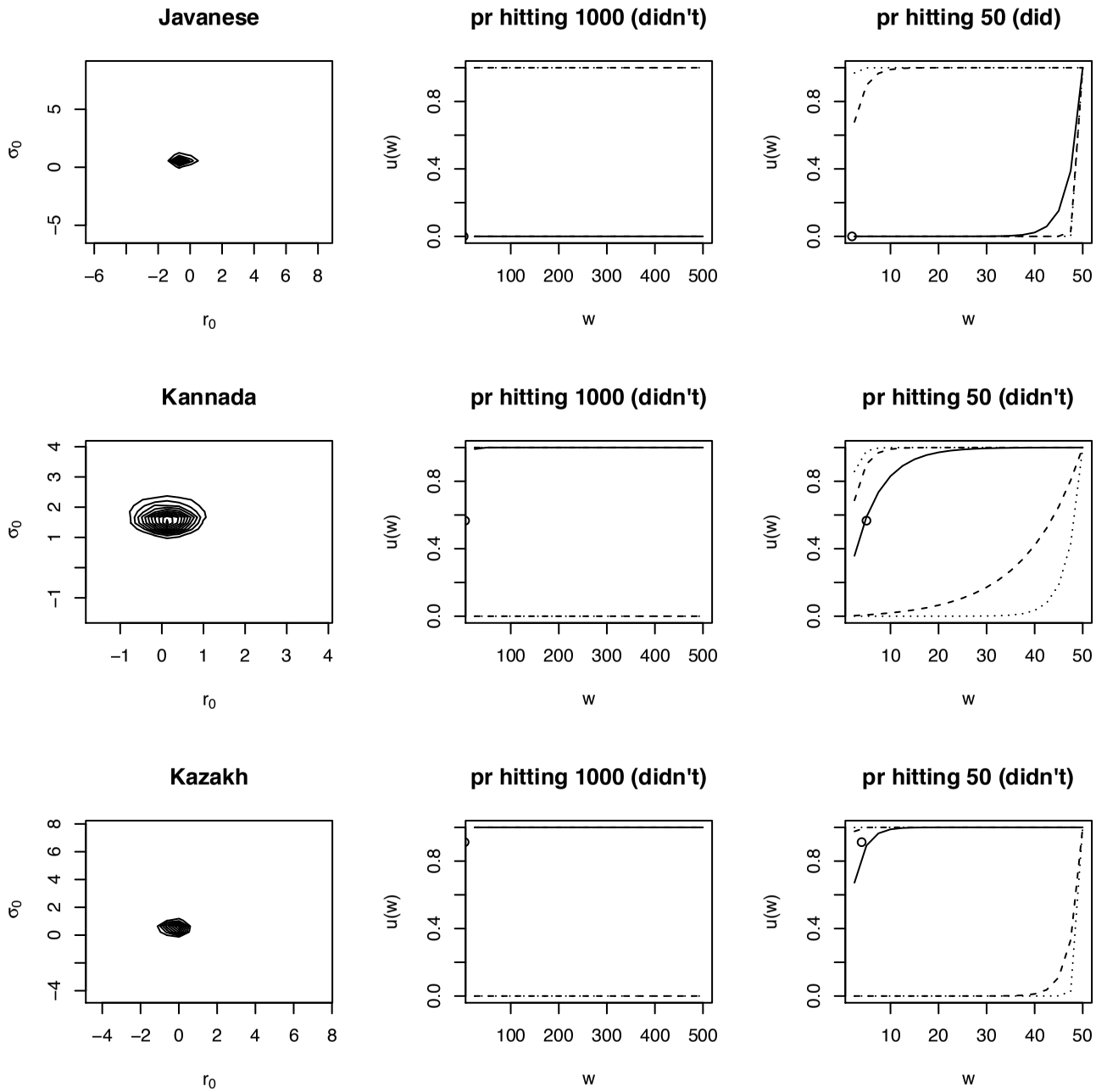
not estimable
not estimable
Ido
σ0
r0
u(w)
w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Kannada
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Kazakh
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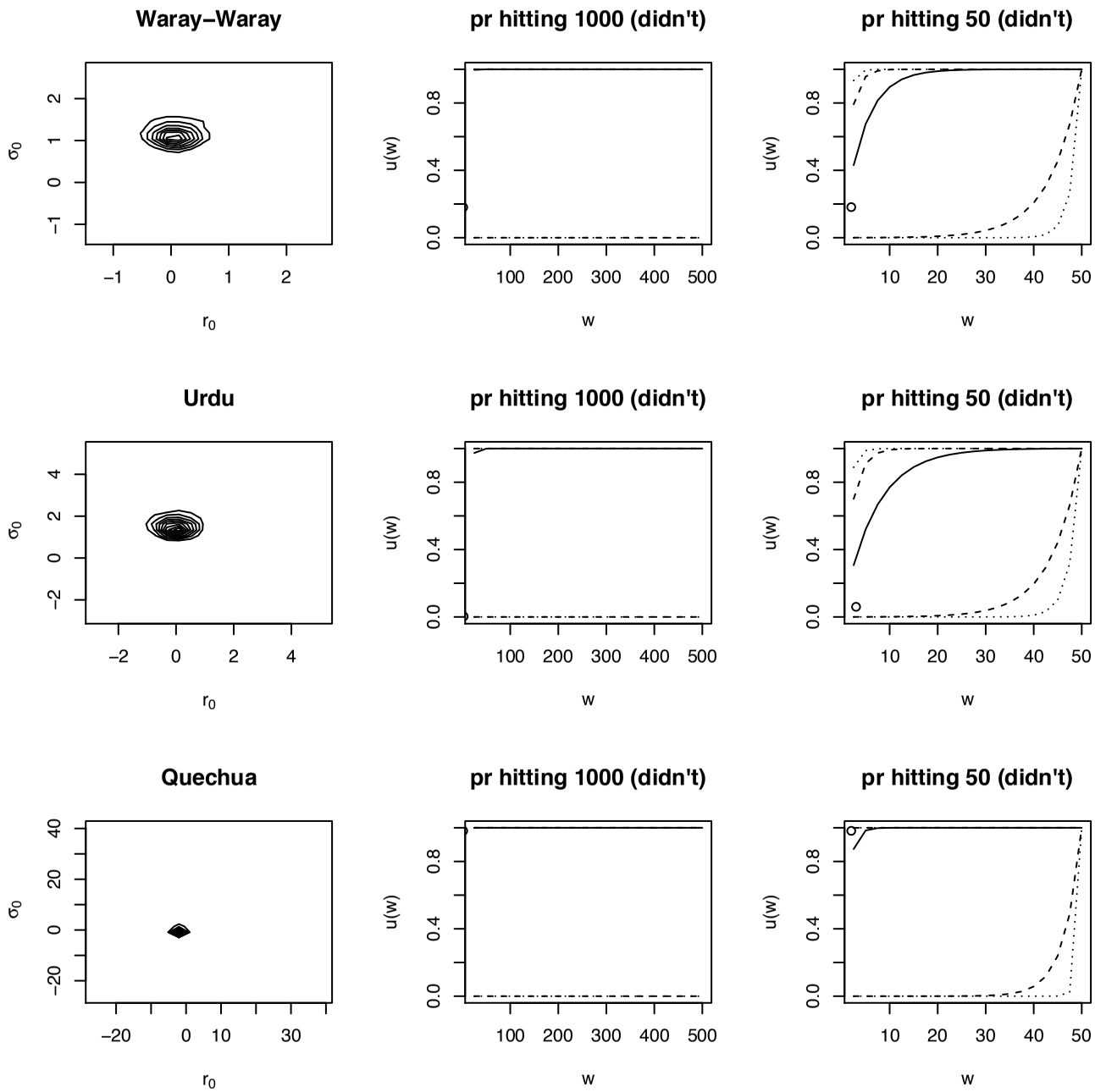
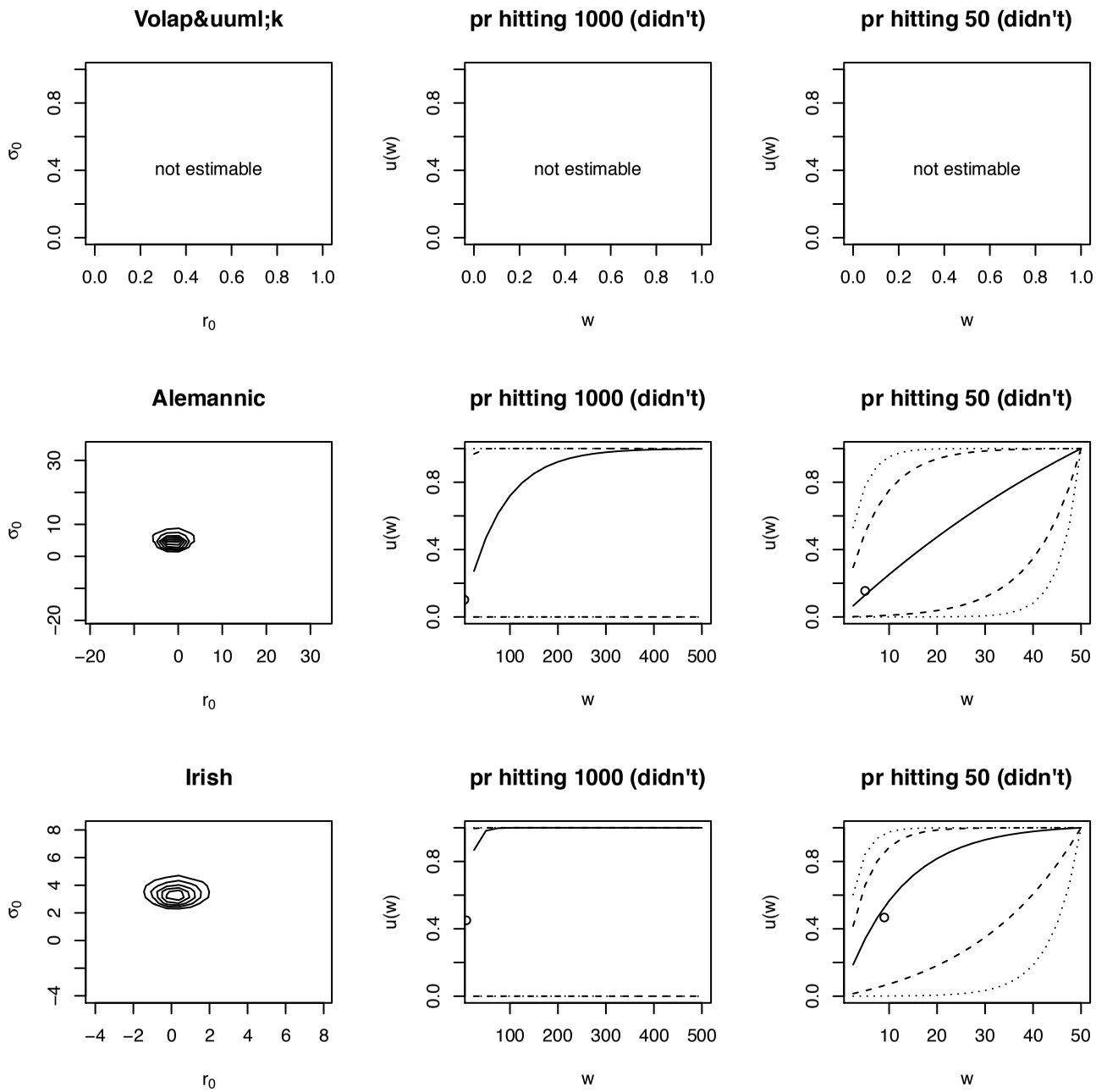
pr hitting 50 (didn't)
σ0
r0
u(w)
w


Waray–Waray
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Urdu
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Quechua
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w


Volapük
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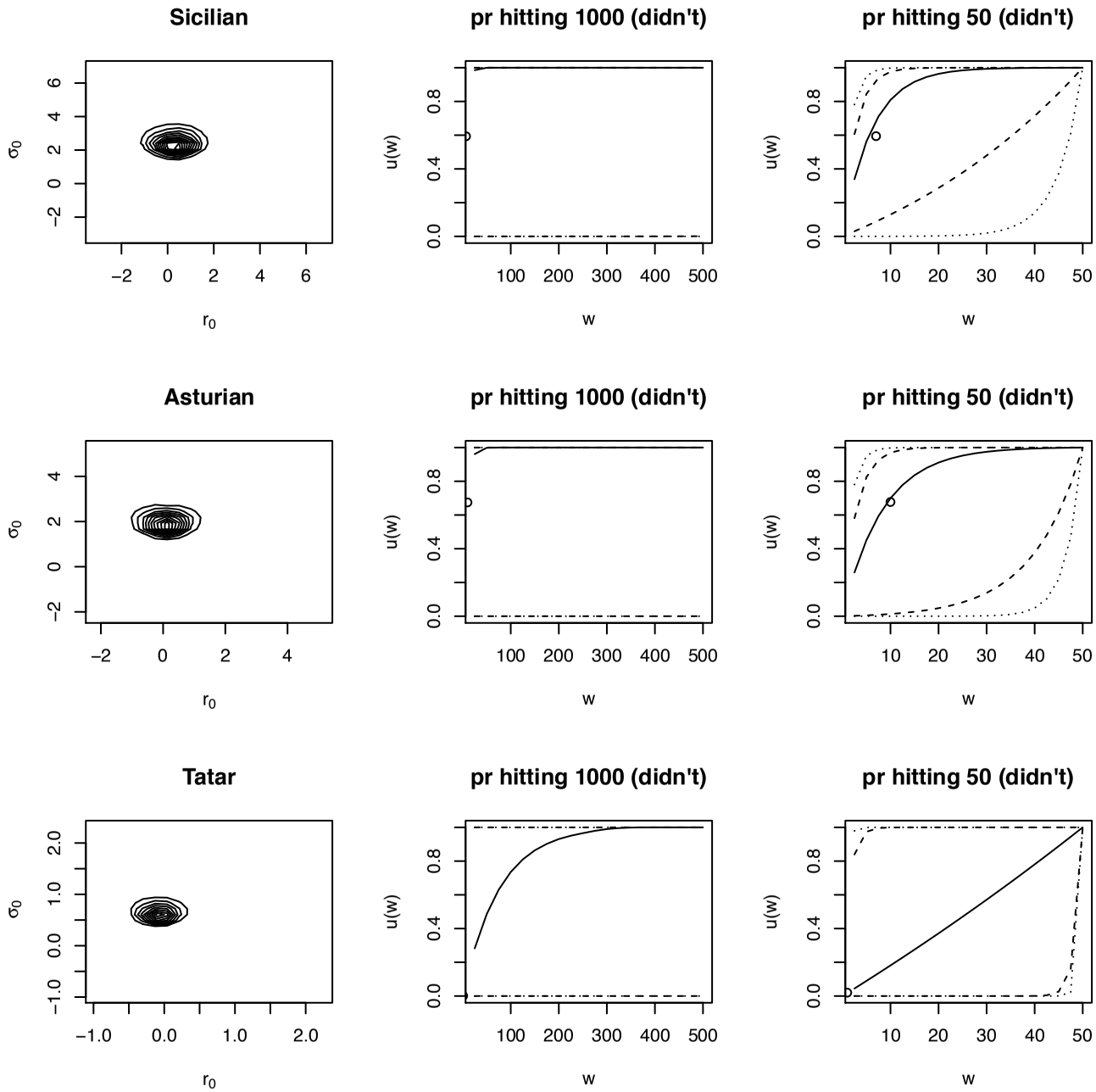
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w
Alemannic
Irish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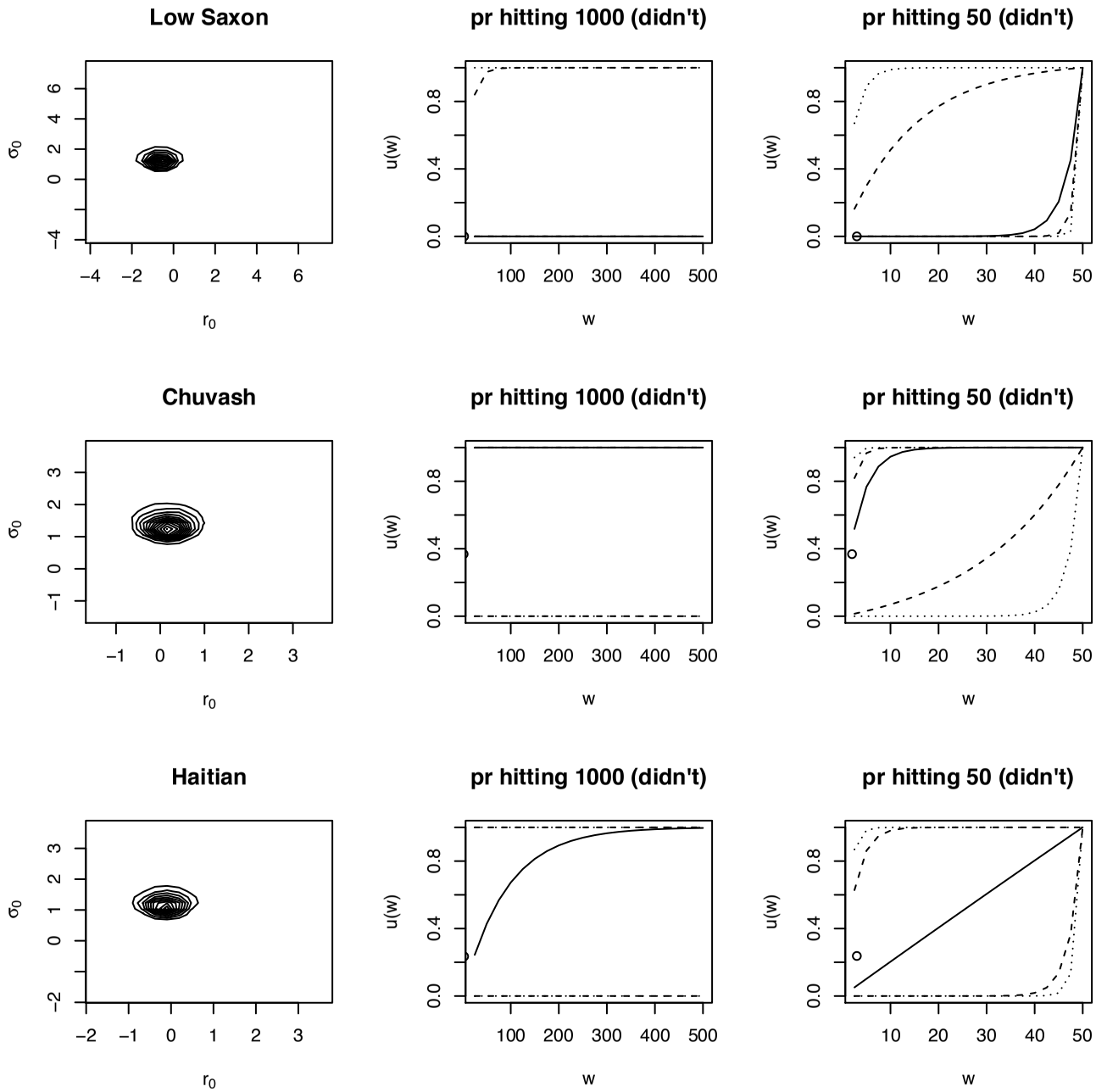
pr hitting 50 (didn't)
Asturian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Tatar
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w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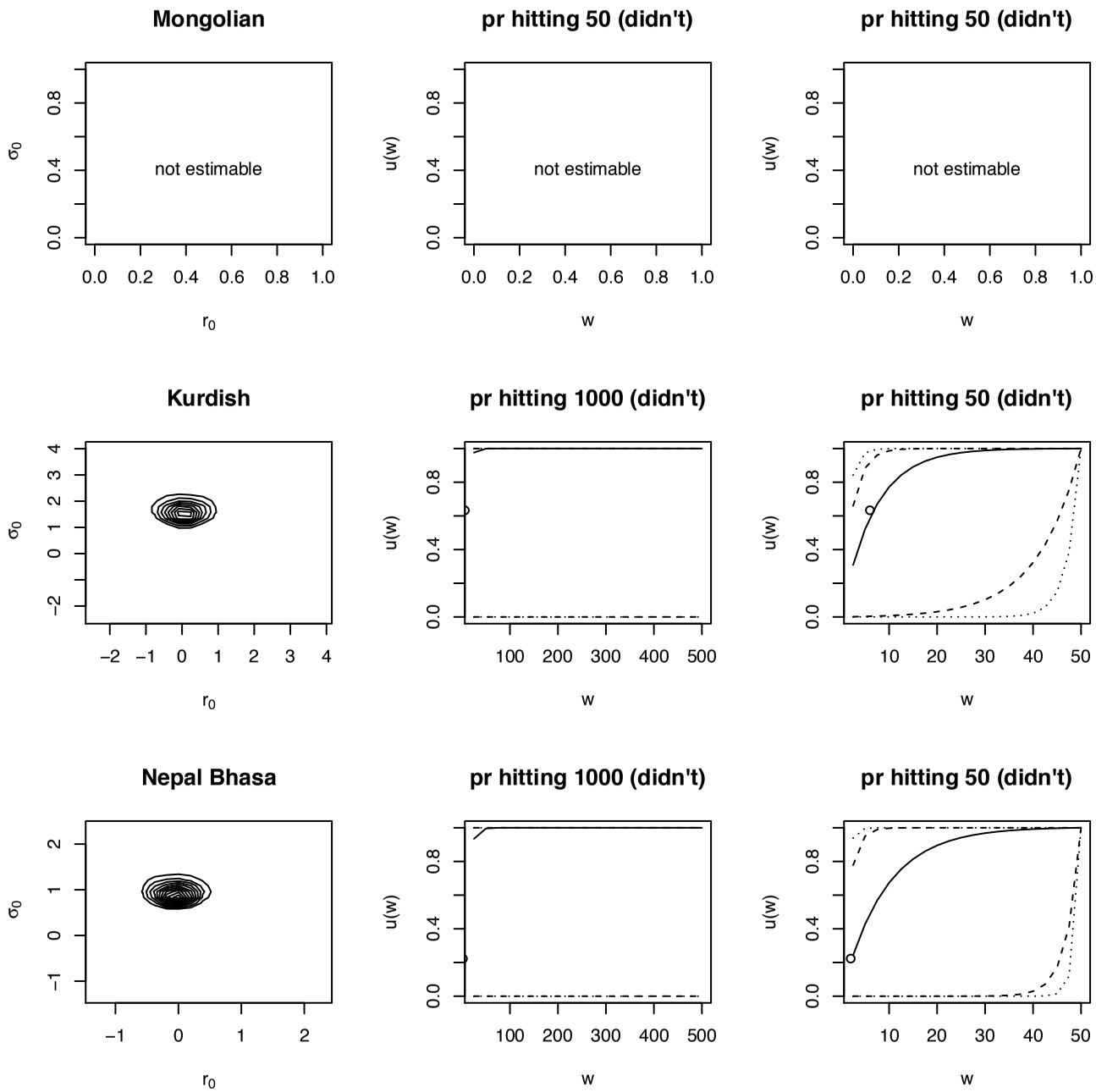
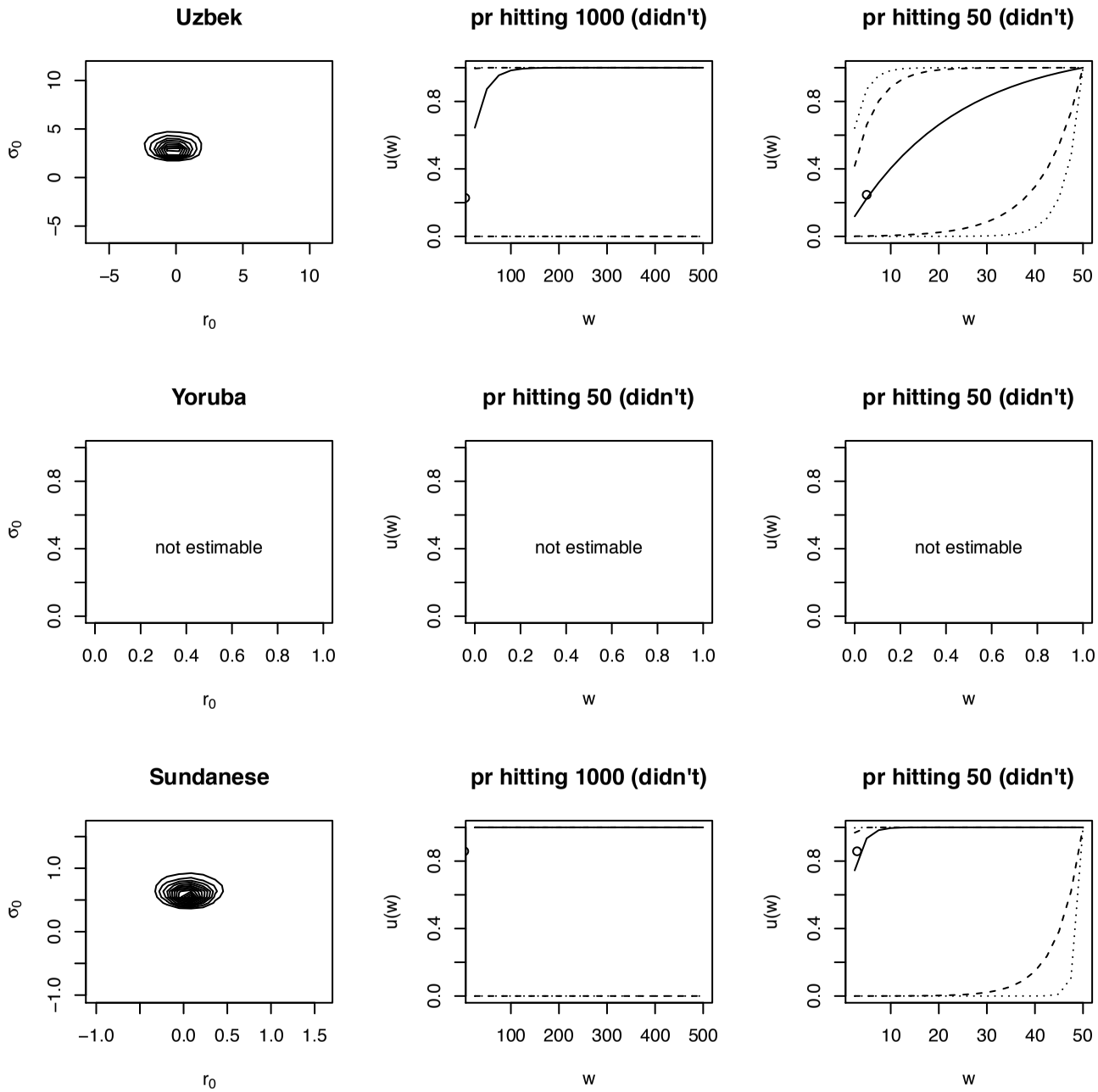
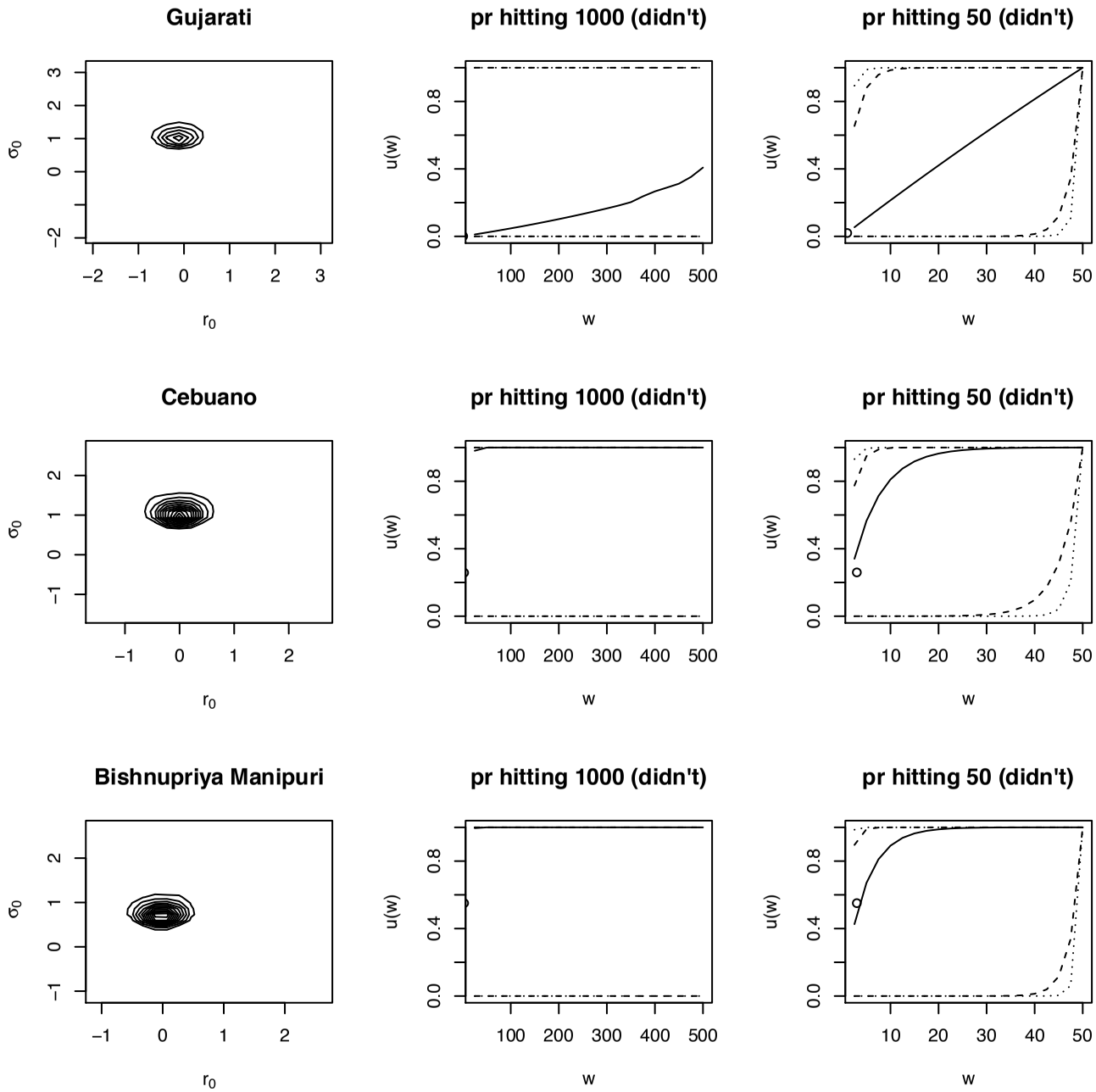
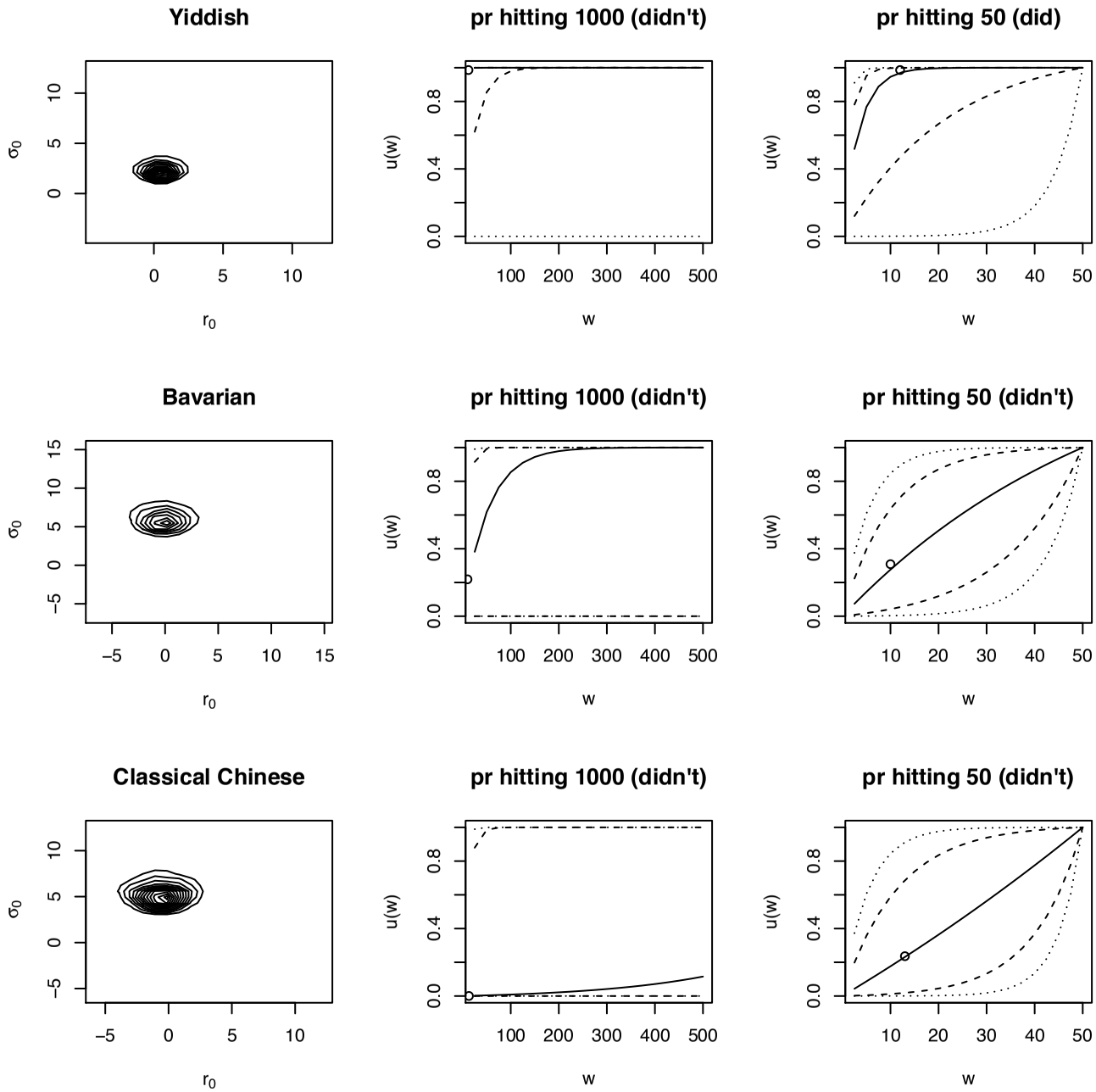
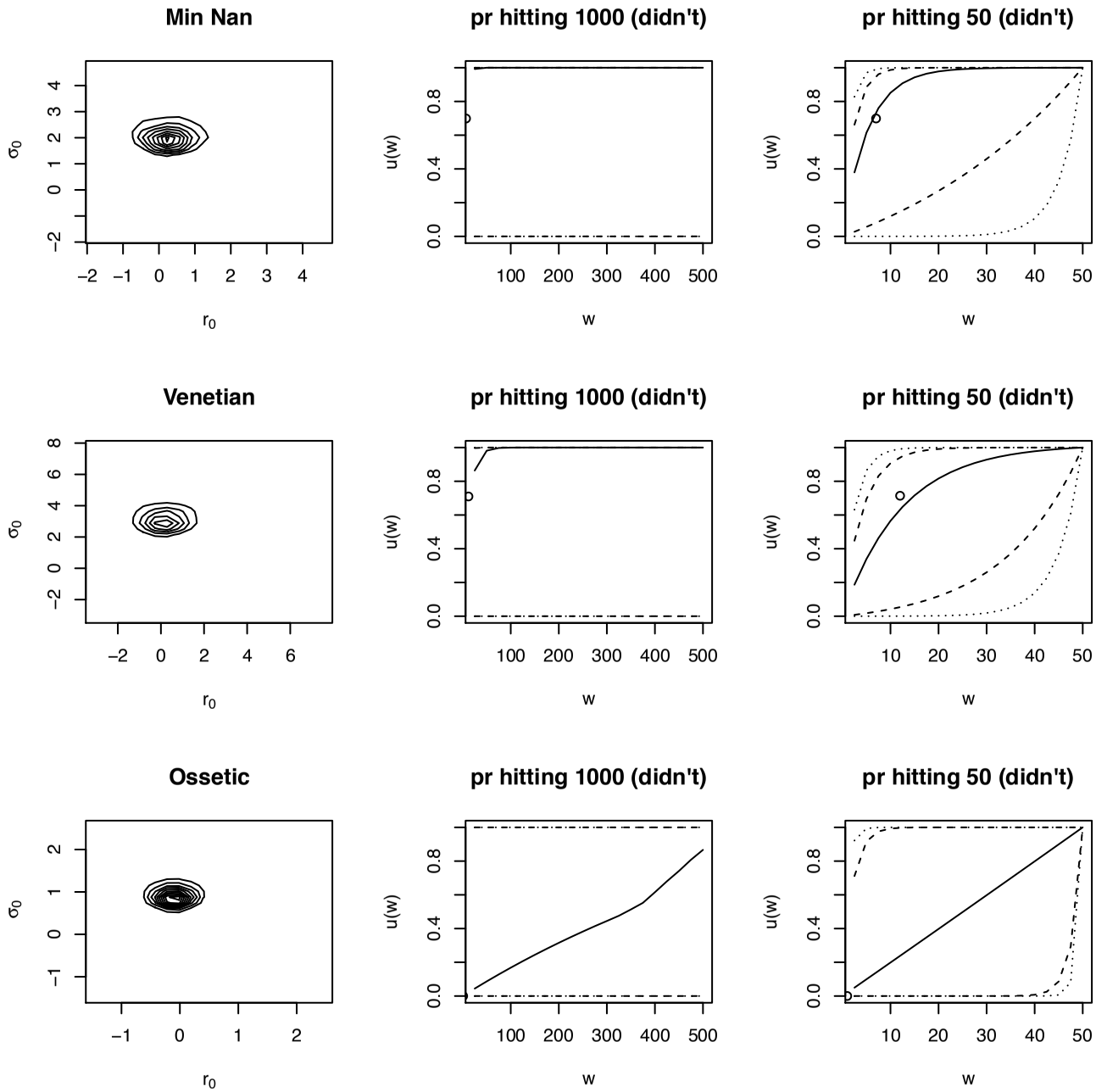
pr hitting 50 (didn't)
Chuvash
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Haitian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

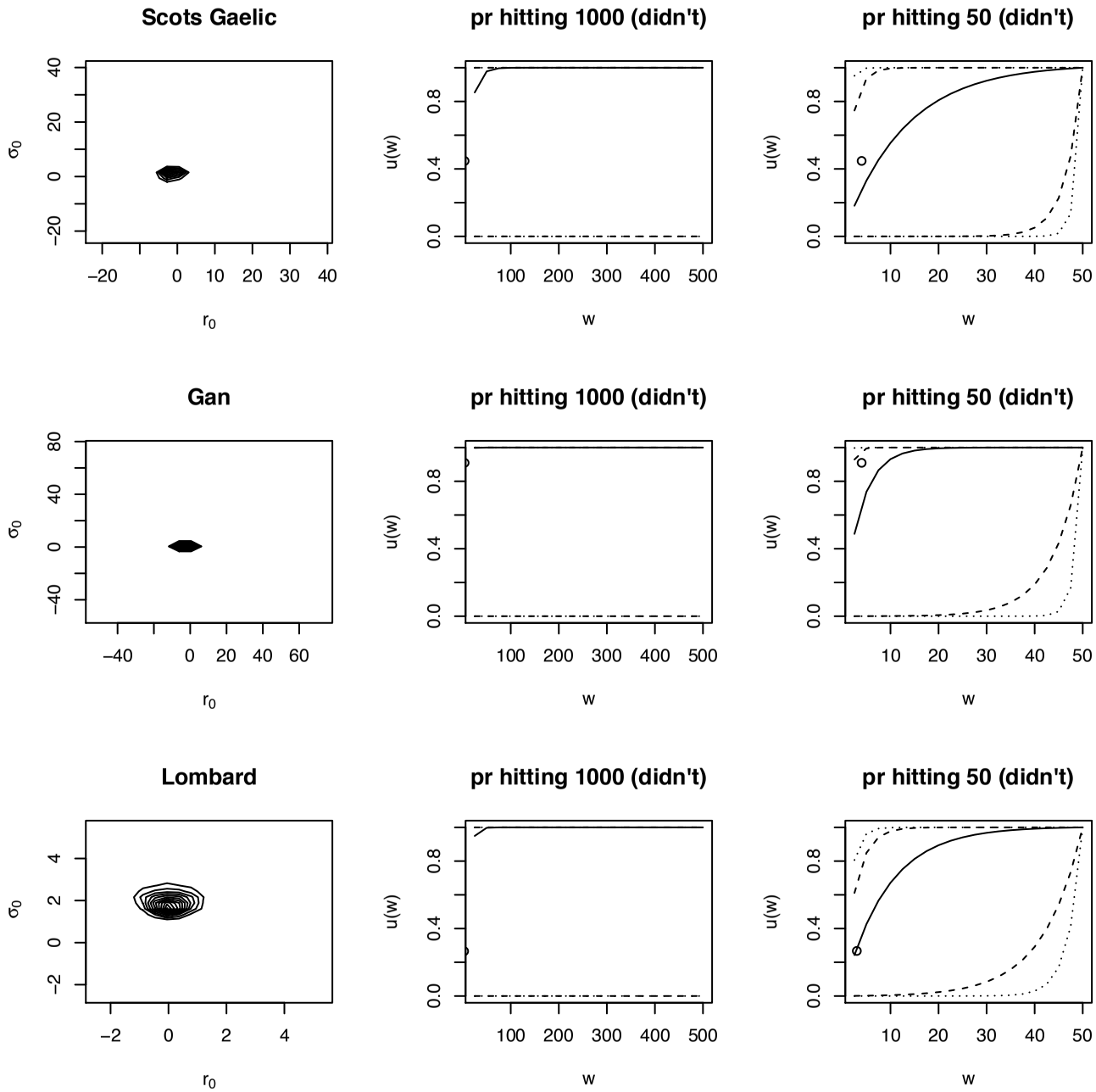

Mongolian
not estimable
σ0
r0
pr hitting 50 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w
Kurdish
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w
Nepal Bhasa
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w

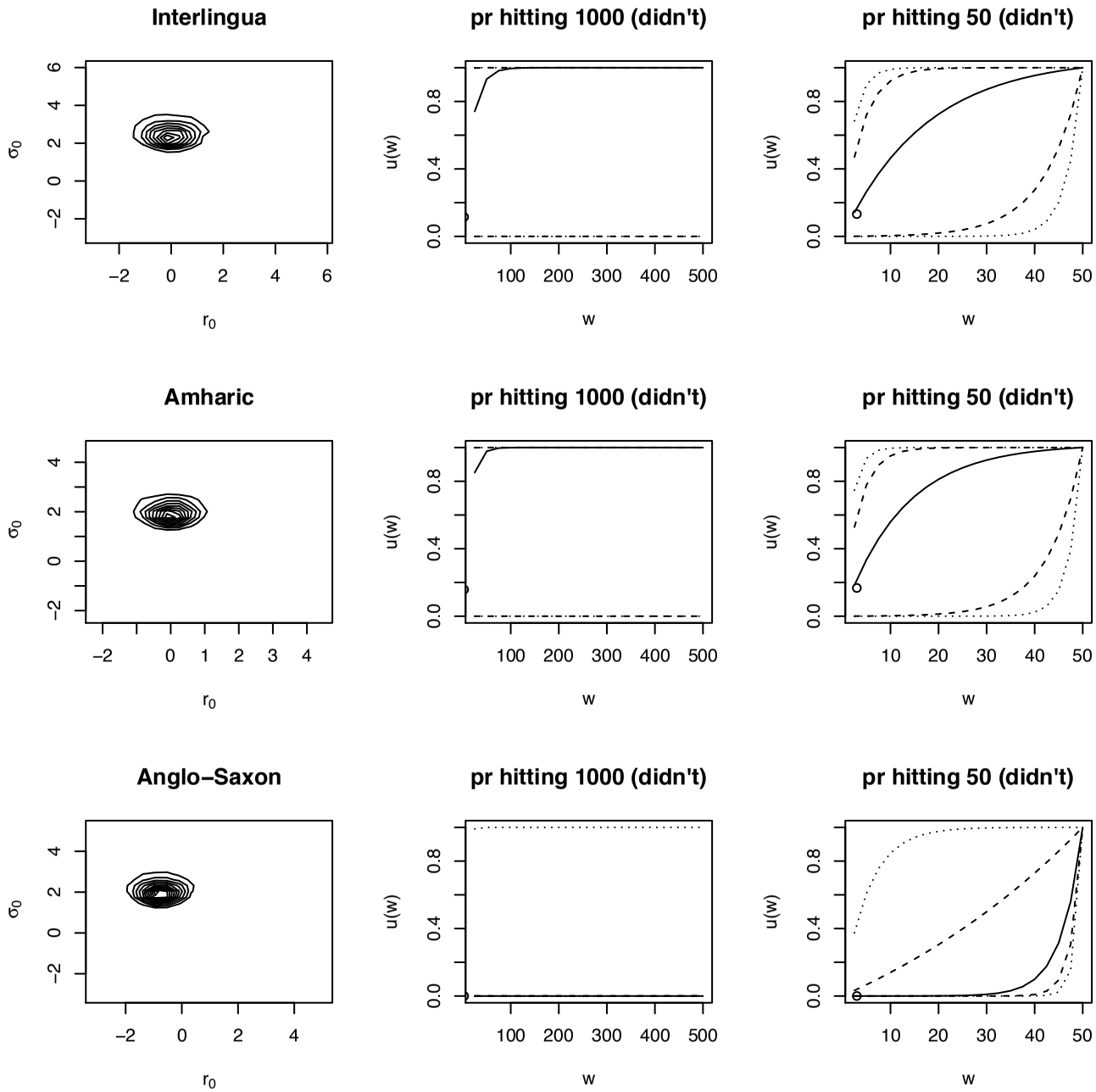

Uzbek
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Yoruba
pr hitting 50 (didn't)
pr hitting 50 (didn't)
Sundanese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

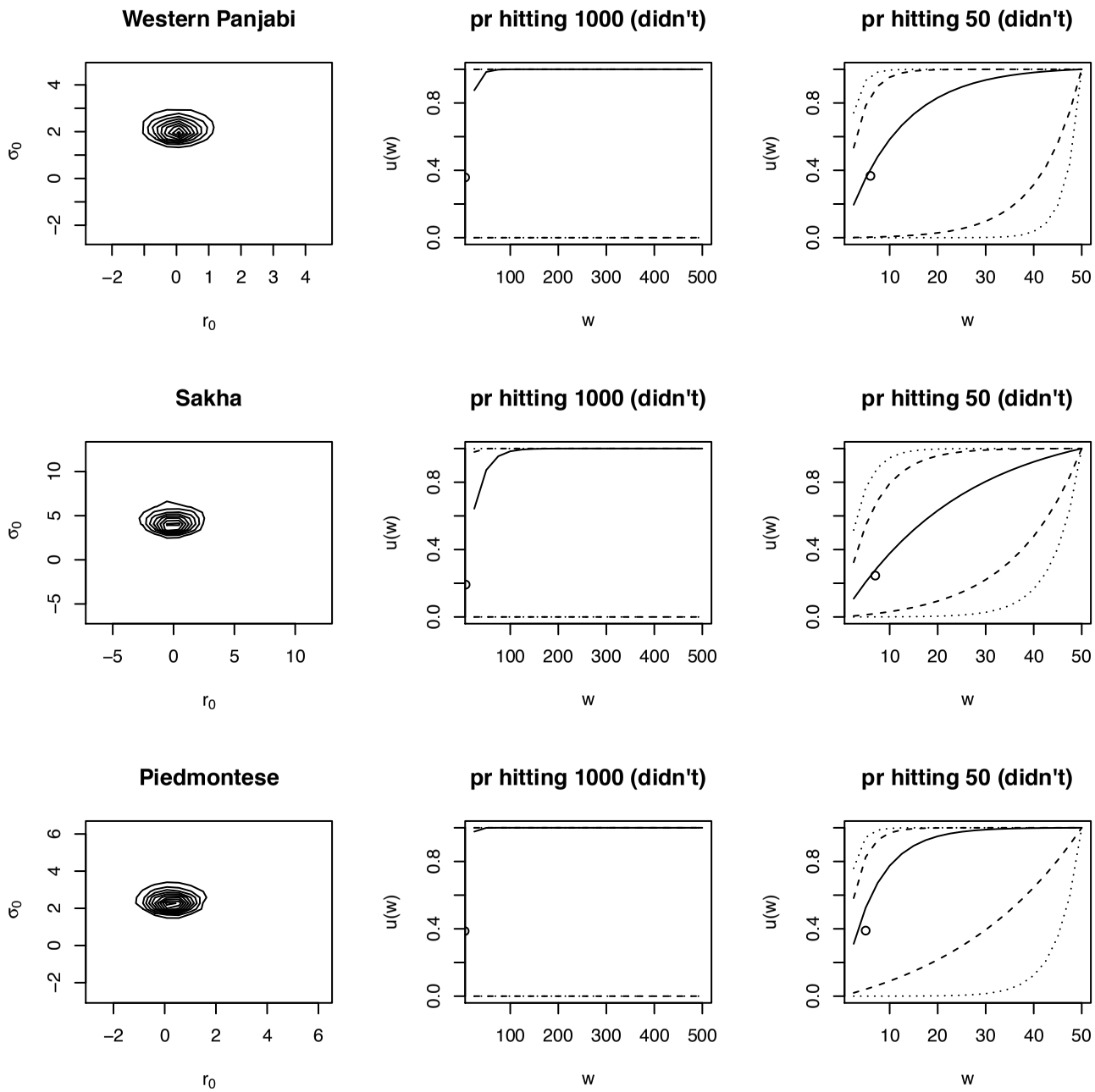

Gujarati
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Cebuano
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Bishnupriya Manipuri
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

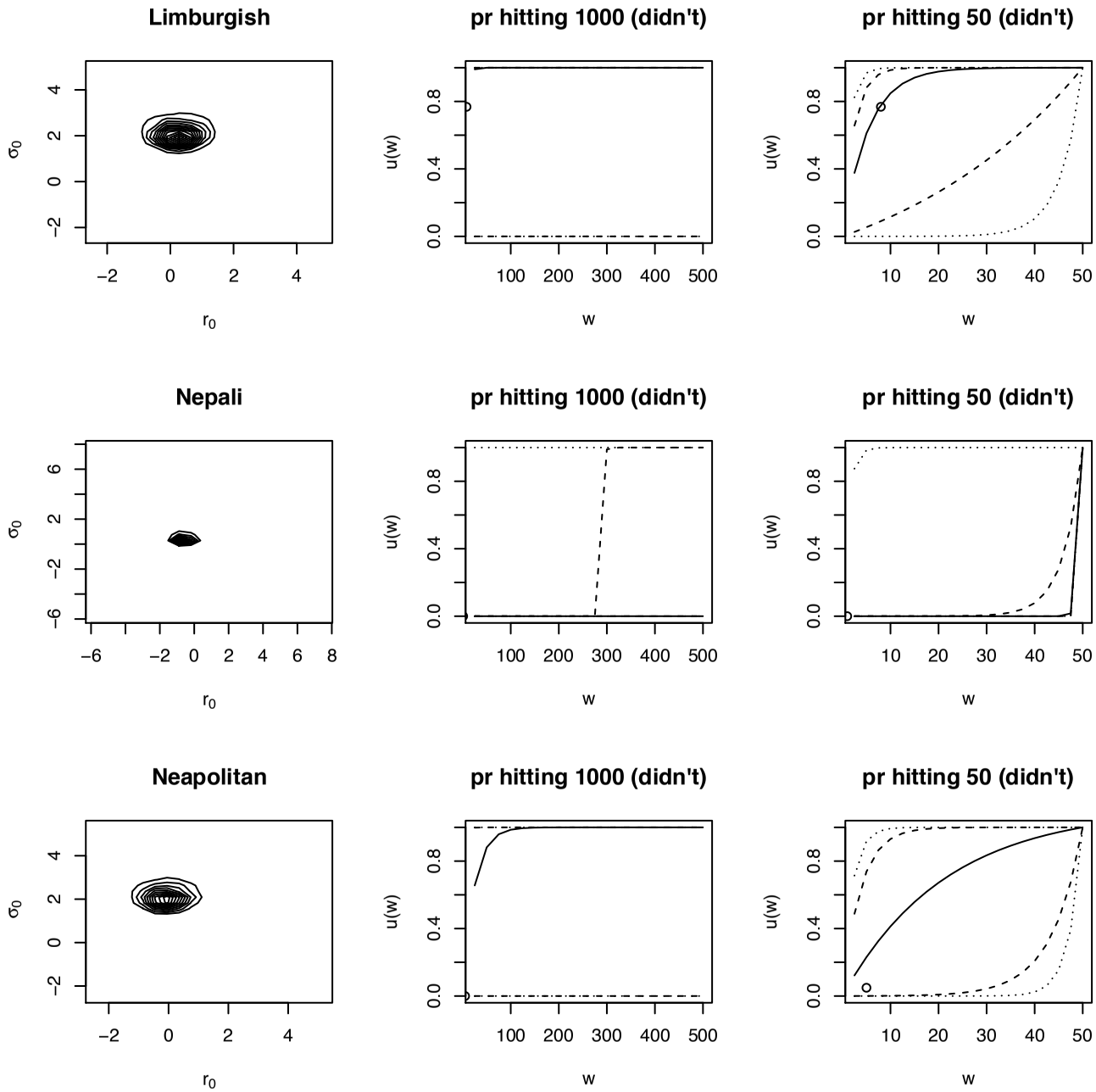

Yiddish
pr hitting 1000 (didn't)
pr hitting 50 (did)
Bavarian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Classical Chinese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

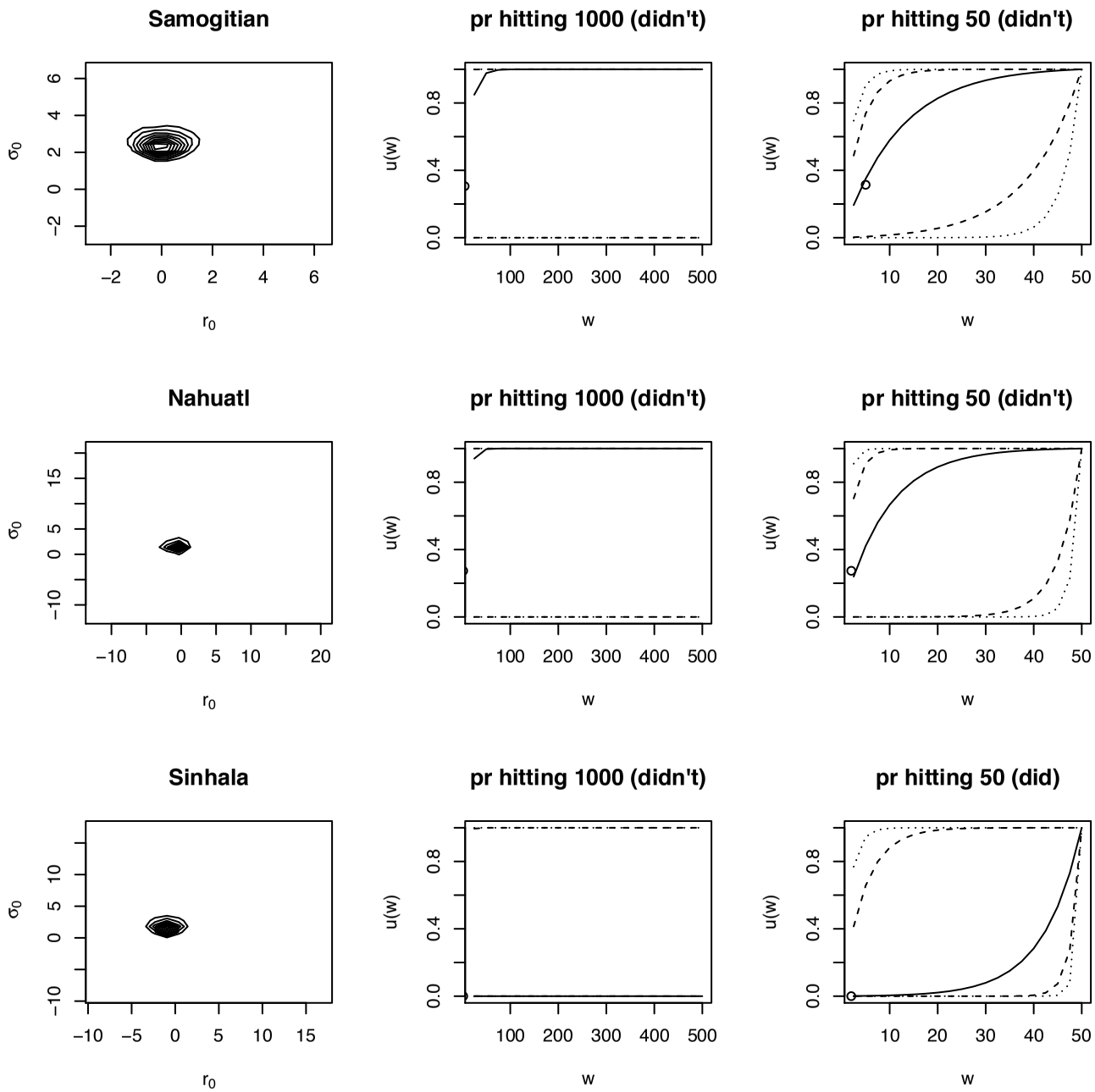

Min Nan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Venetian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Ossetic
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

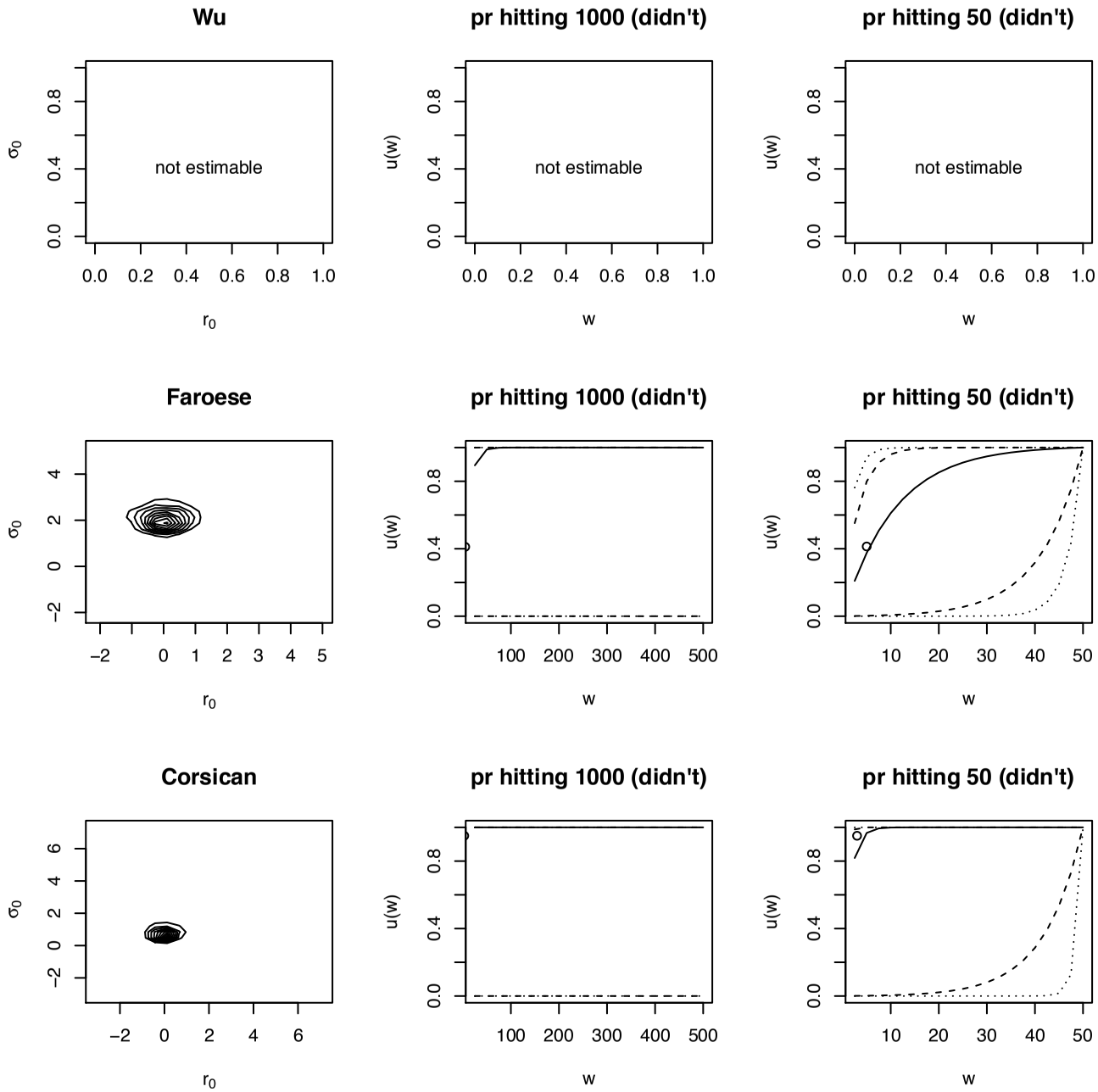

Scots Gaelic
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Gan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Lombard
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

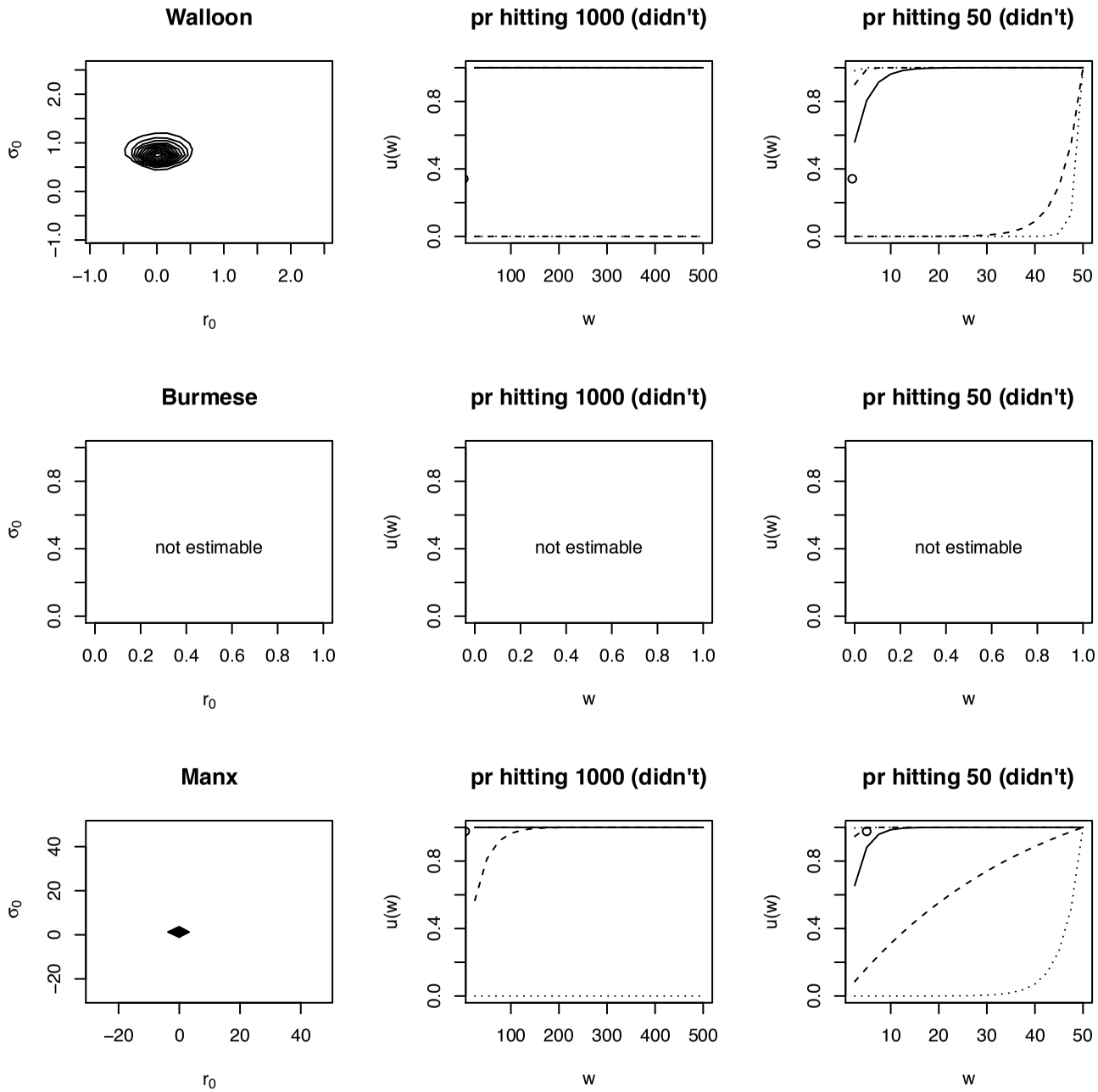

Interlingua
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Amharic
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Anglo−Saxon
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

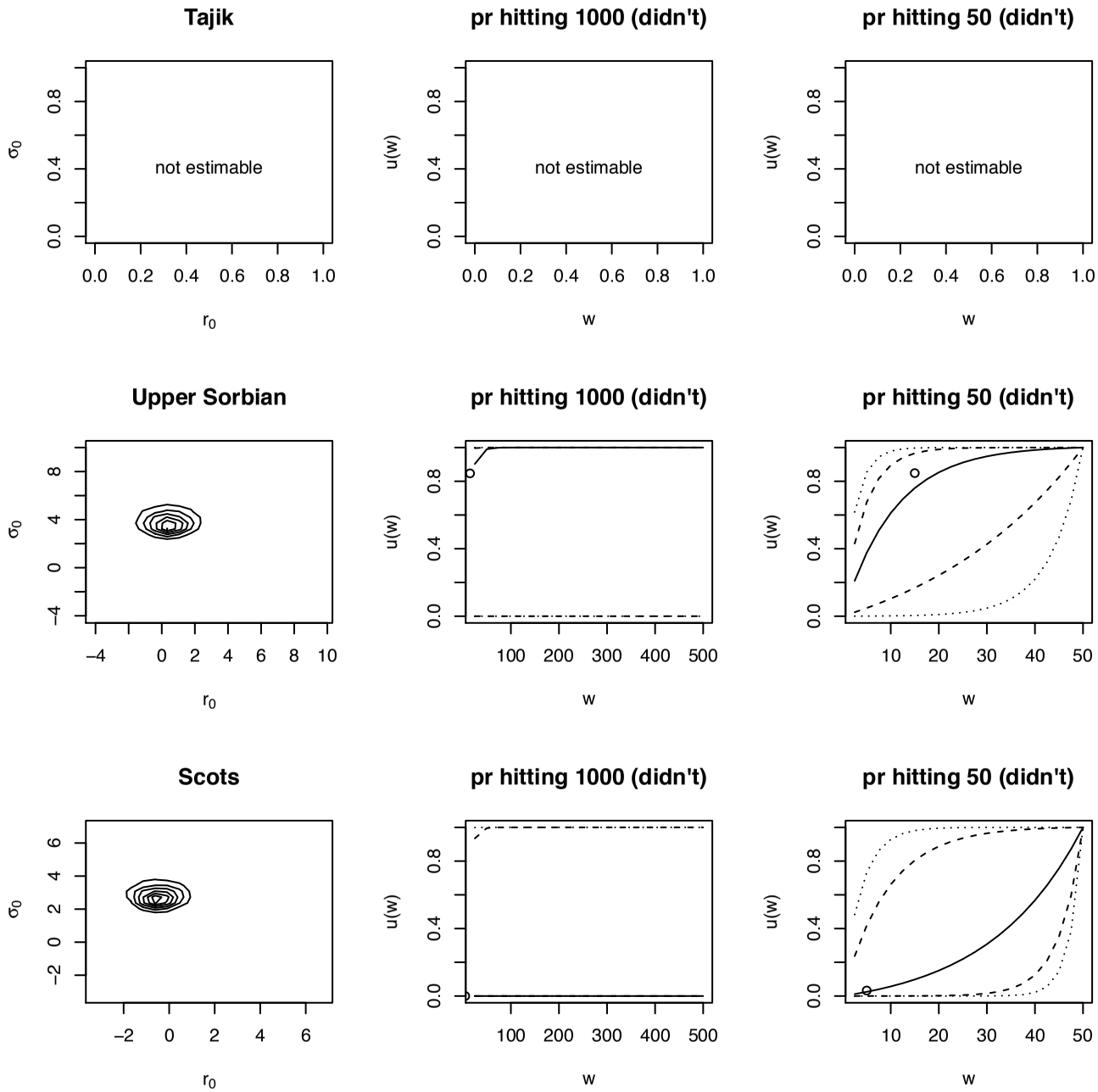

Western Panjabi
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Sakha
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Piedmontese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

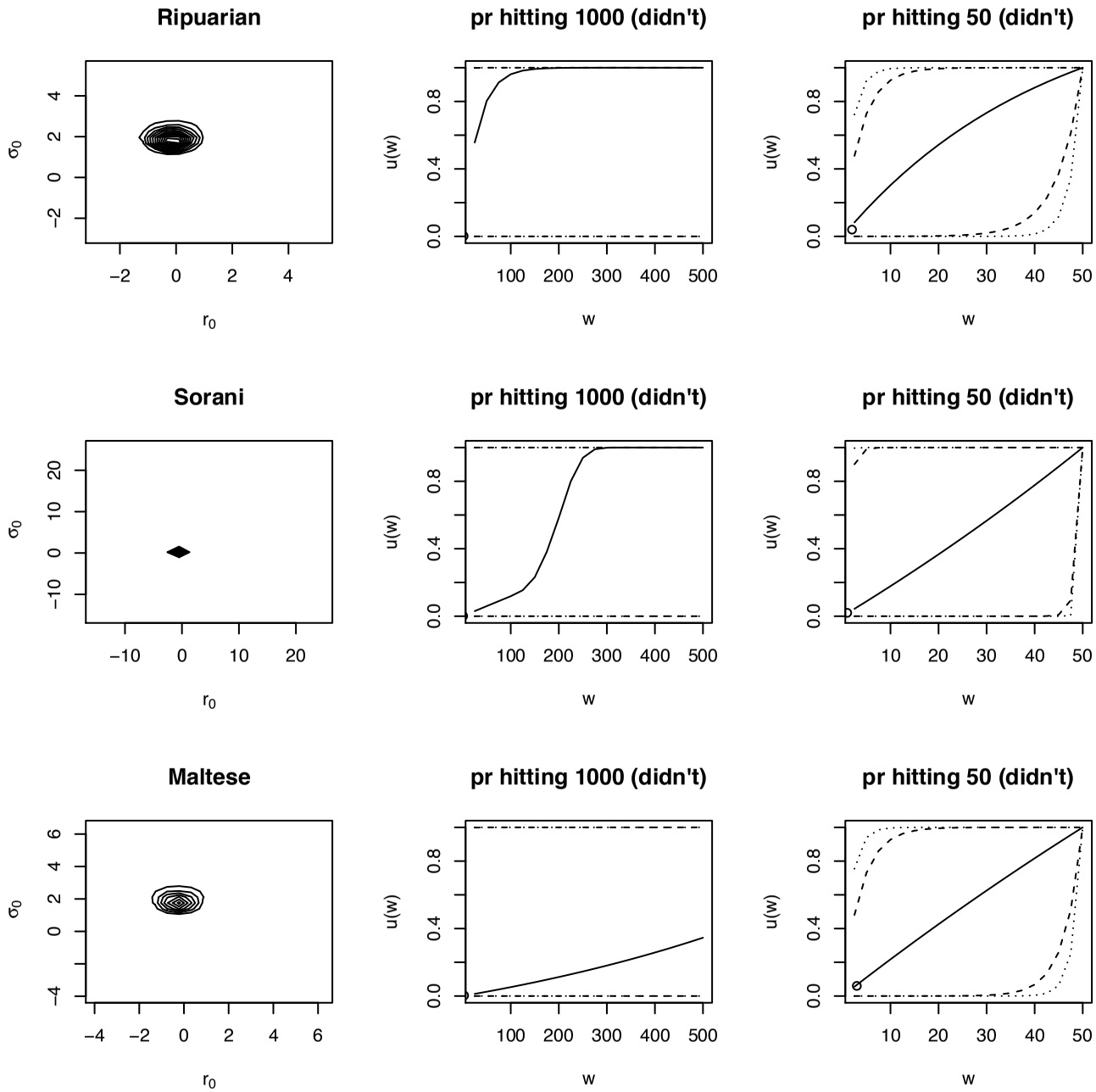

Limburgish
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Nepali
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Neapolitan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

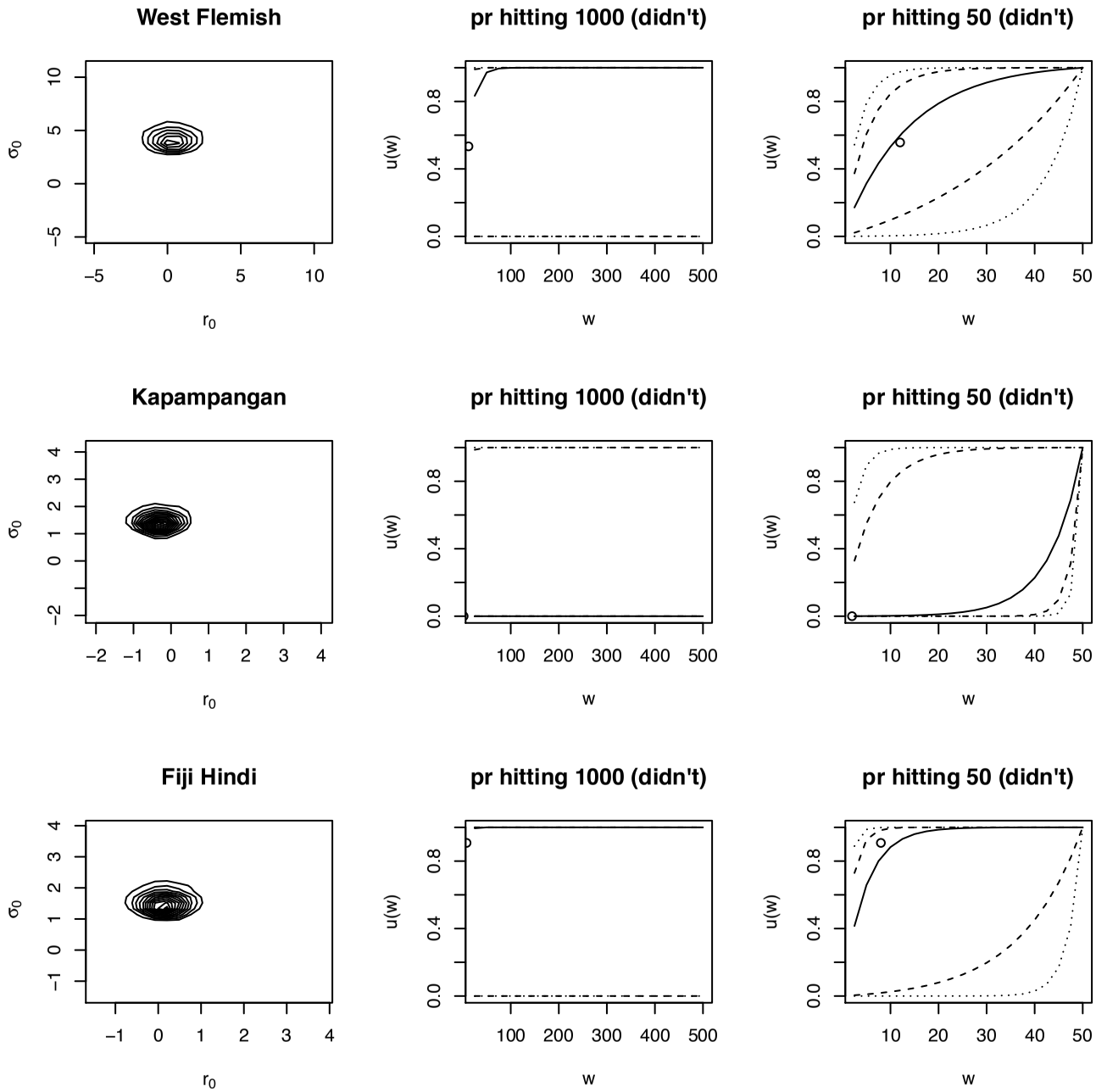

Samogitian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Nahuatl
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Sinhala
pr hitting 1000 (didn't)
pr hitting 50 (did)
σ0
r0
u(w)
w

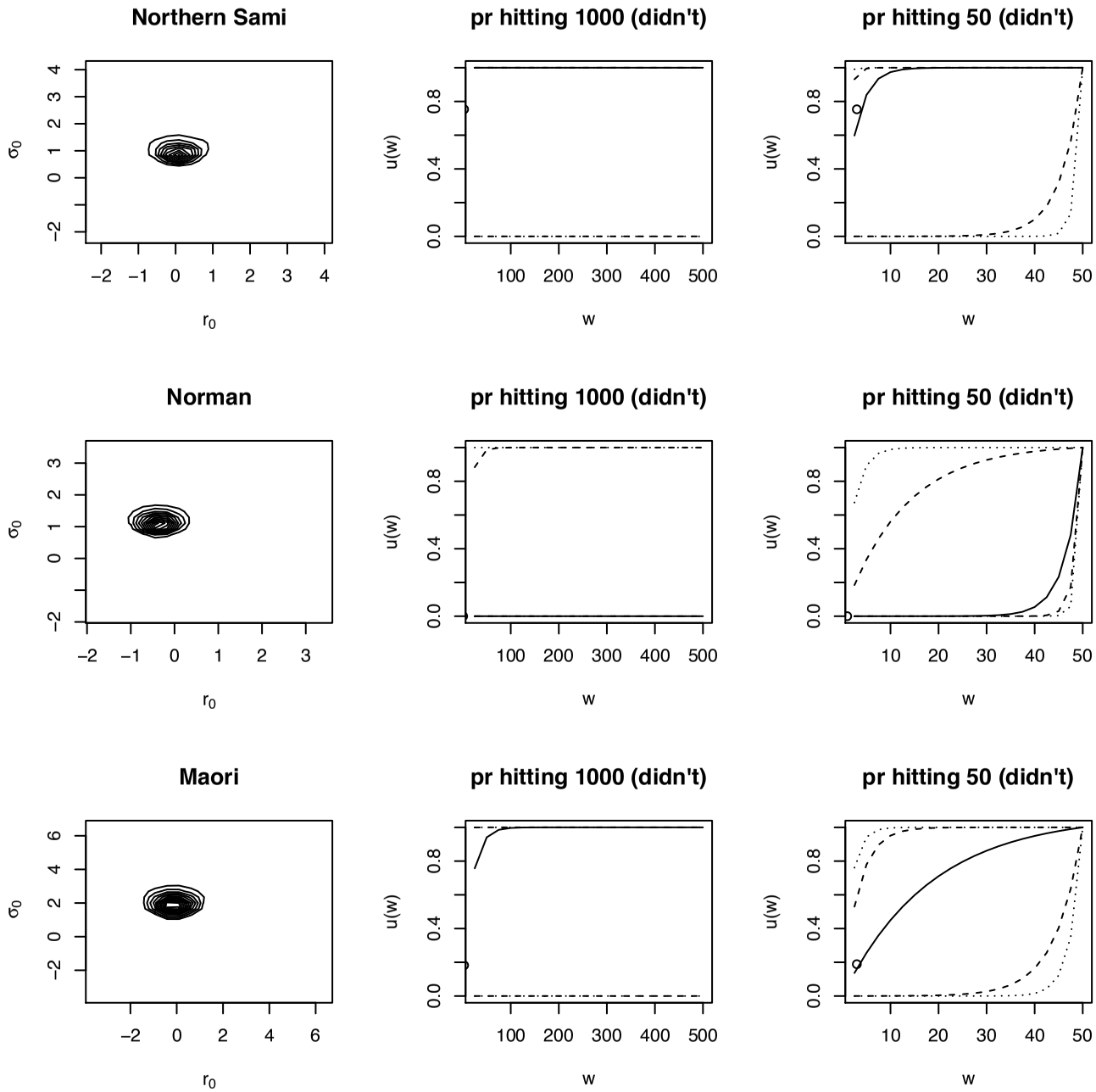

Wu
not estimable
σ0
r0
pr hitting 1000 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w
Faroese
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w
Corsican
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w

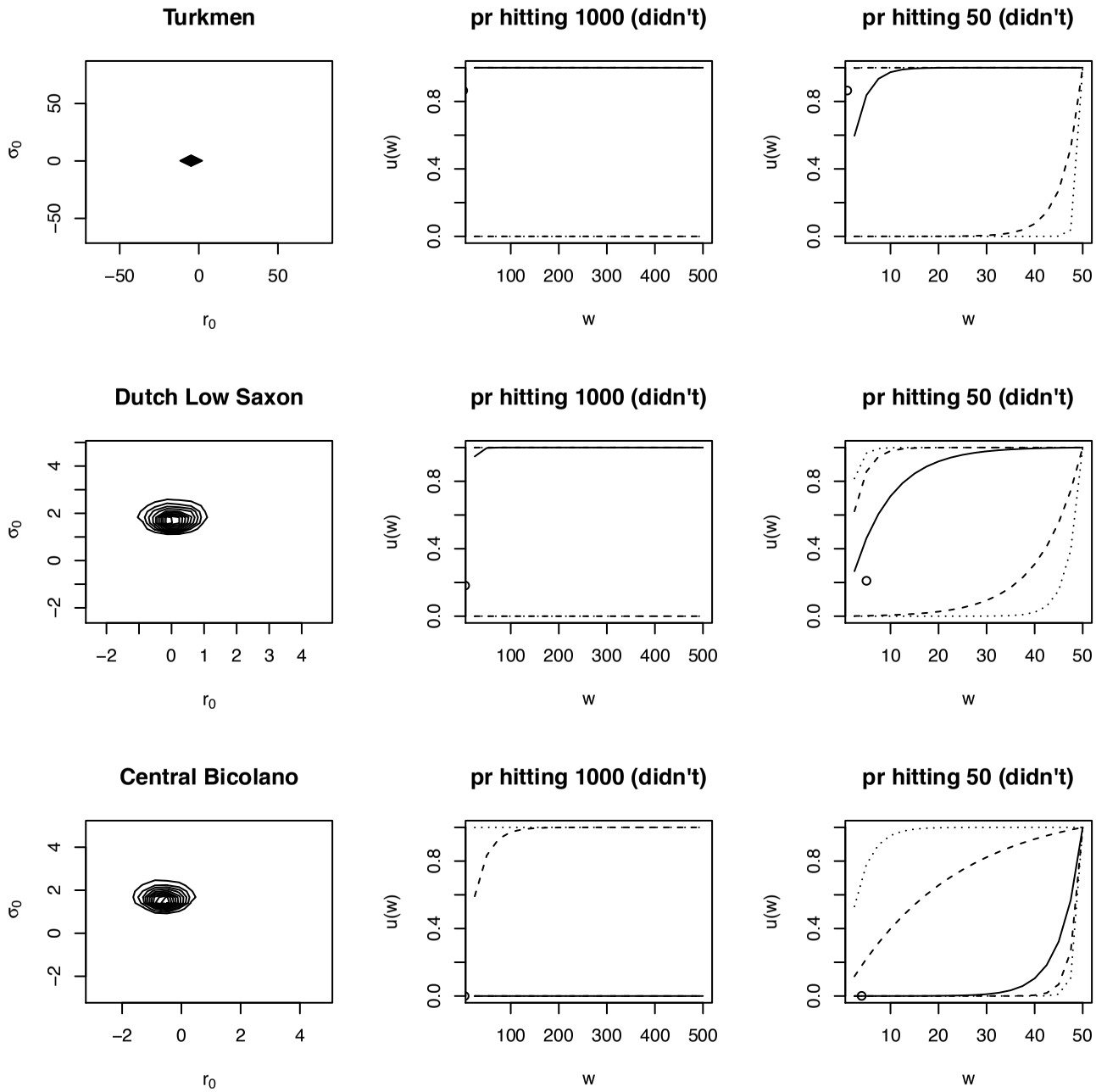

Walloon
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Burmese
not estimable
not estimable
not estimable
Manx
σ0
r0
u(w)
w

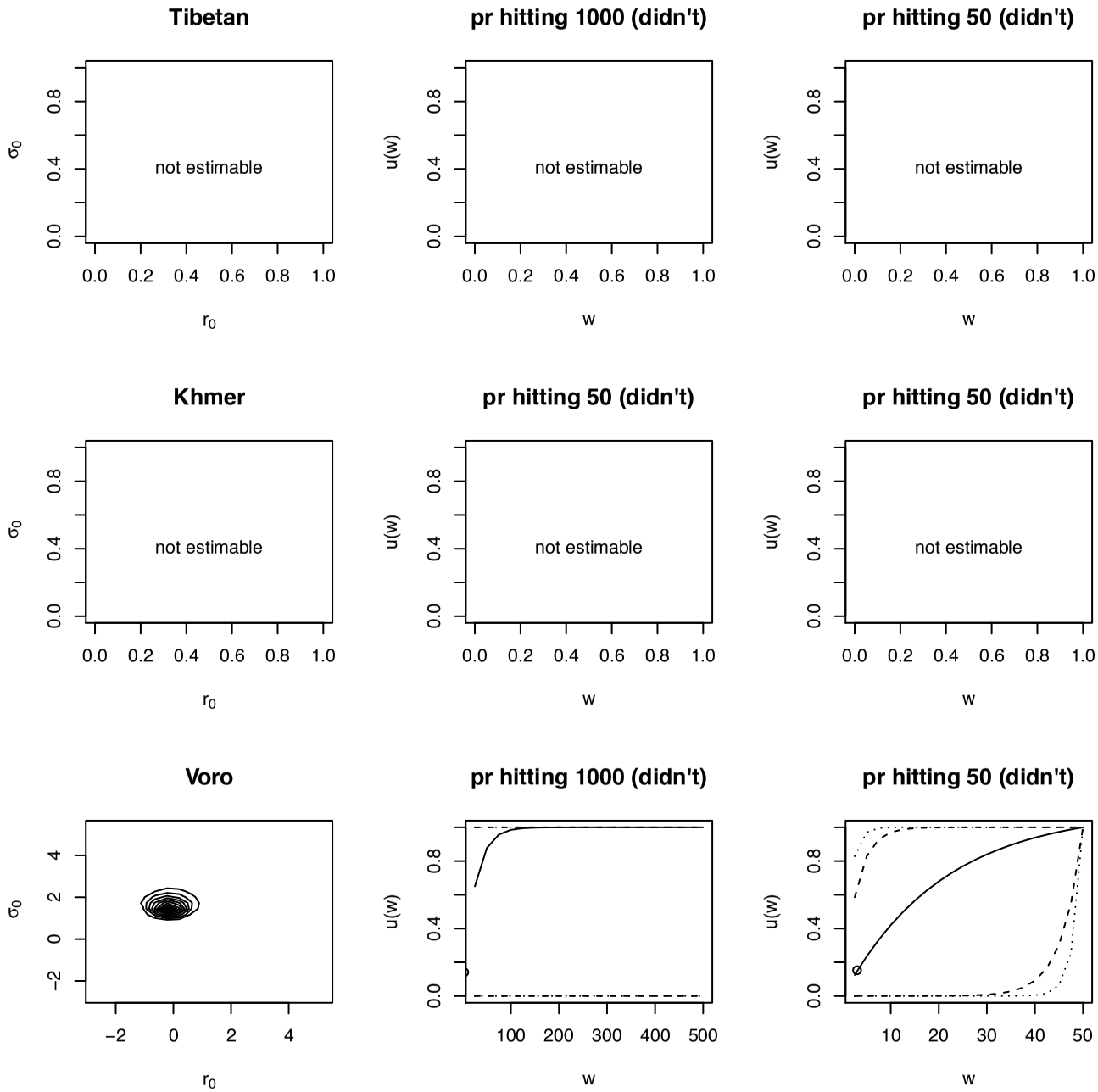

Tajik
not estimable
σ0
r0
pr hitting 1000 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w
Upper Sorbian
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w
Scots
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w

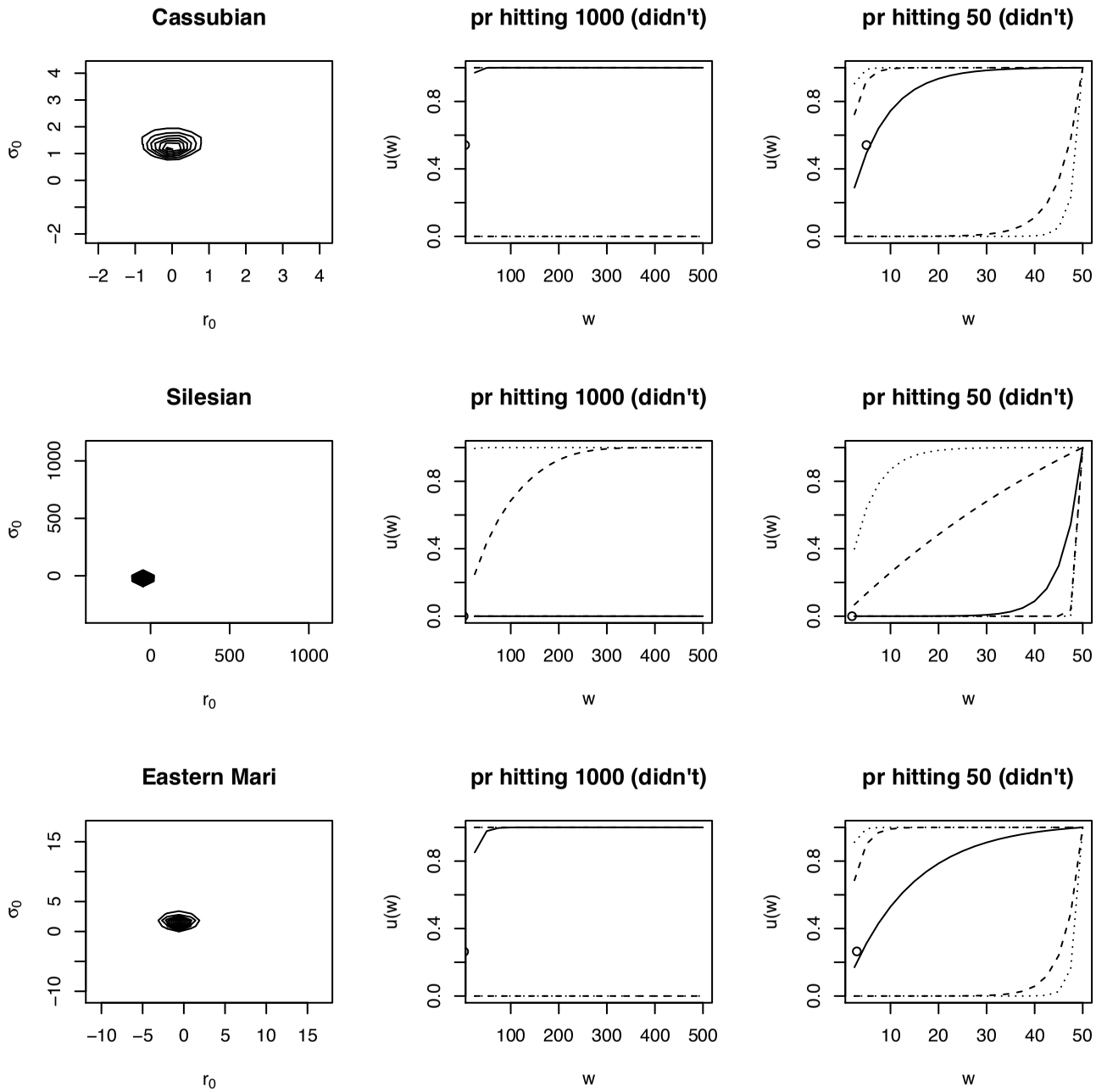

Ripuarian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Sorani
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Maltese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

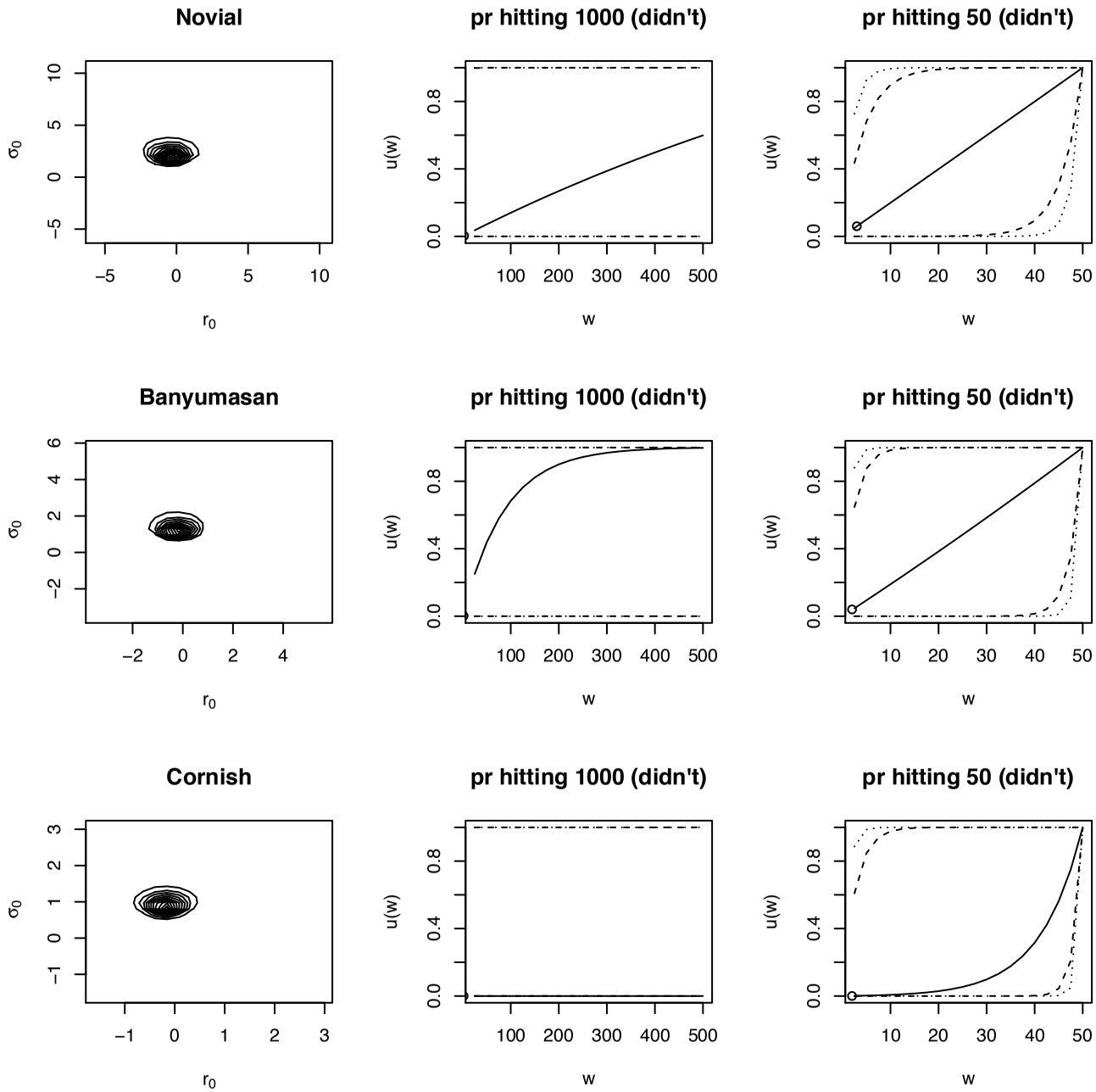

West Flemish
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Kapampangan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Fiji Hindi
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

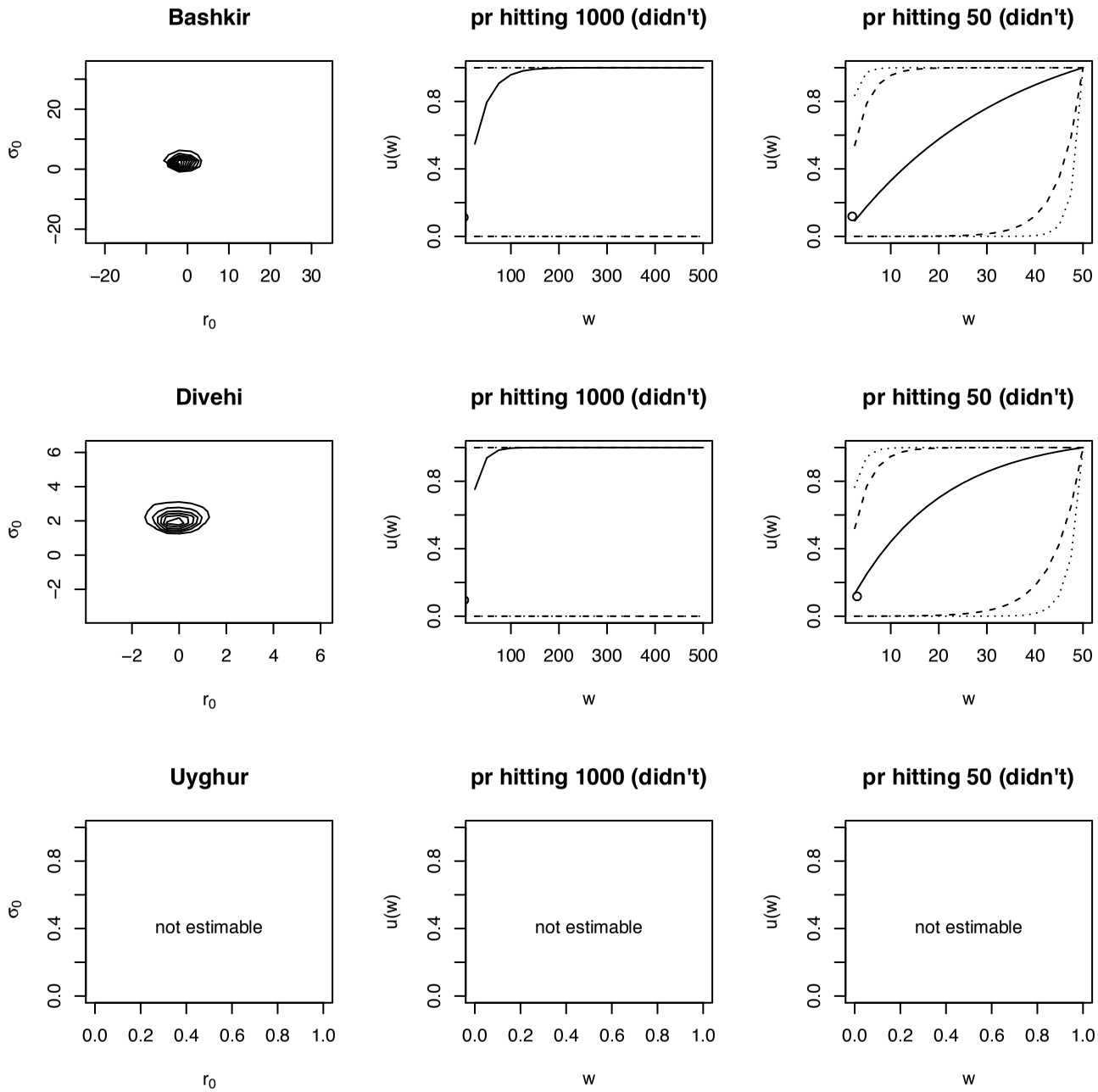

Northern Sami
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Norman
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Maori
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

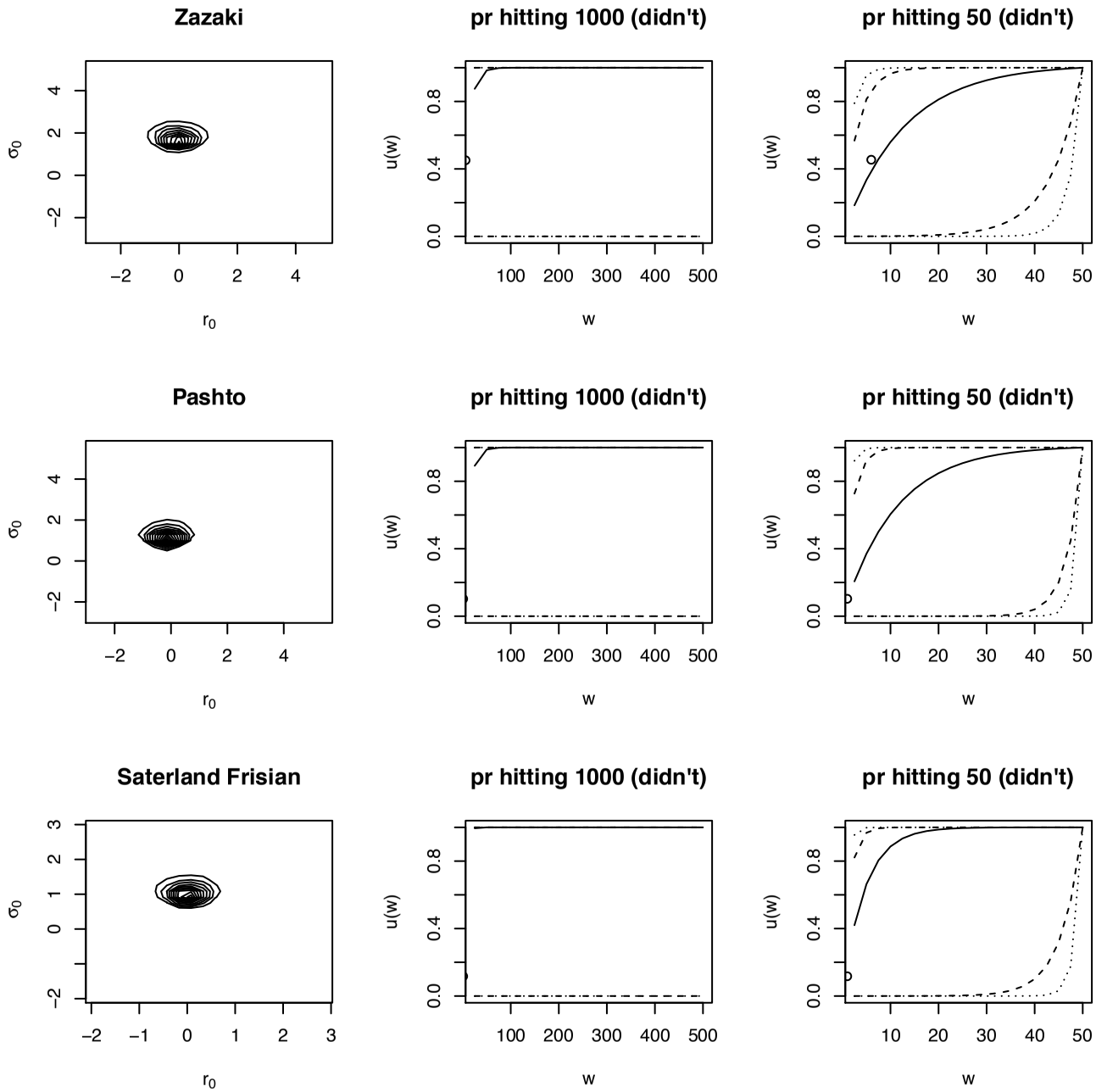

Turkmen
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Dutch Low Saxon
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Central Bicolano
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

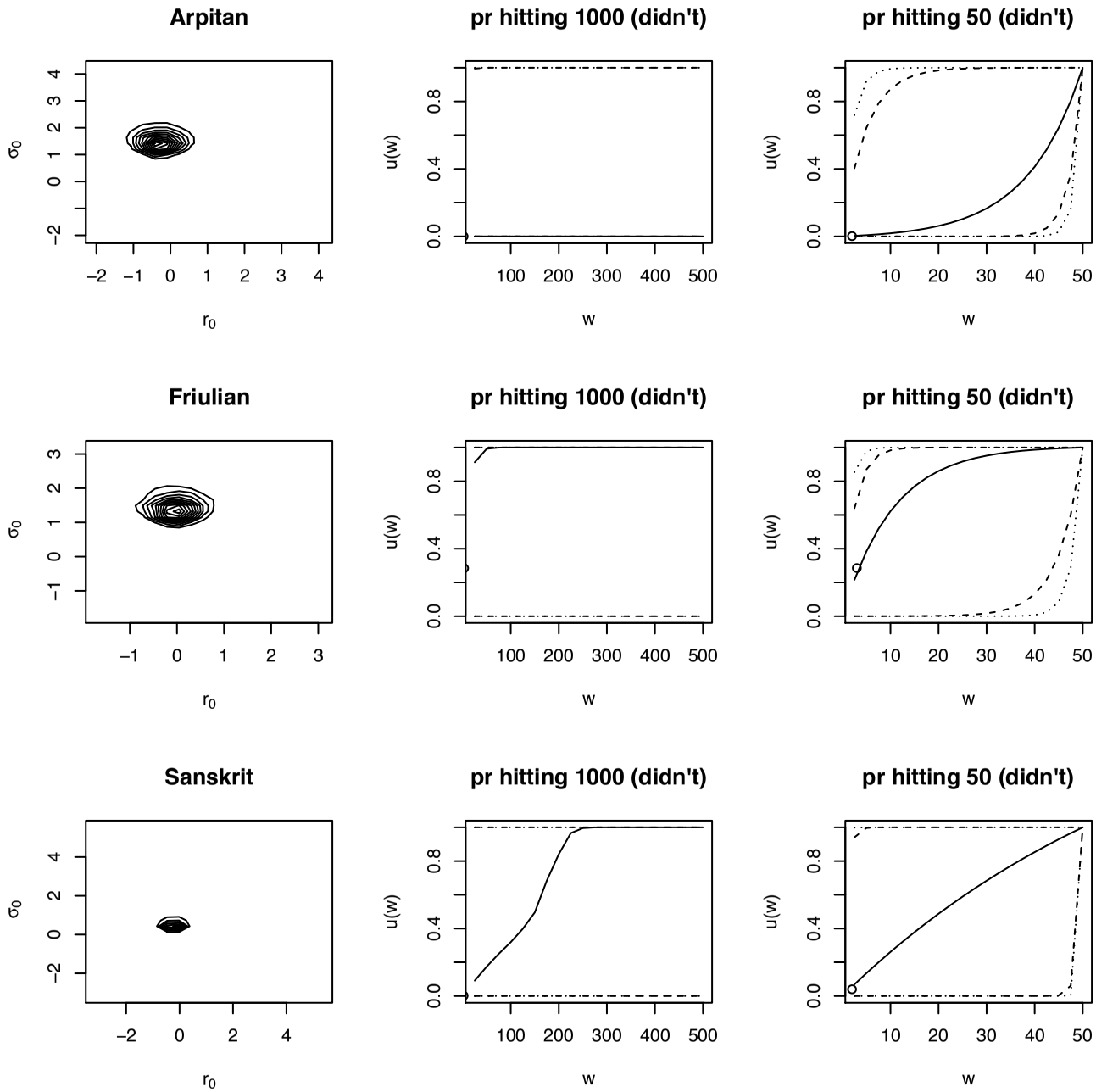

Tibetan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Khmer
pr hitting 50 (didn't)
pr hitting 50 (didn't)
Voro
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w

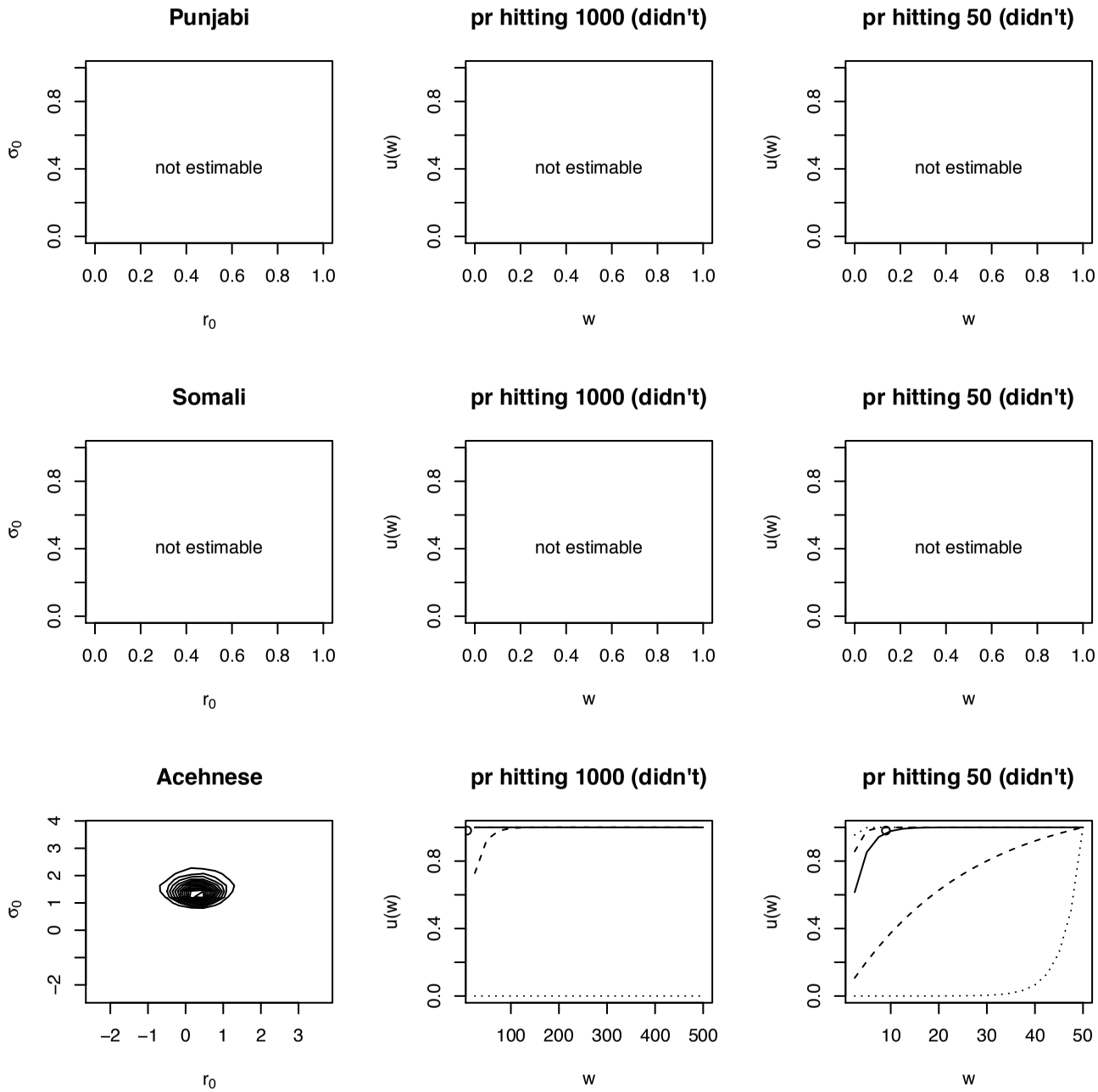

Cassubian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Silesian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Eastern Mari
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

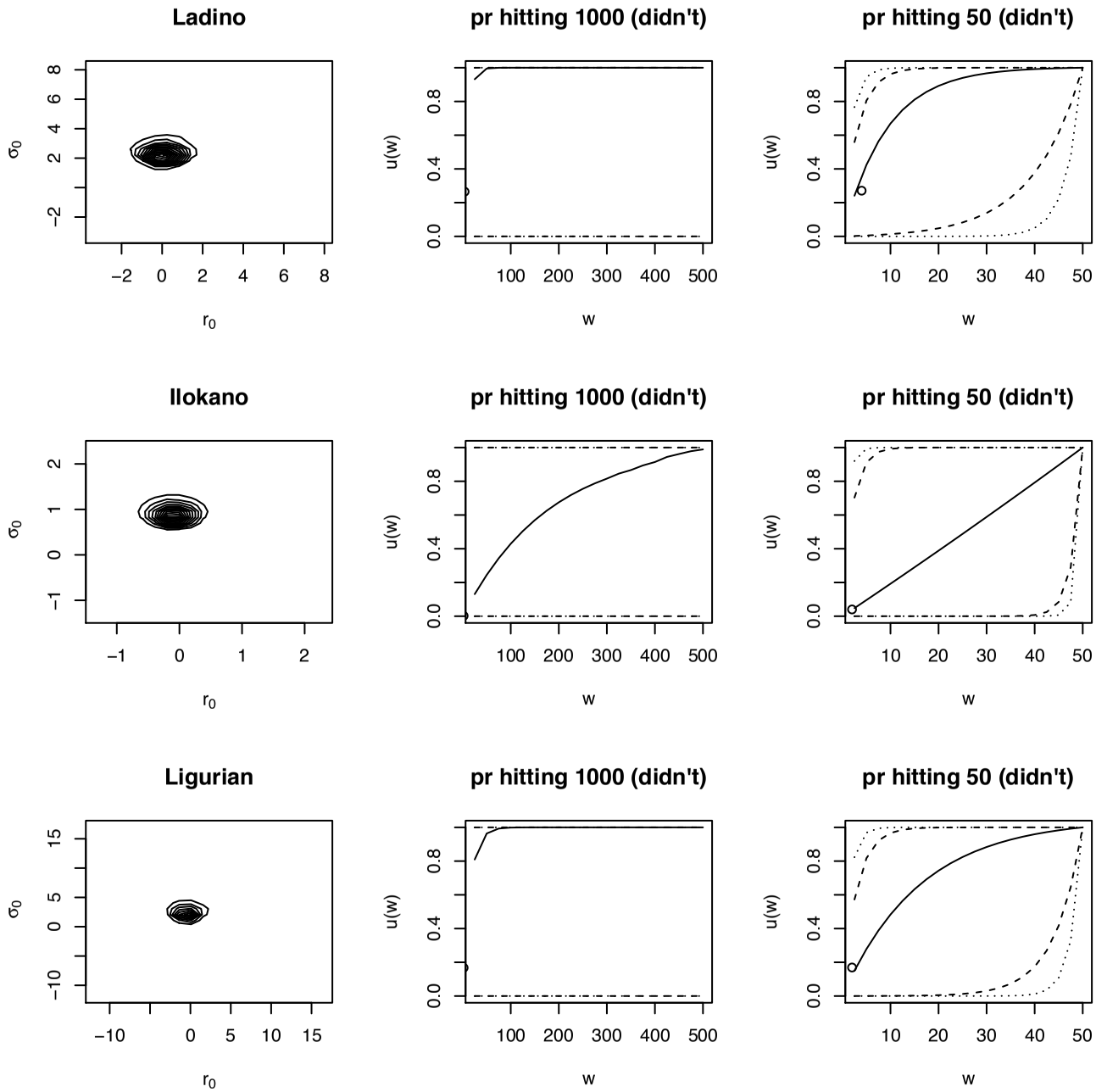

Novial
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Banyumasan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Cornish
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

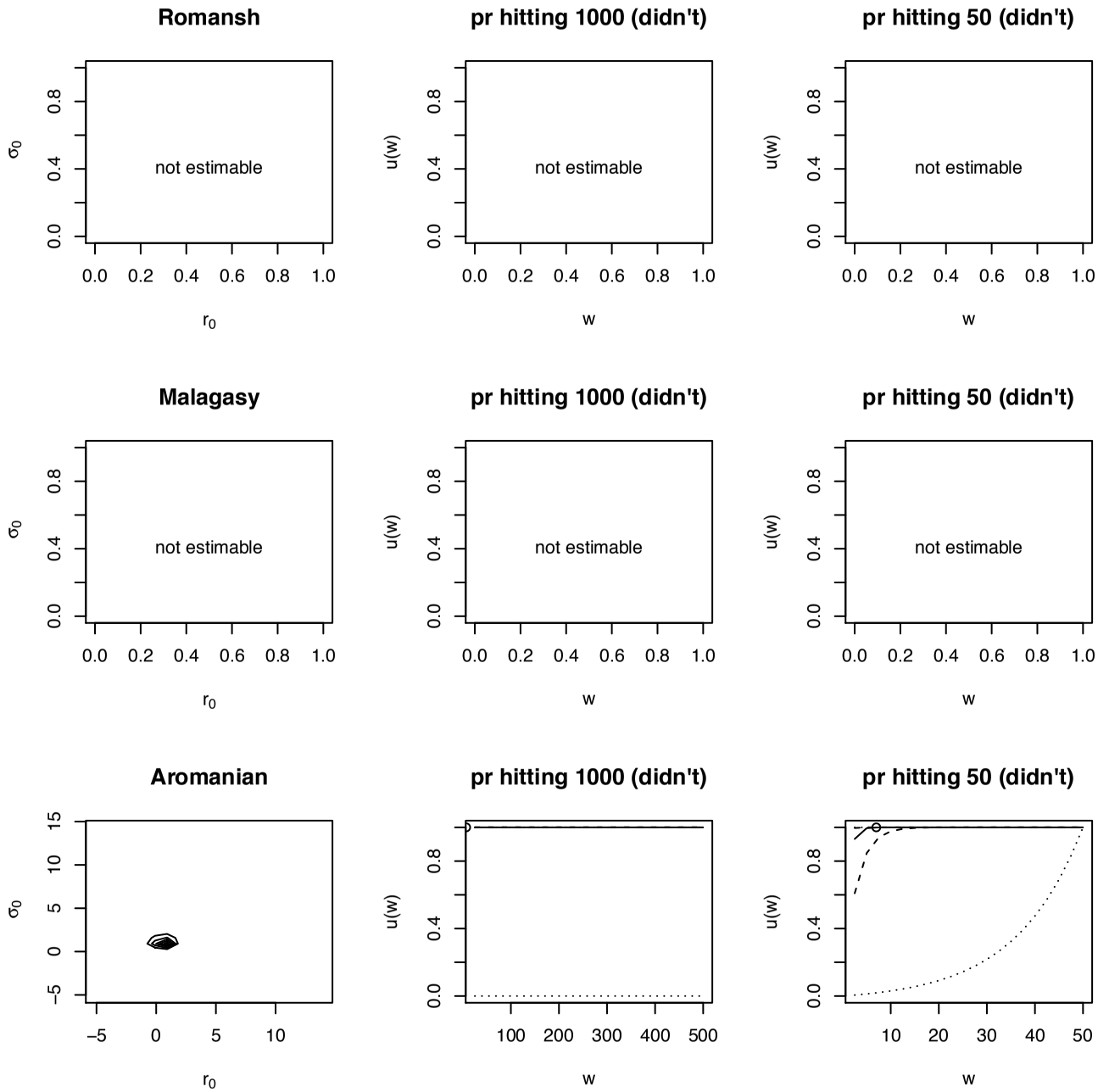

Bashkir
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Divehi
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Uyghur
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

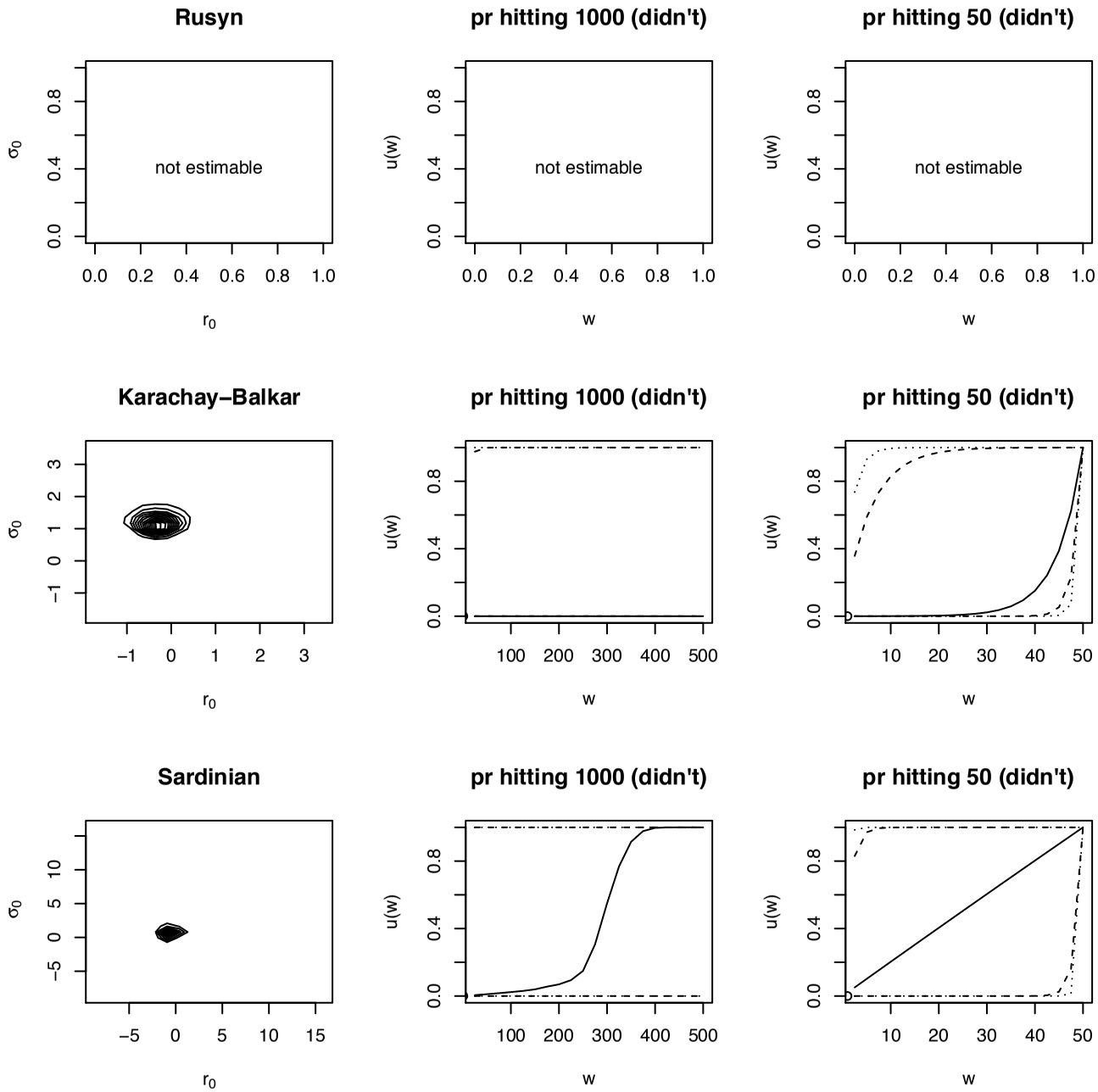

Zazaki
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Pashto
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Saterland Frisian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

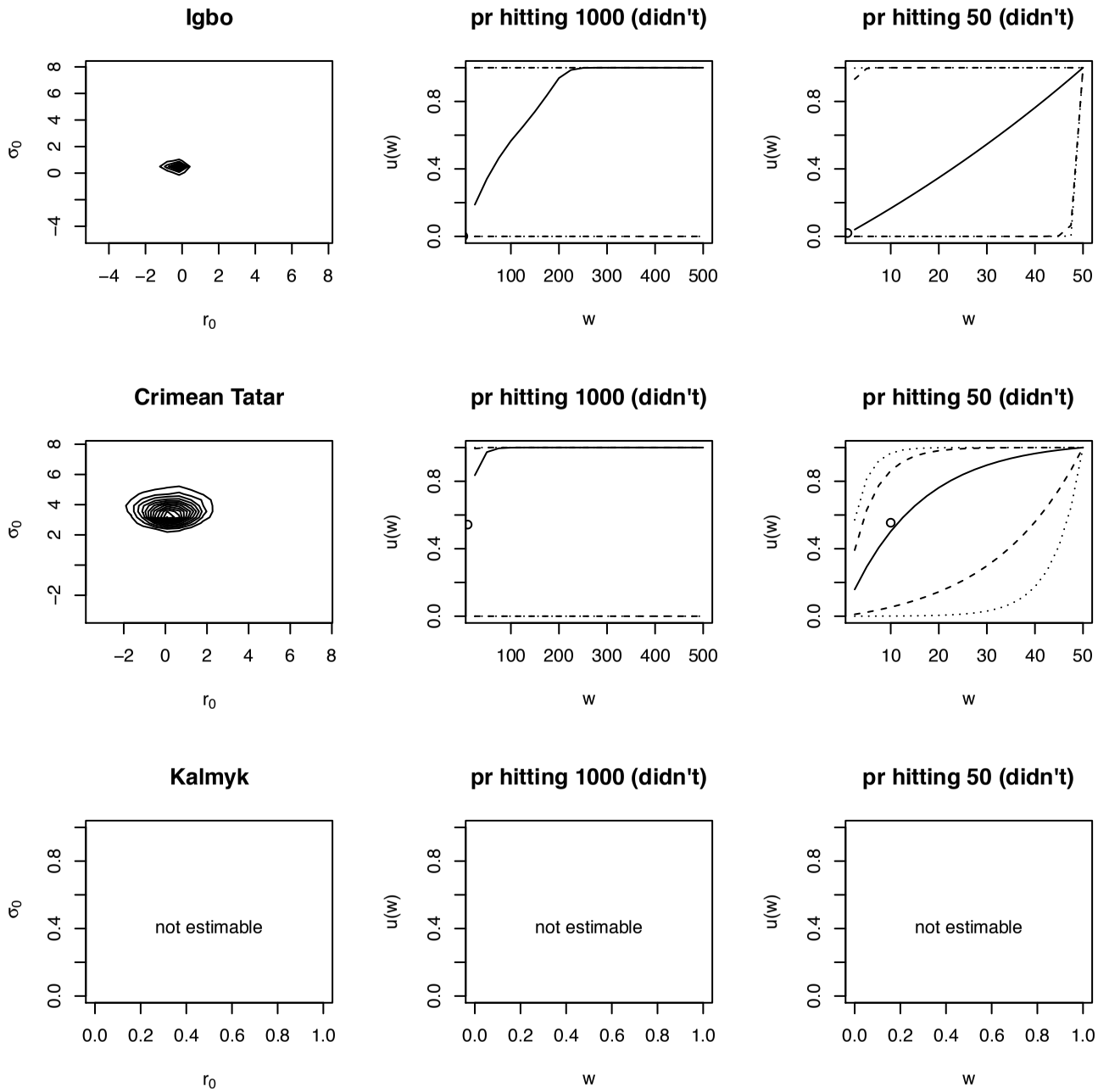

Arpitan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Friulian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Sanskrit
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

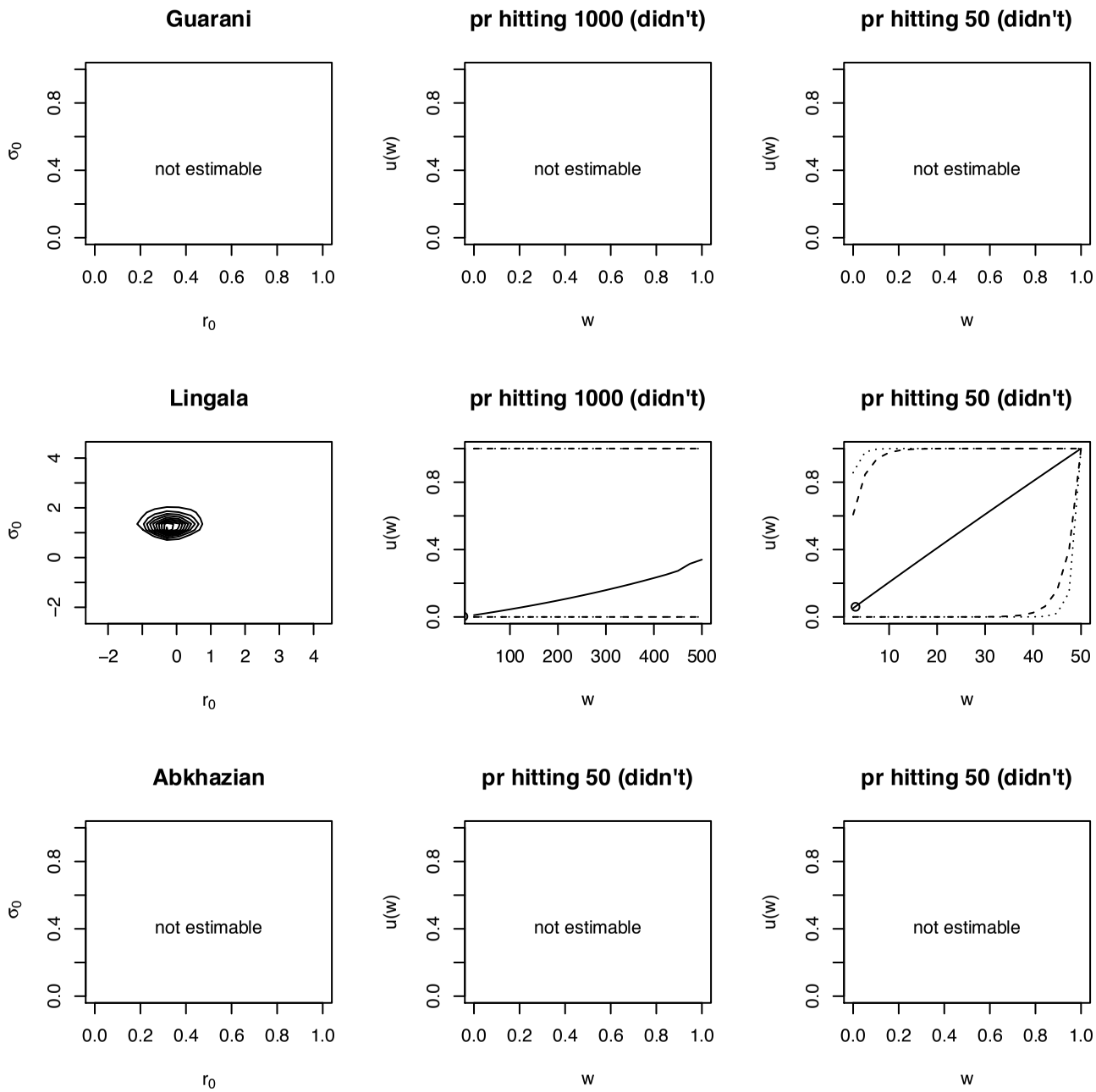

Punjabi
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Somali
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Acehnese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w

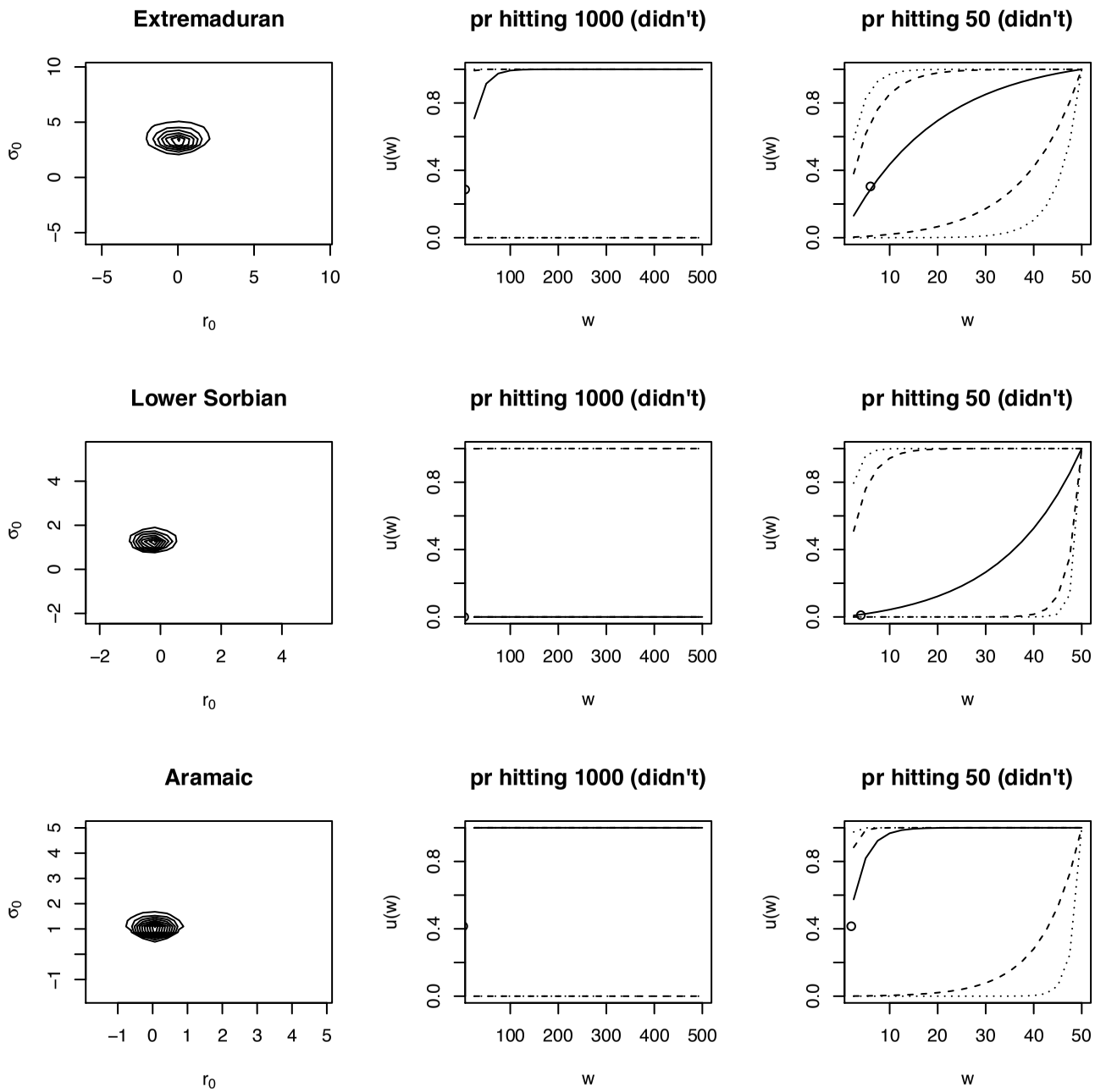

Ladino
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Ilokano
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Ligurian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

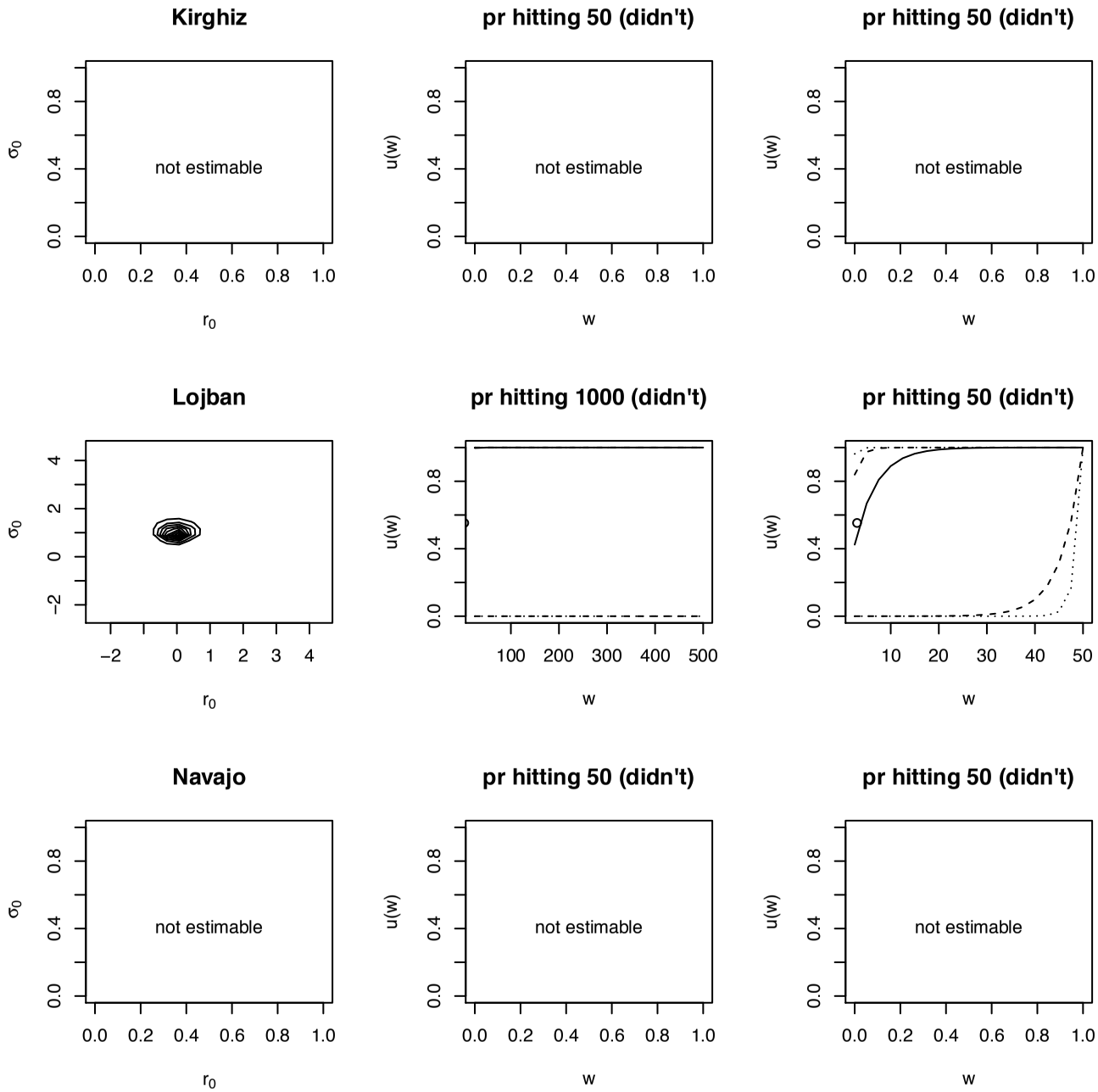

Romansh
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
u(w)
u(w)
r0
w
w
Malagasy
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
u(w)
u(w)
r0
w
w
Aromanian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
u(w)
u(w)
r0
w
w

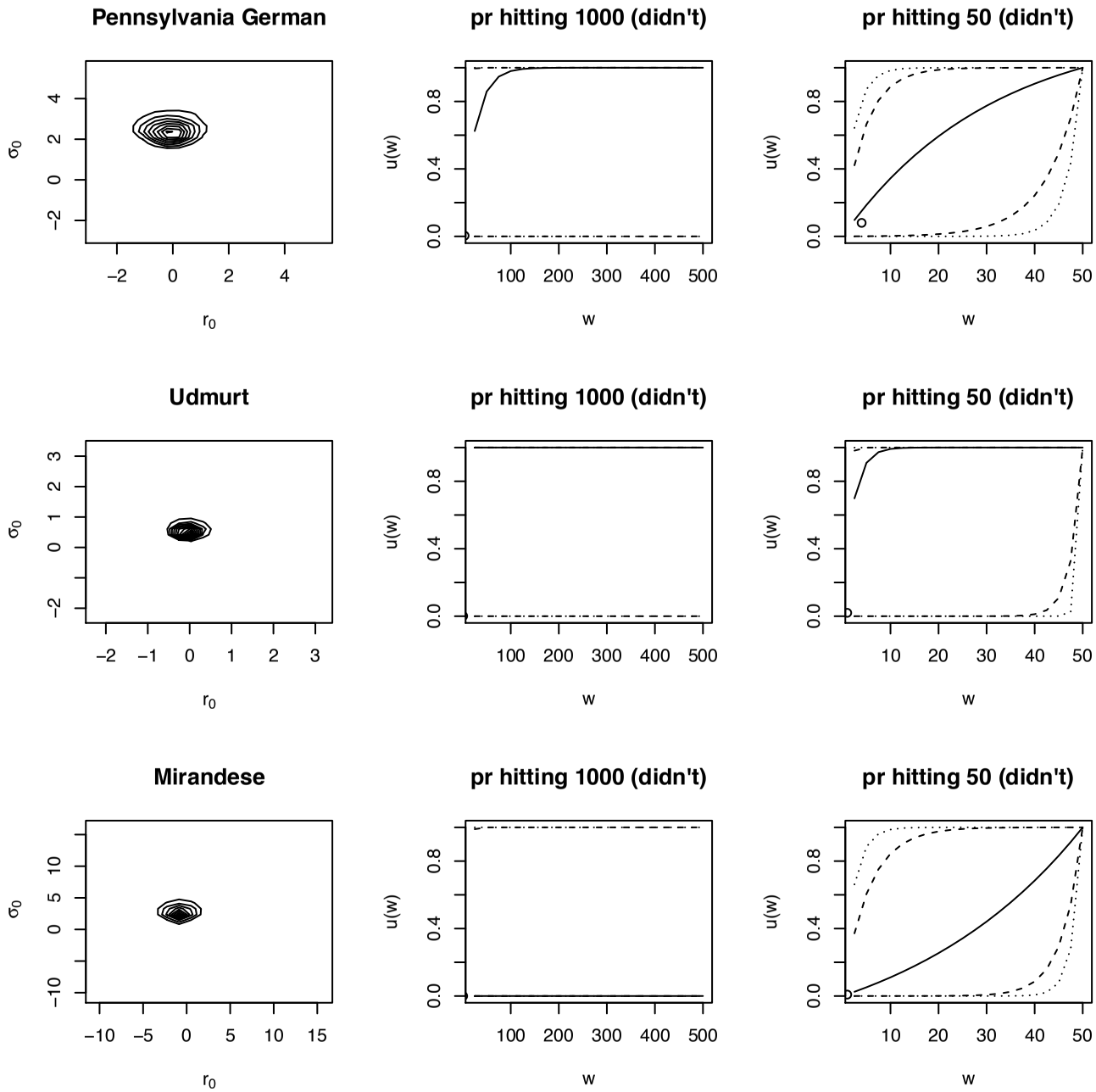

Rusyn
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w
Karachay–Balkar
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Sardinian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)

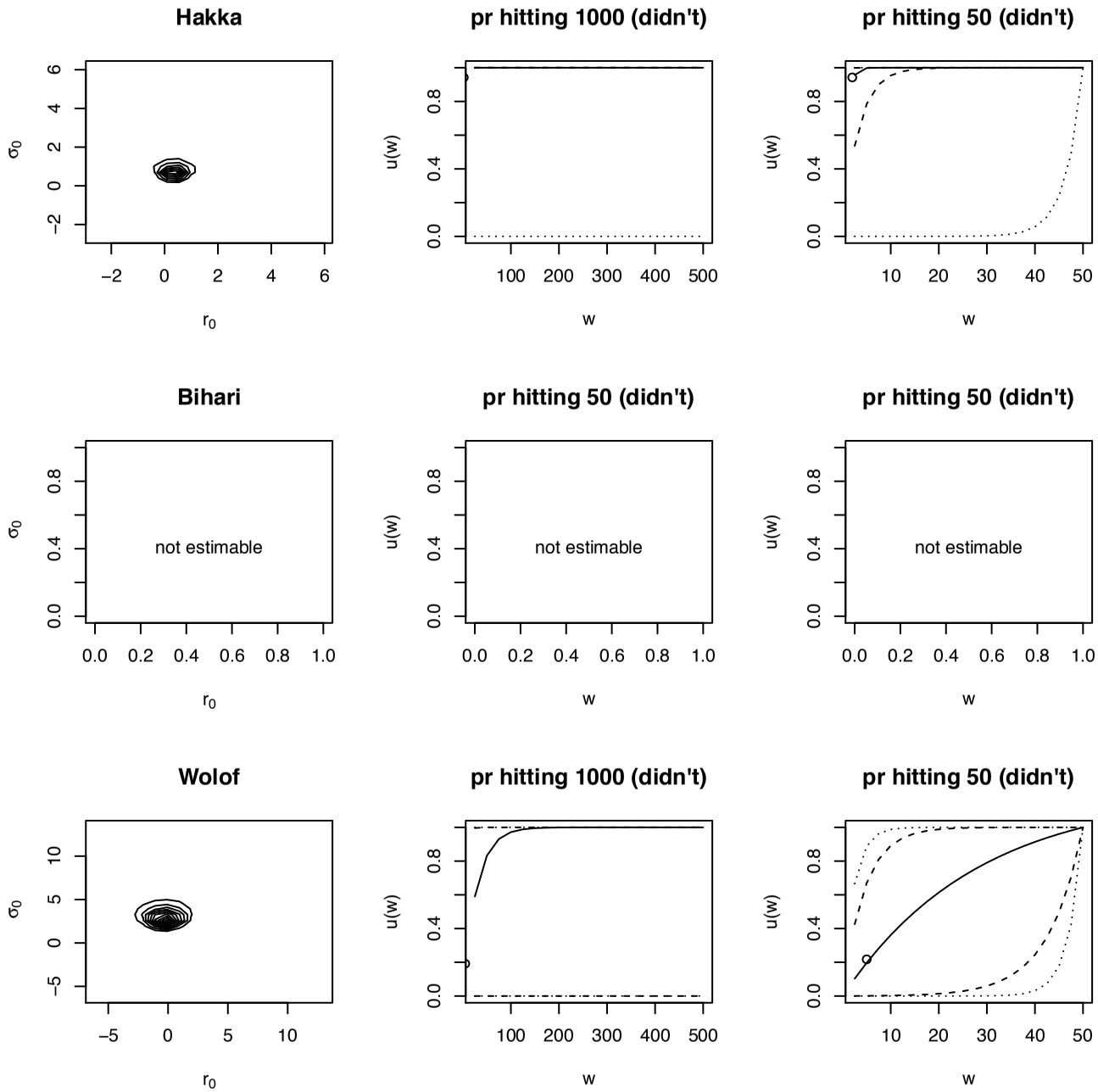

Igbo
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Crimean Tatar
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Kalmyk
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

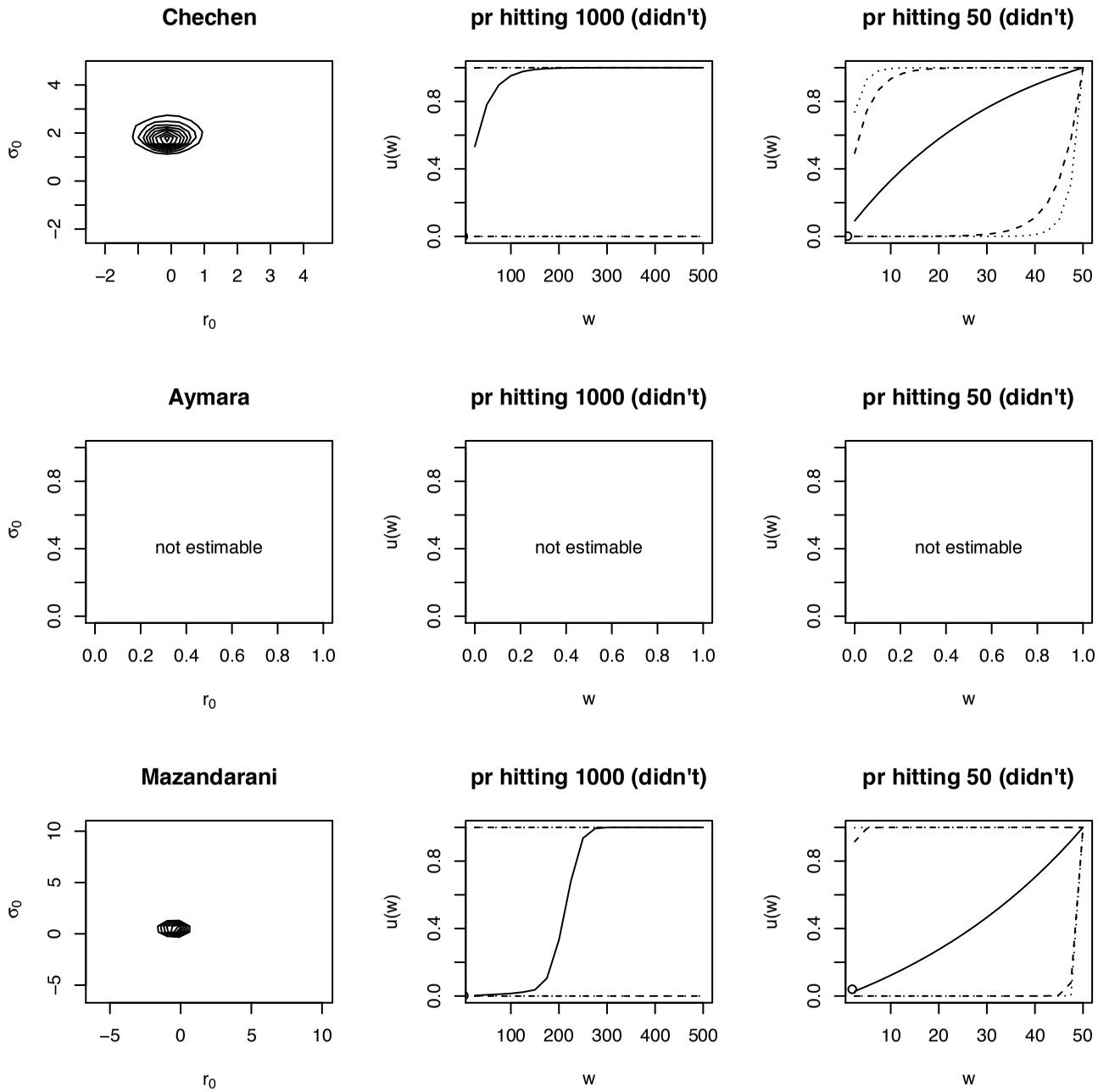

Guarani
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
u(w)
u(w)
r0
w
w
Lingala
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
u(w)
u(w)
r0
w
w
Abkhazian
pr hitting 50 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
u(w)
u(w)
r0
w
w

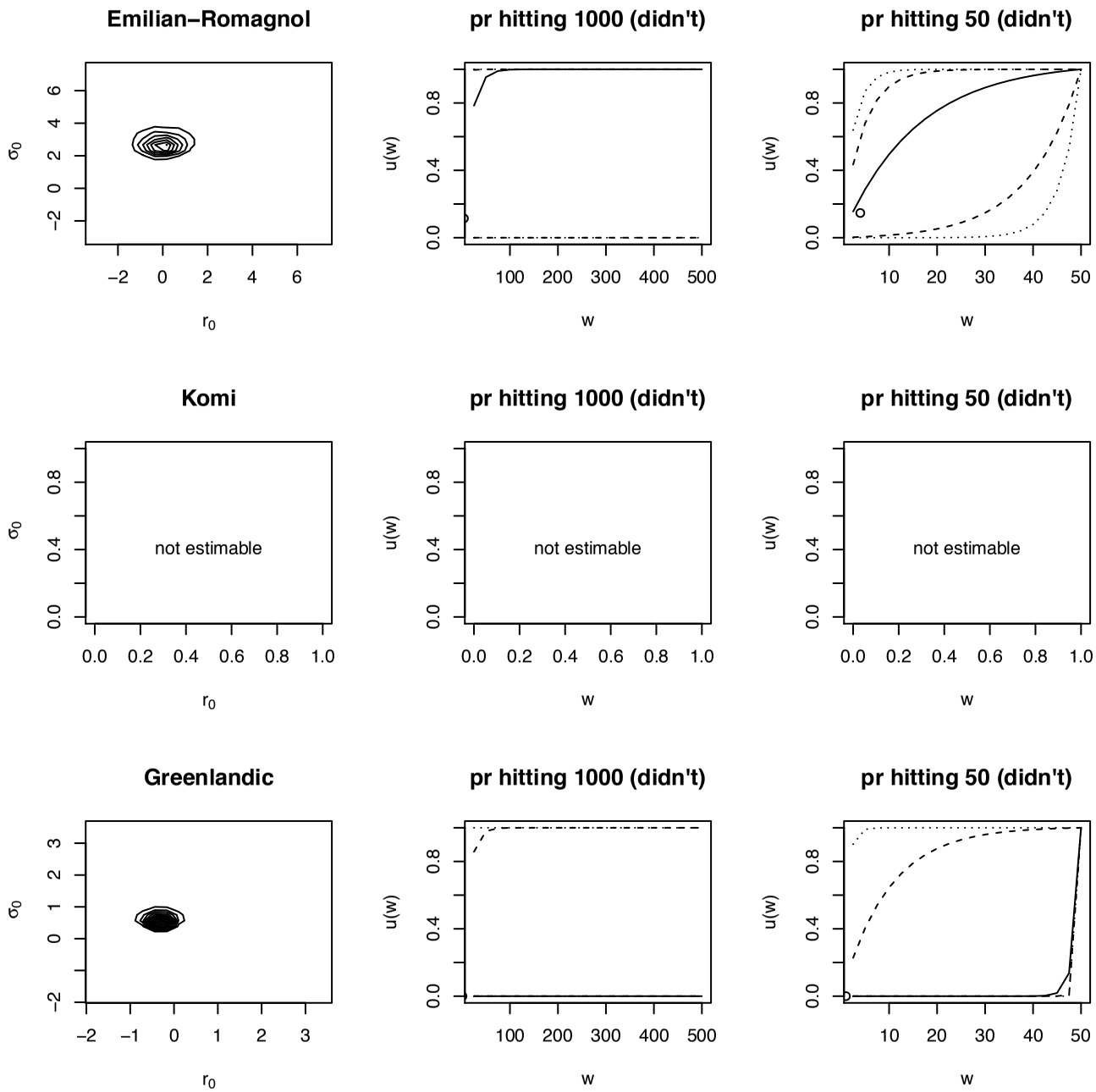

Extremaduran
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Lower Sorbian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Aramaic
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

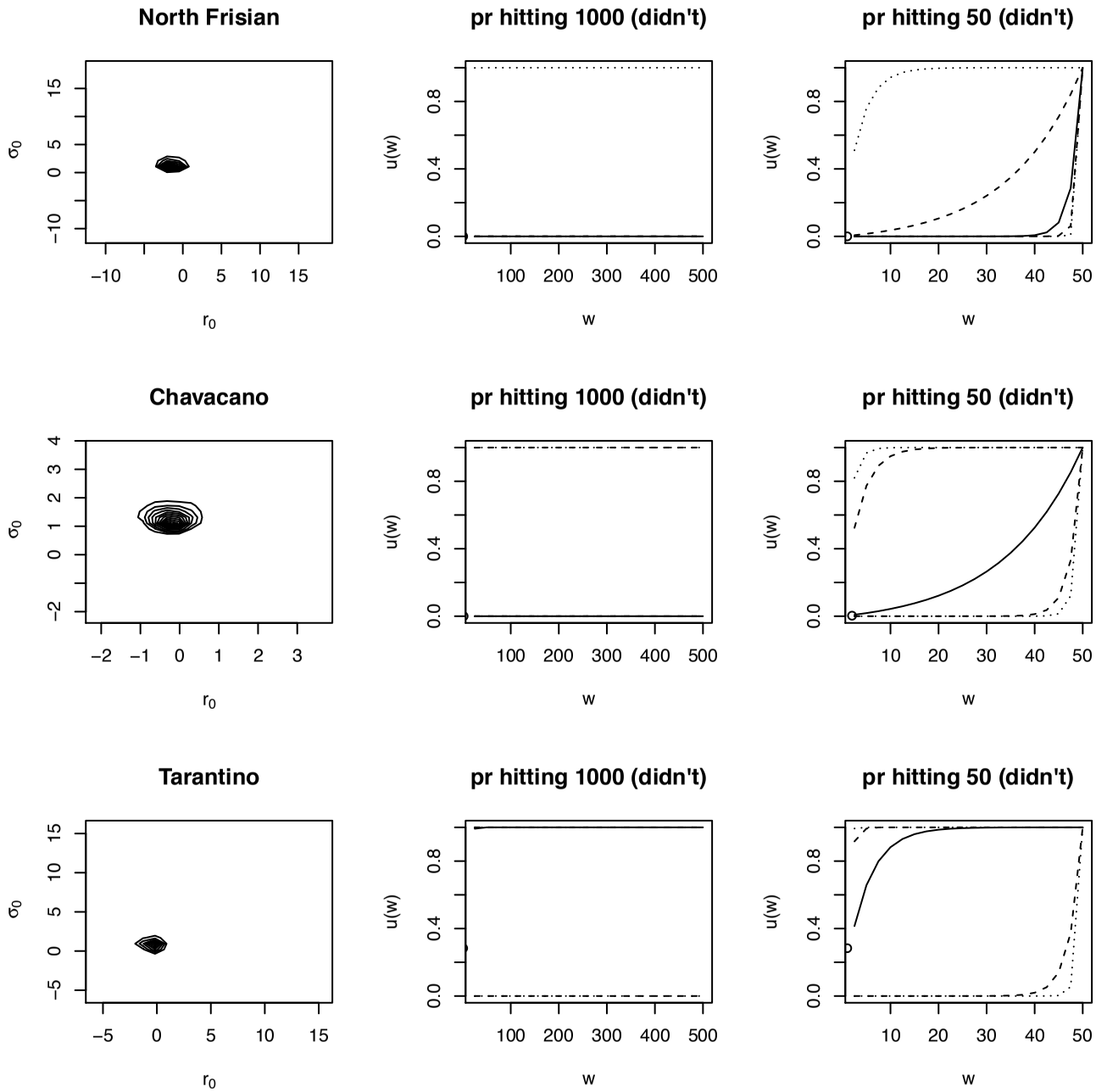

Kirghiz
not estimable
σ0
r0
pr hitting 50 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w
Lojban
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w
Navajo
not estimable
σ0
r0
pr hitting 50 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w

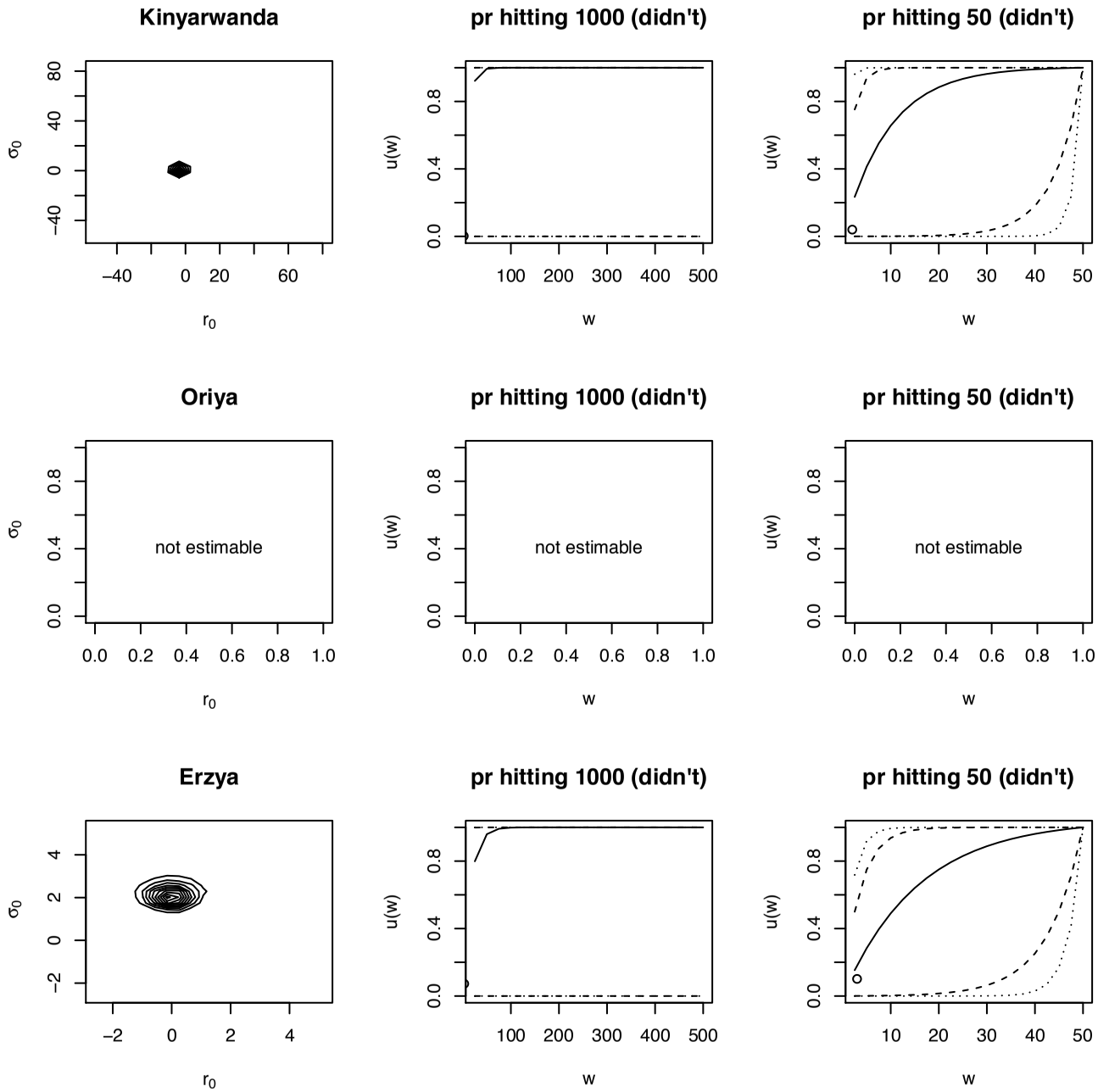

Pennsylvania German
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Udmurt
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Mirandese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

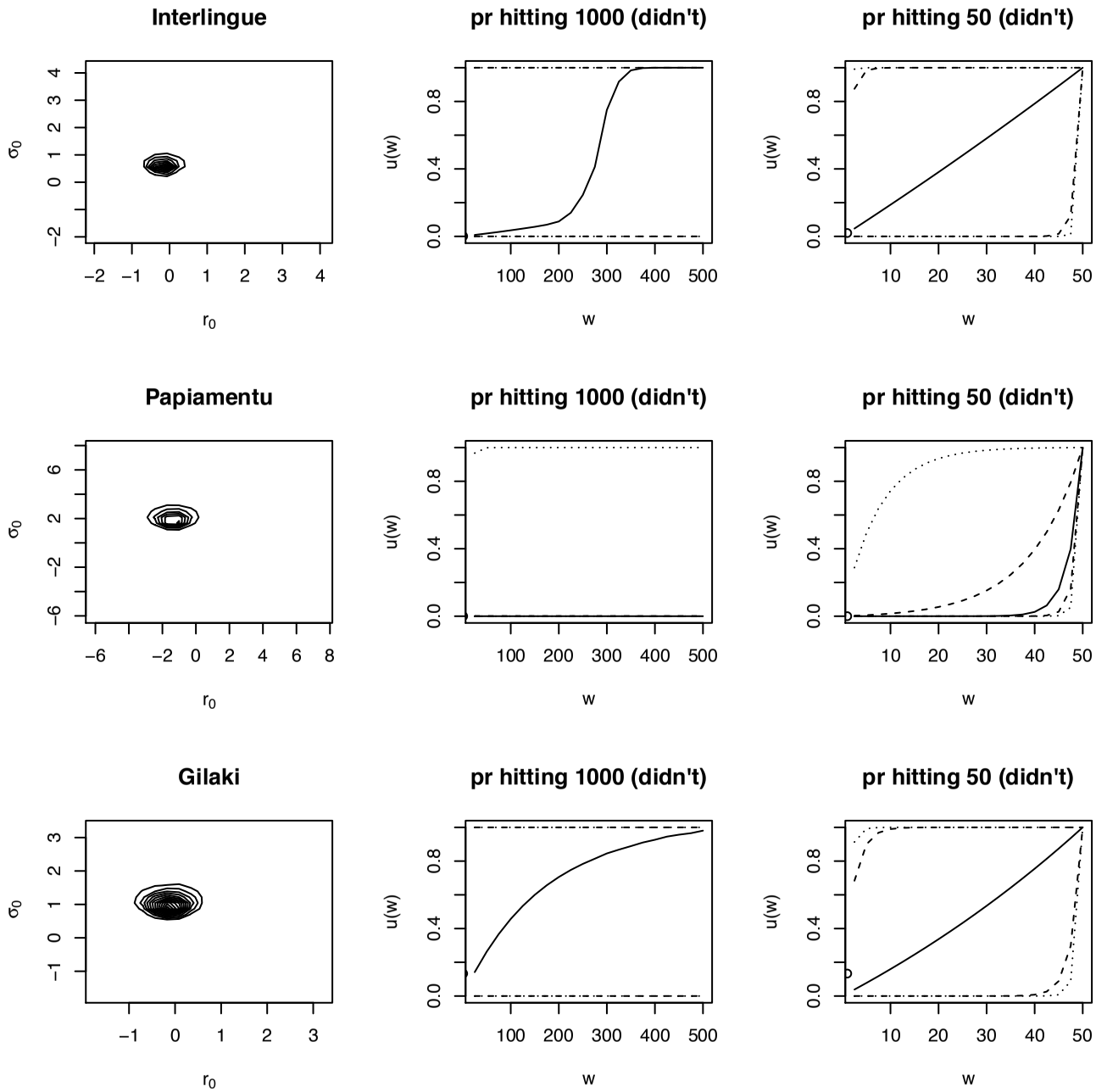

Hakka
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Bihari
pr hitting 50 (didn't)
pr hitting 50 (didn't)
Wolof
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

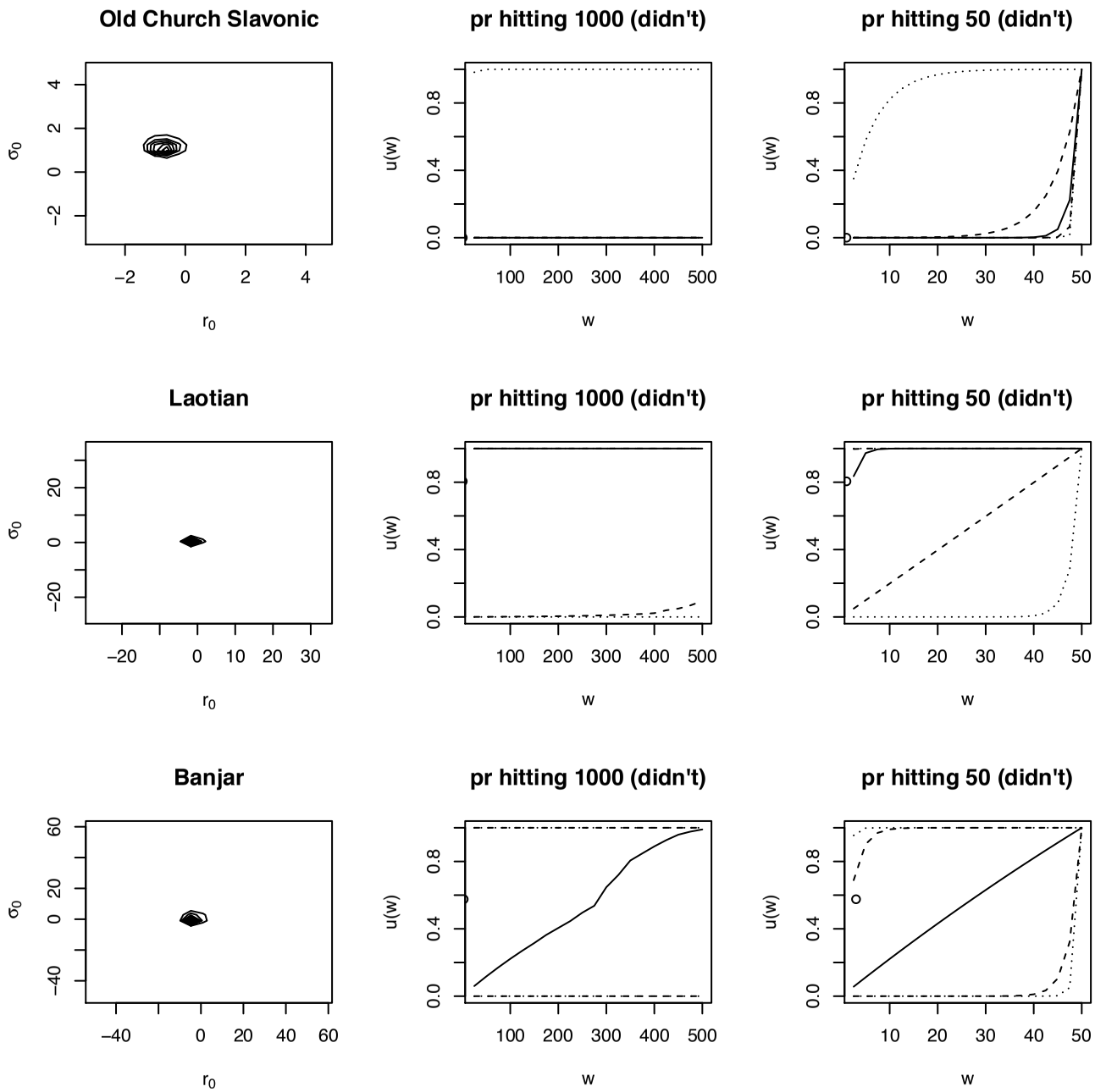

Chechen
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Aymara
not estimable
not estimable
not estimable
Mazandarani
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

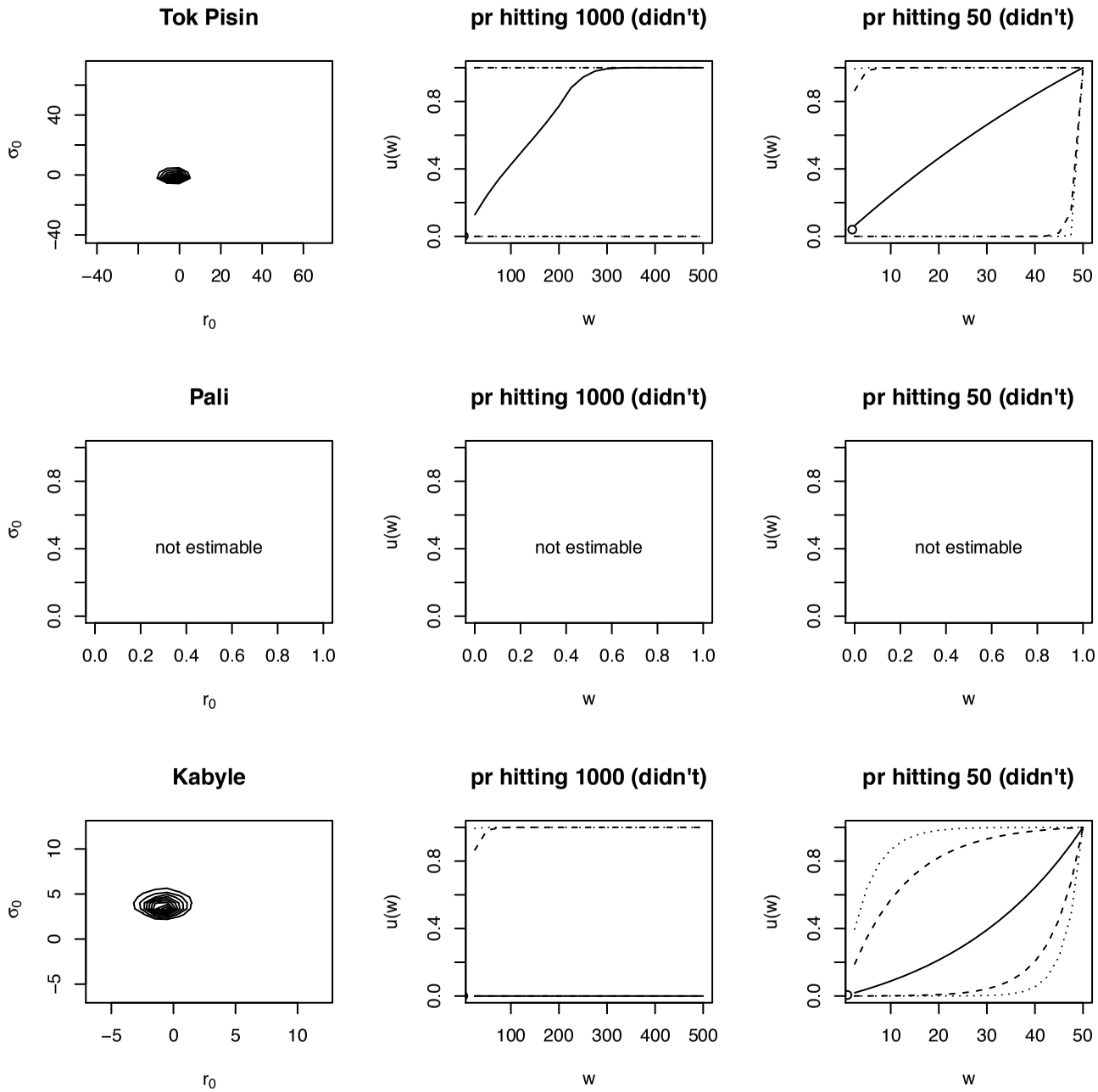

Emilian−Romagnol
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Komi
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Greenlandic
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

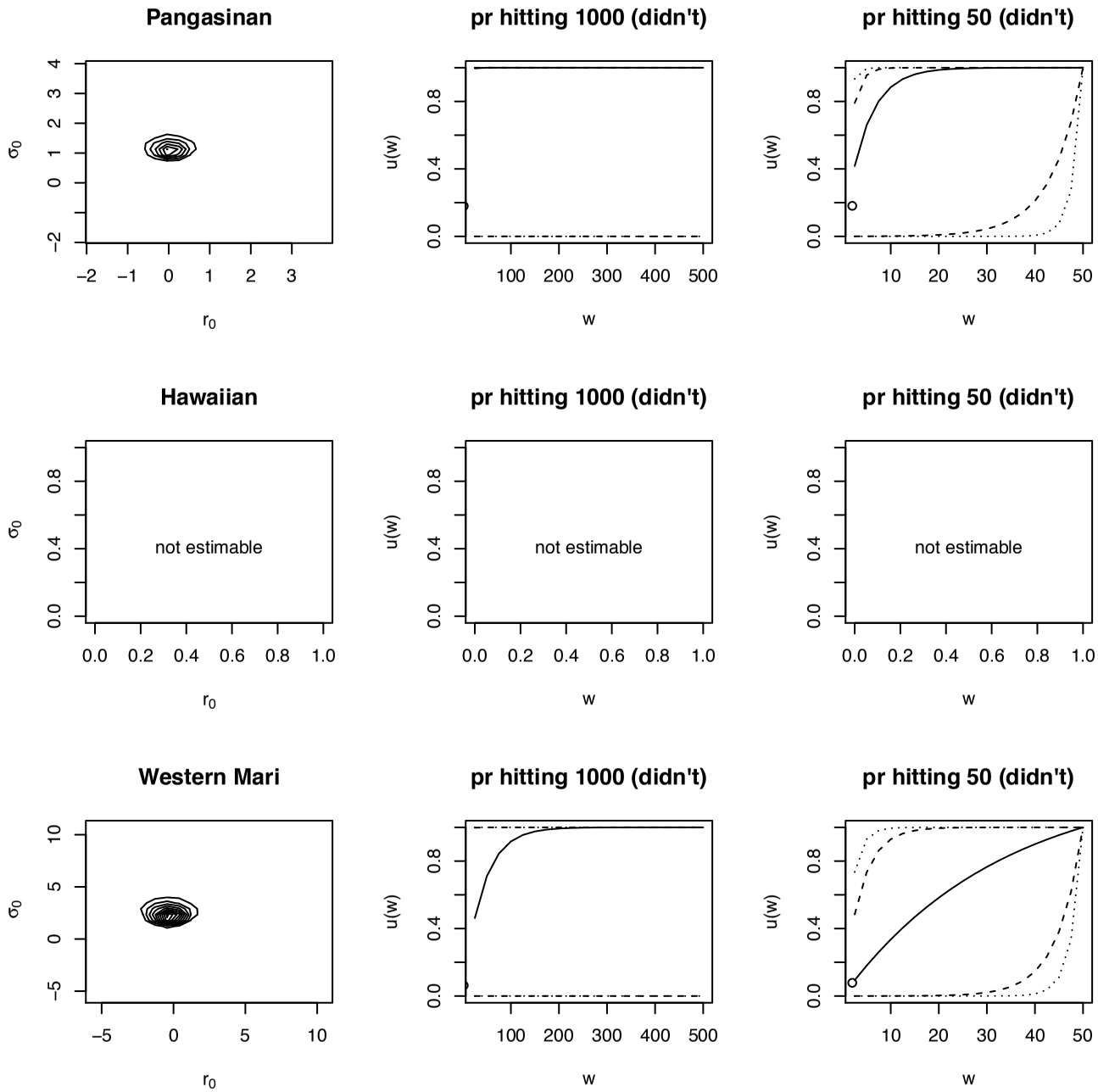

North Frisian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Chavacano
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Tarantino
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

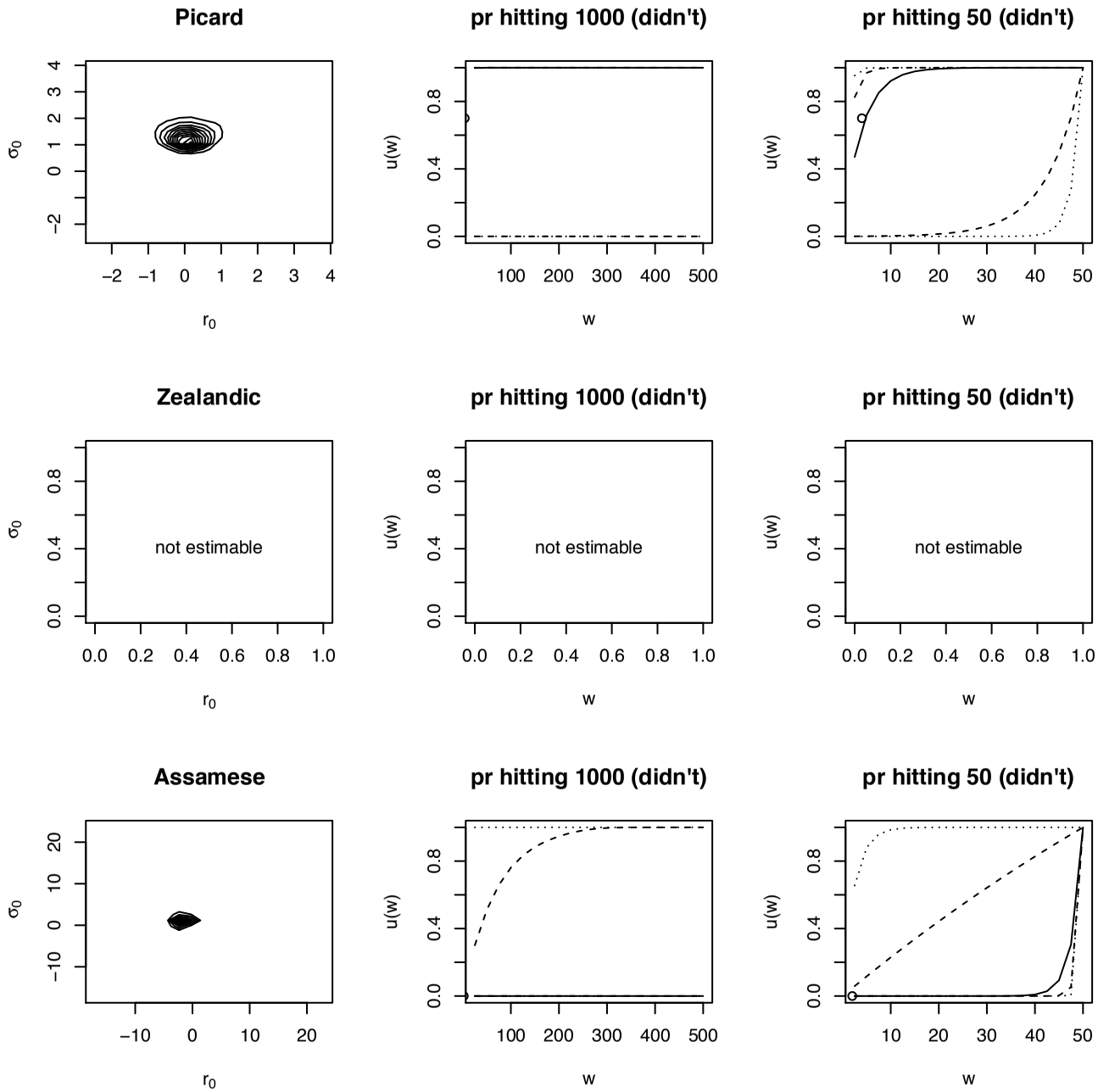

Kinyarwanda
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Oriya
not estimable
not estimable
not estimable
Erzya
σ0
r0
u(w)
w

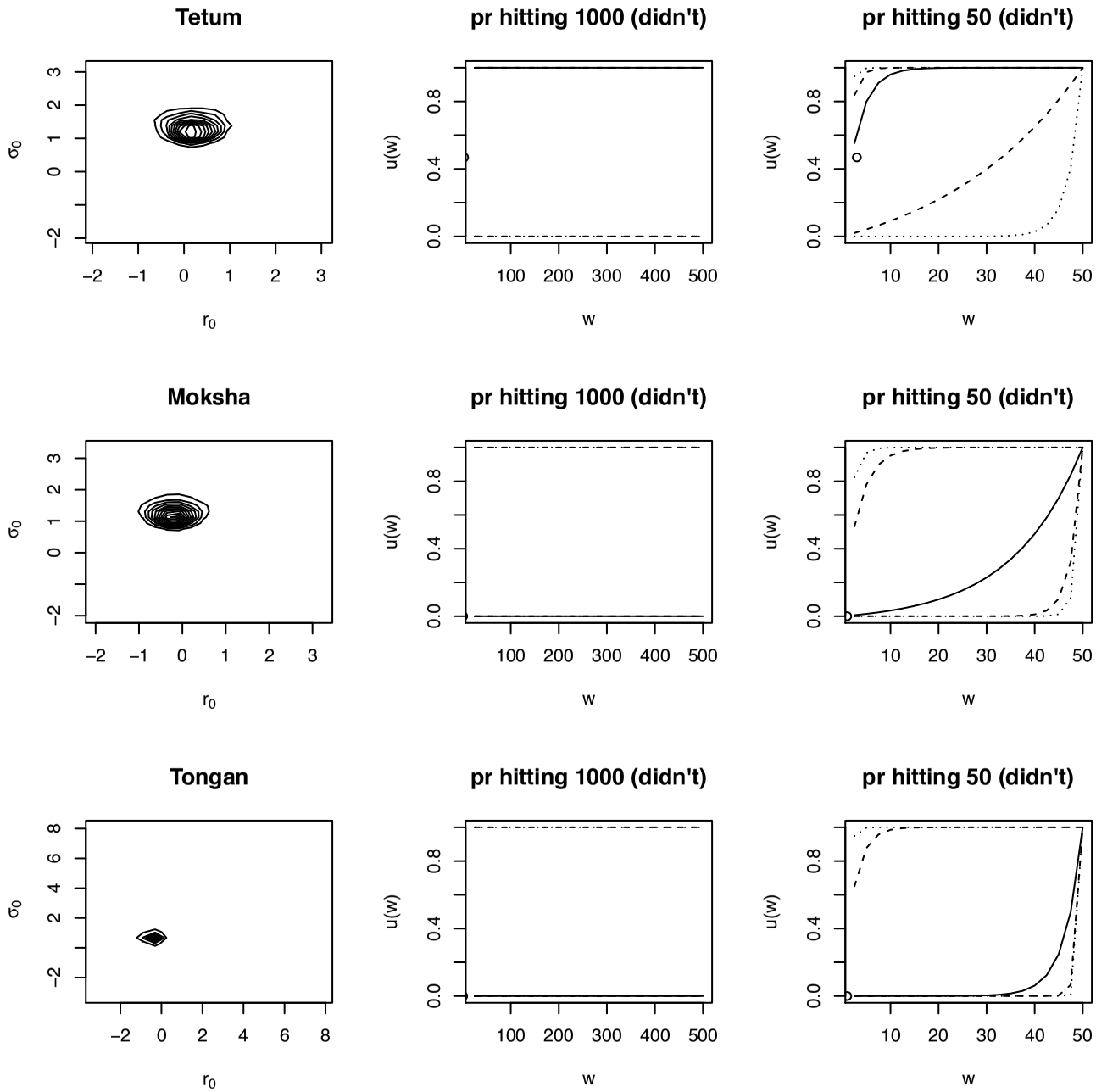

Interlingue
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Papiamentu
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Gilaki
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

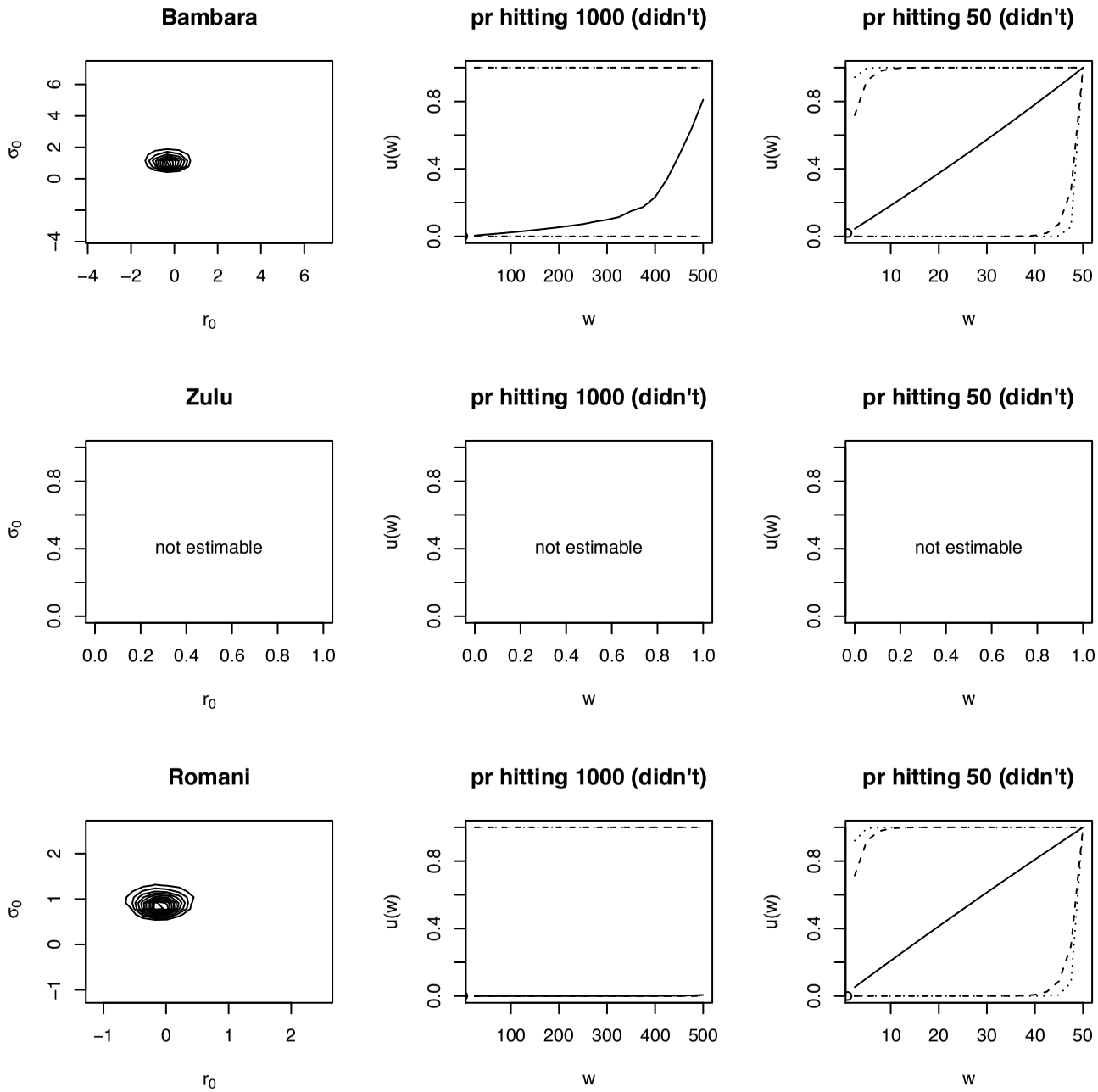

Old Church Slavonic
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Laotian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Banjar
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

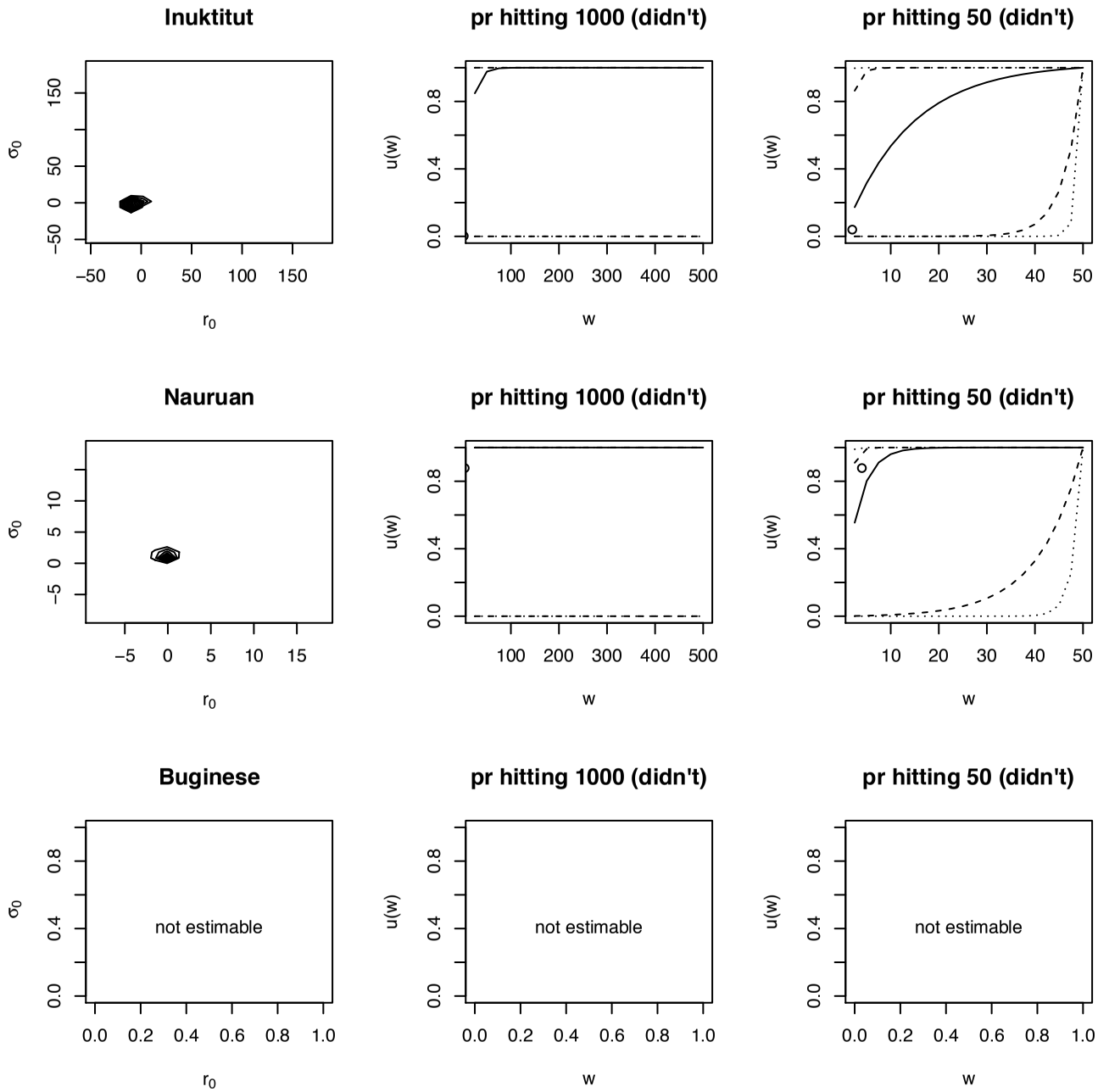

Tok Pisin
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Pali
not estimable
not estimable
not estimable
Kabyle
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

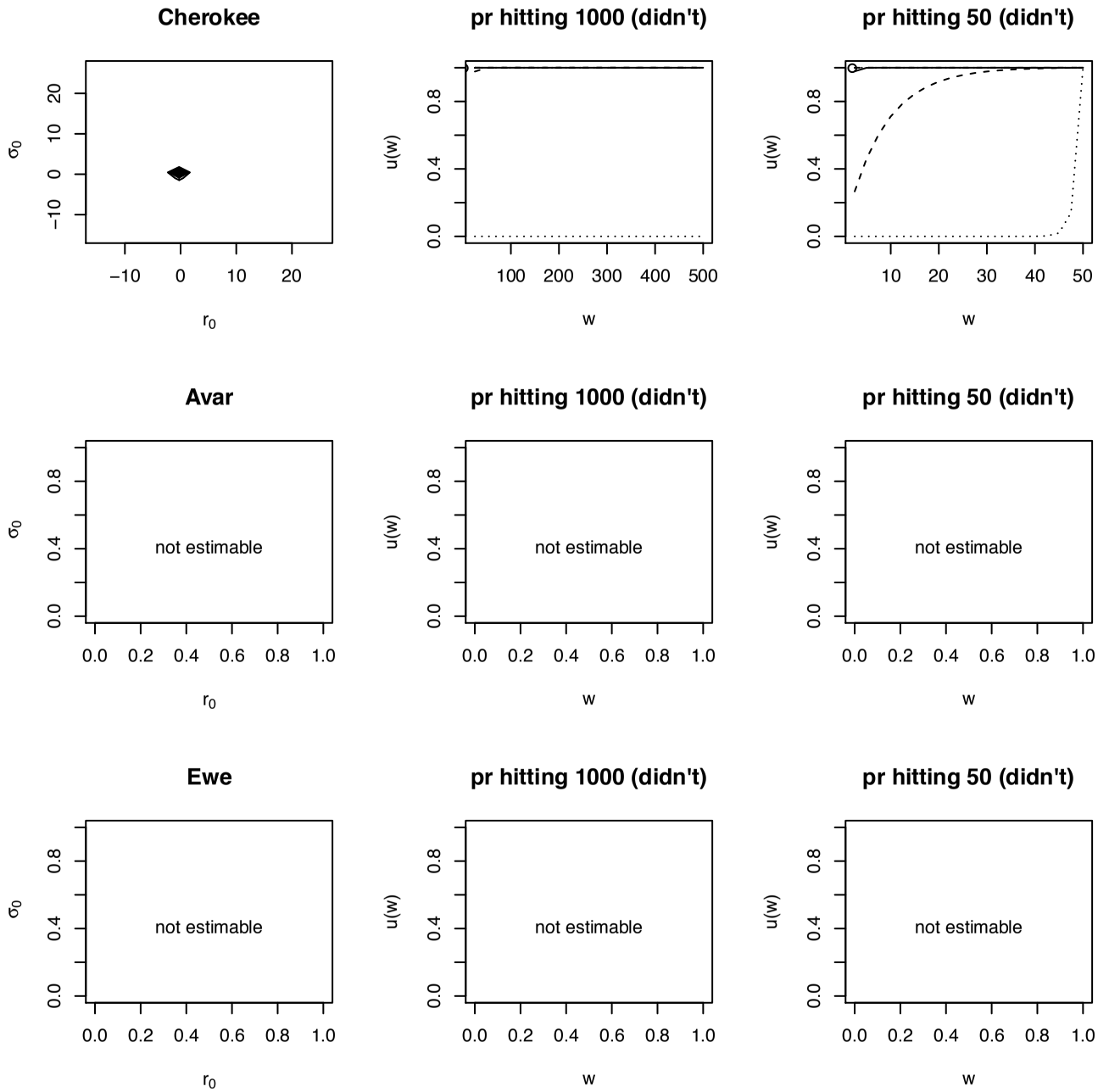

Pangasinan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Hawaiian
not estimable
not estimable
not estimable
Western Mari
σ0
r0
u(w)
w

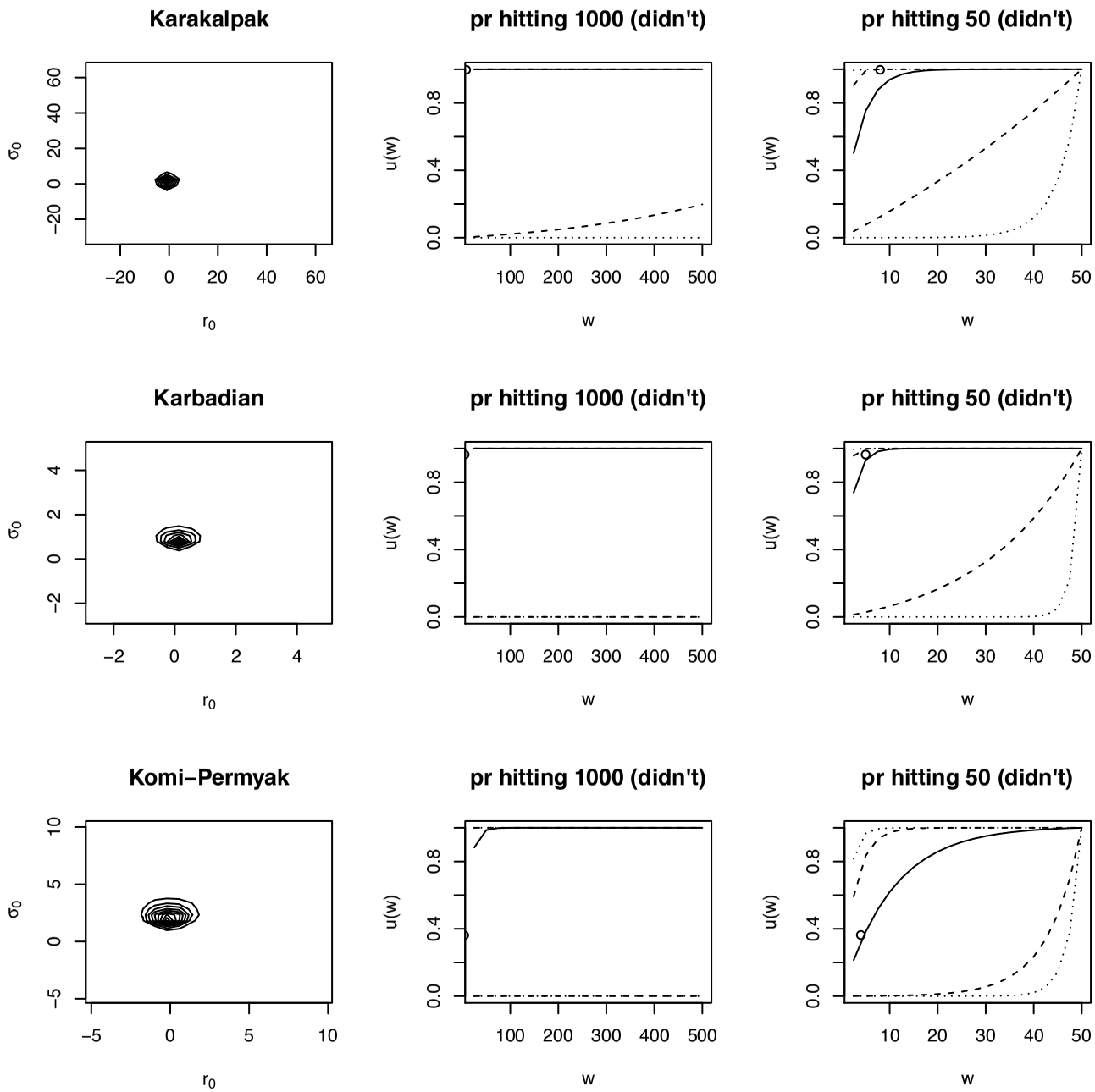

Picard
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Zealandic
not estimable
Assamese
σ0
r0
u(w)
w

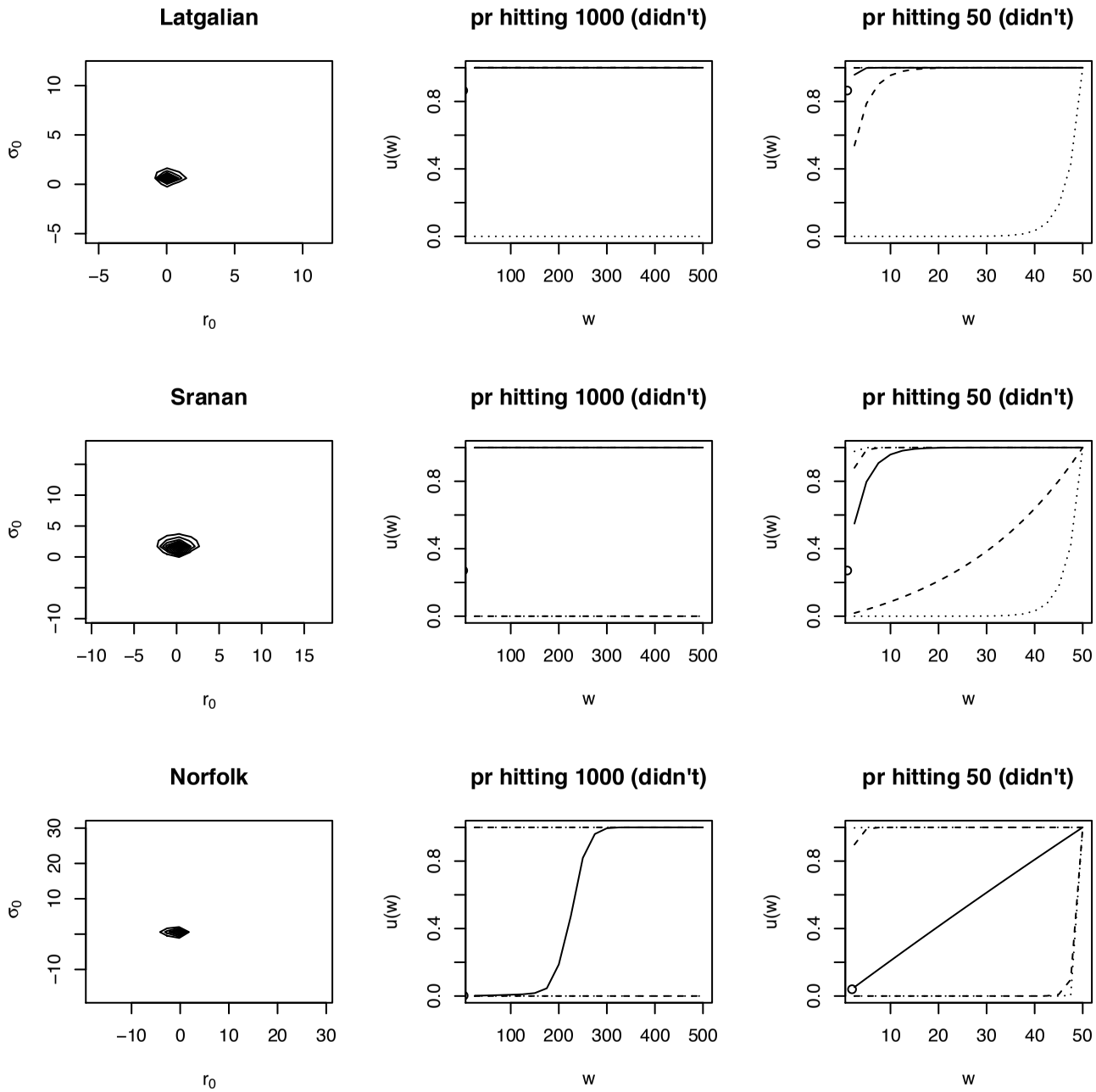

Tetum
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Moksha
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Tongan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

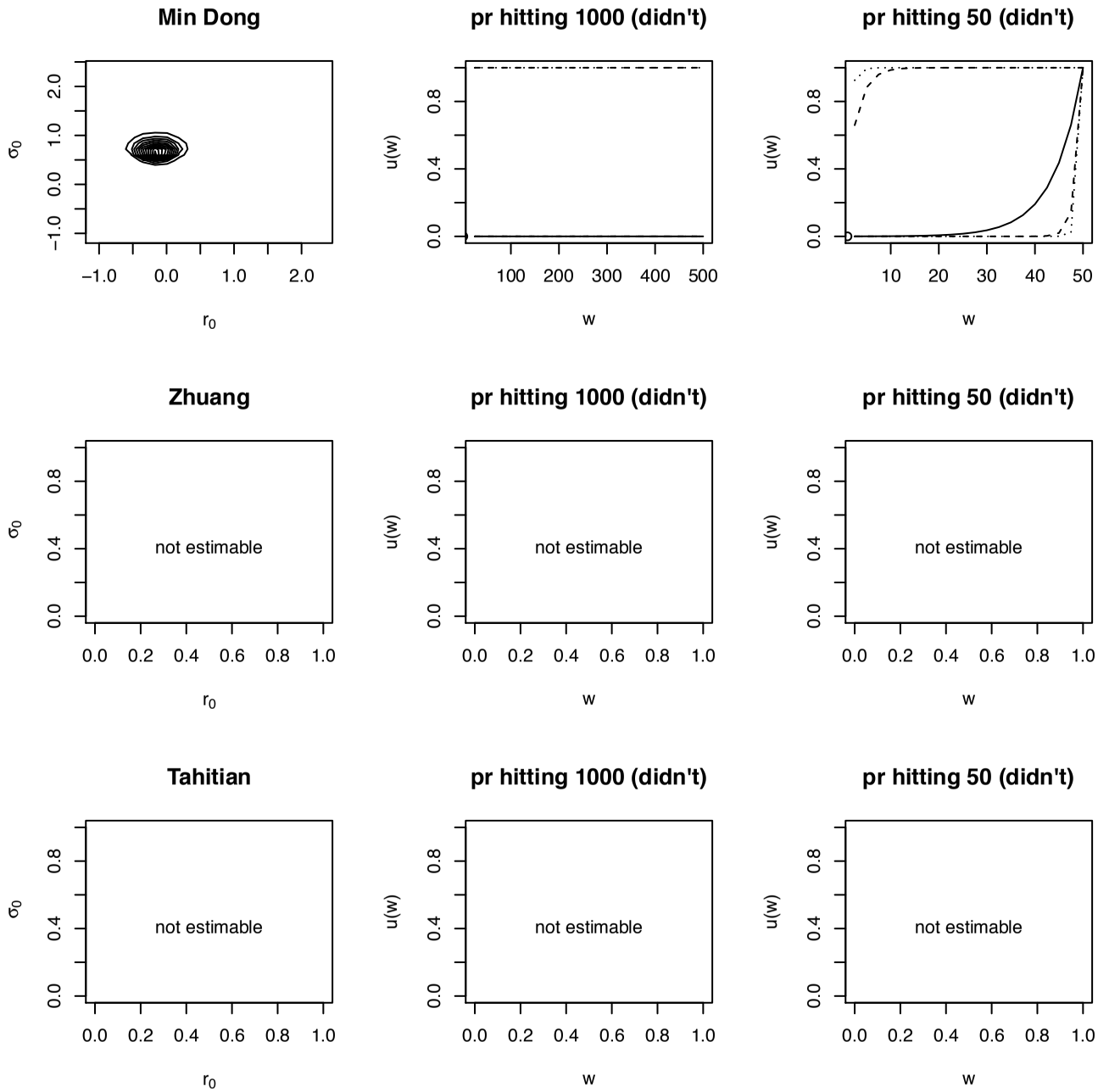

Bambara
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Zulu
not estimable
not estimable
not estimable
Romani
σ_0
r_0
u(w)
w

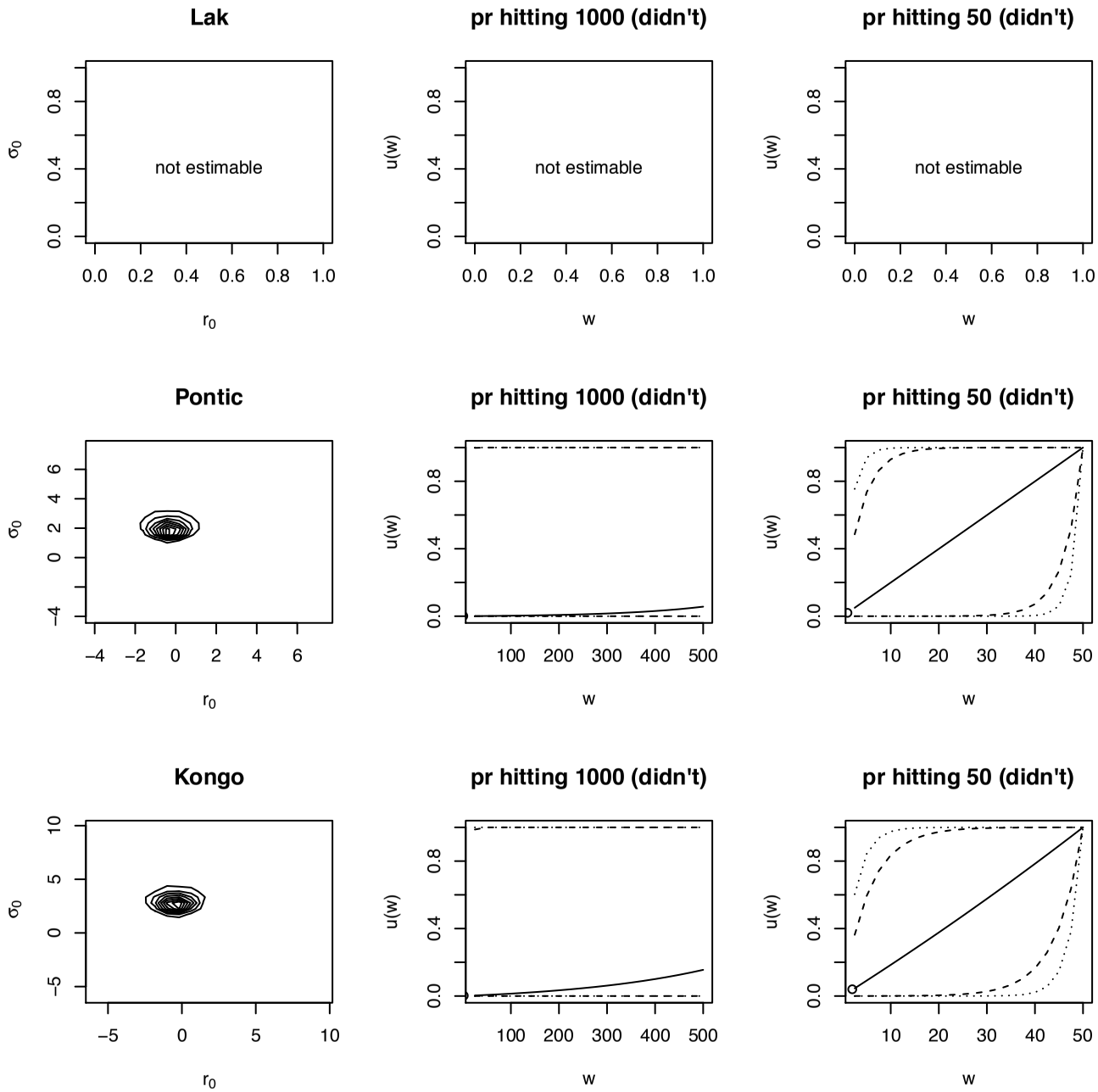

Inuktitut
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Nauruan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Buginese
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

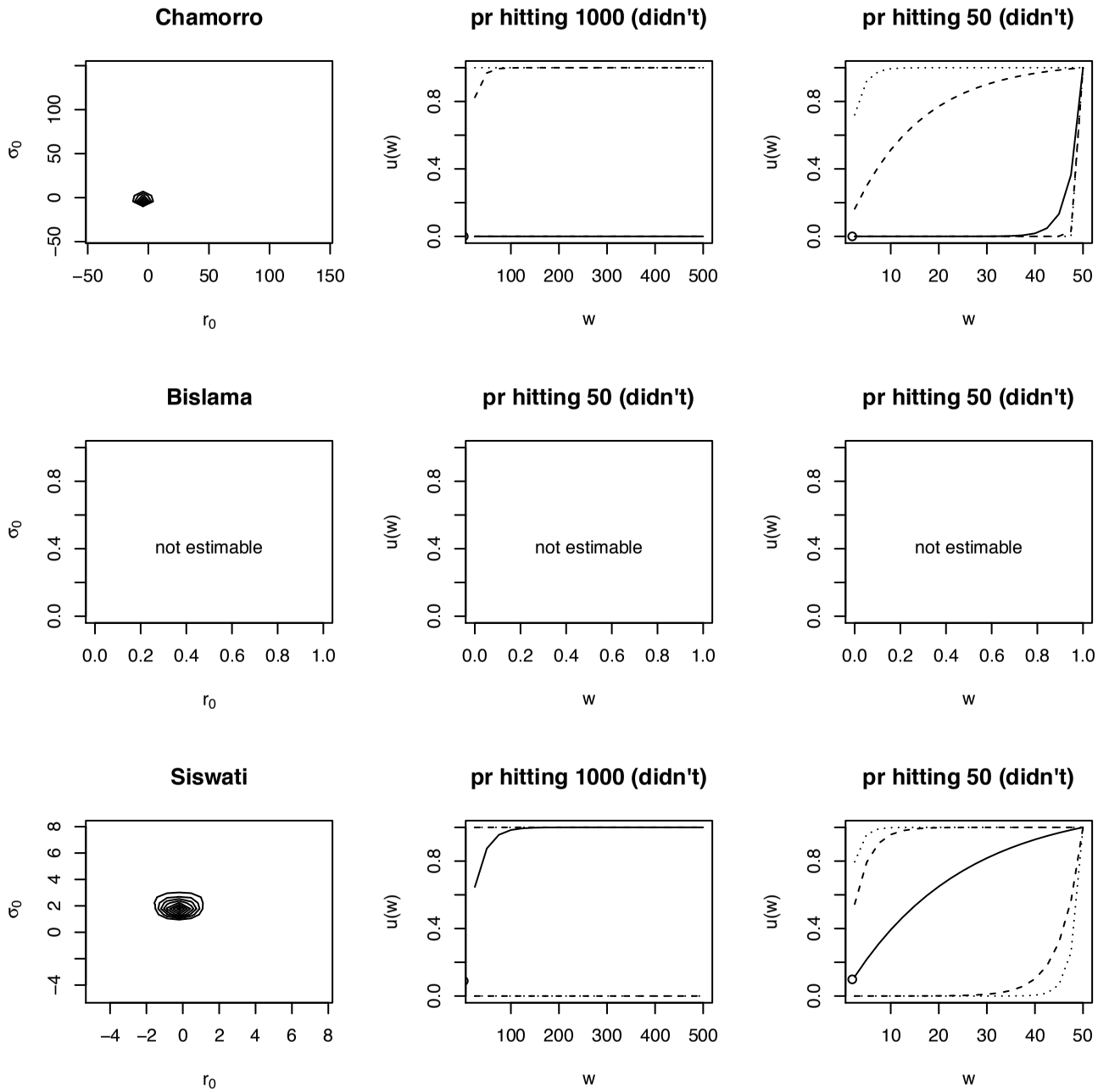

Cherokee
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Avar
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Ewe
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w

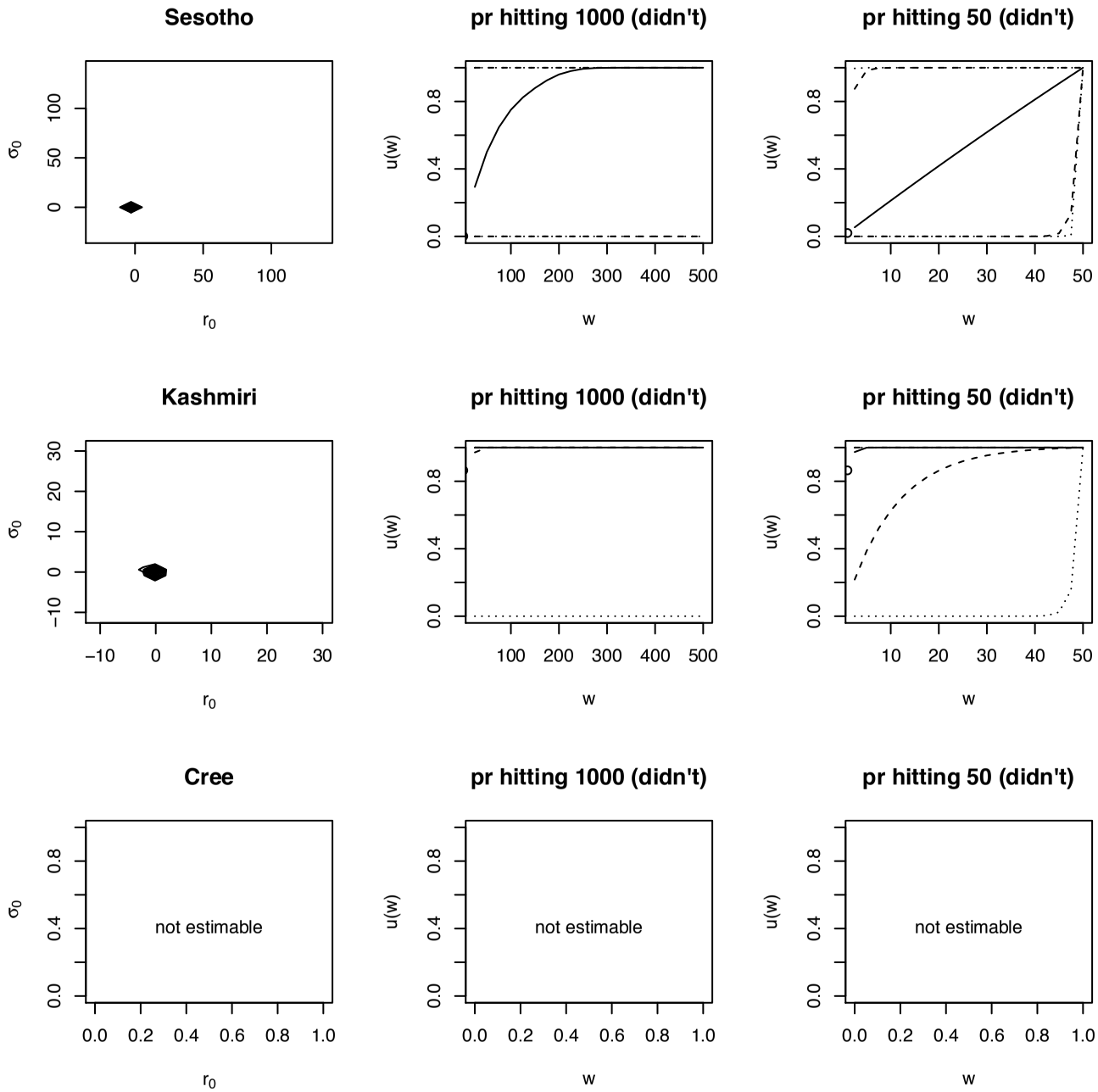

Karakalpak
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Karbadian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Komi−Permyak
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

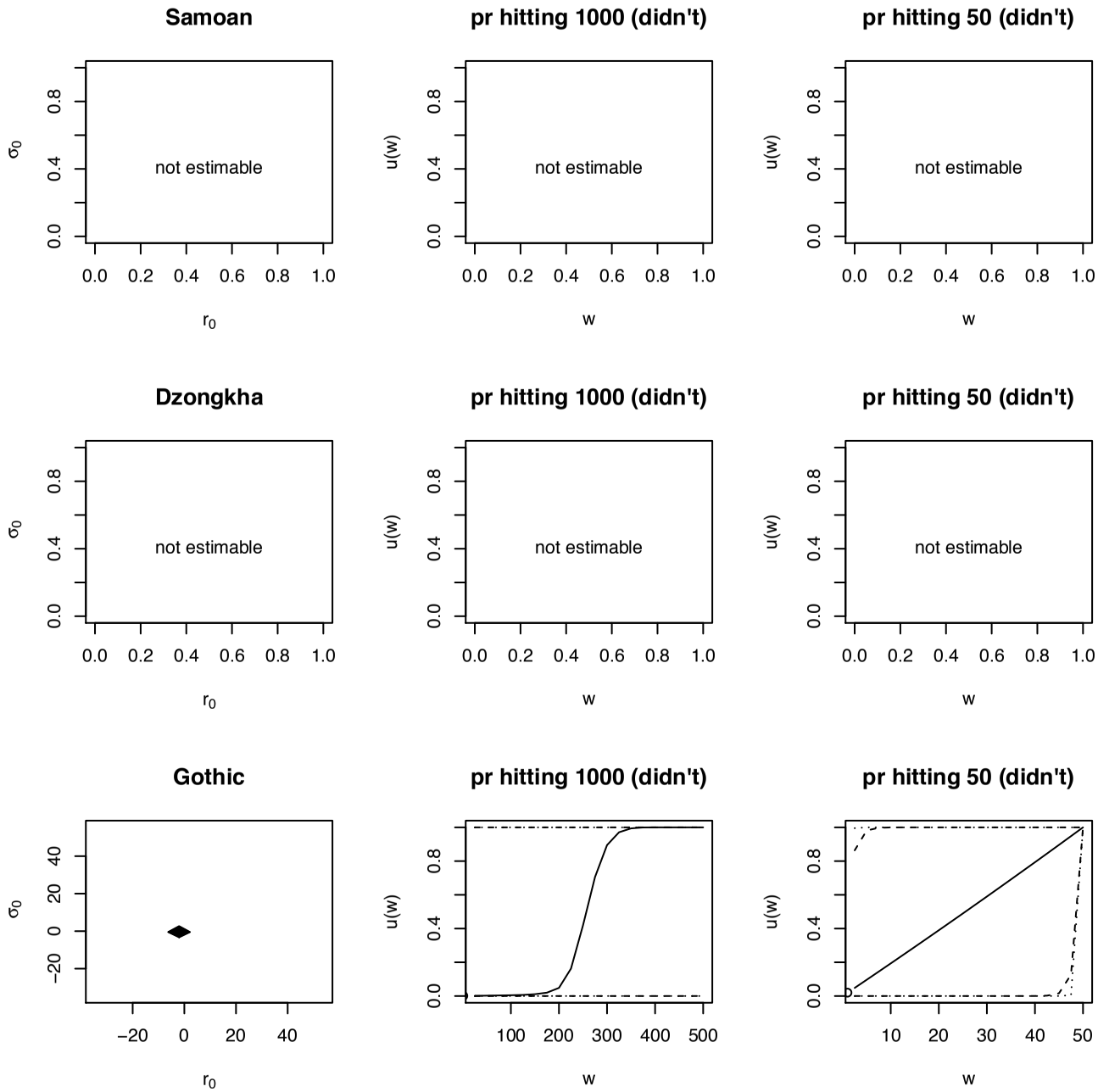

Latgalian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Sranan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Norfolk
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
σ0
r0
u(w)
w

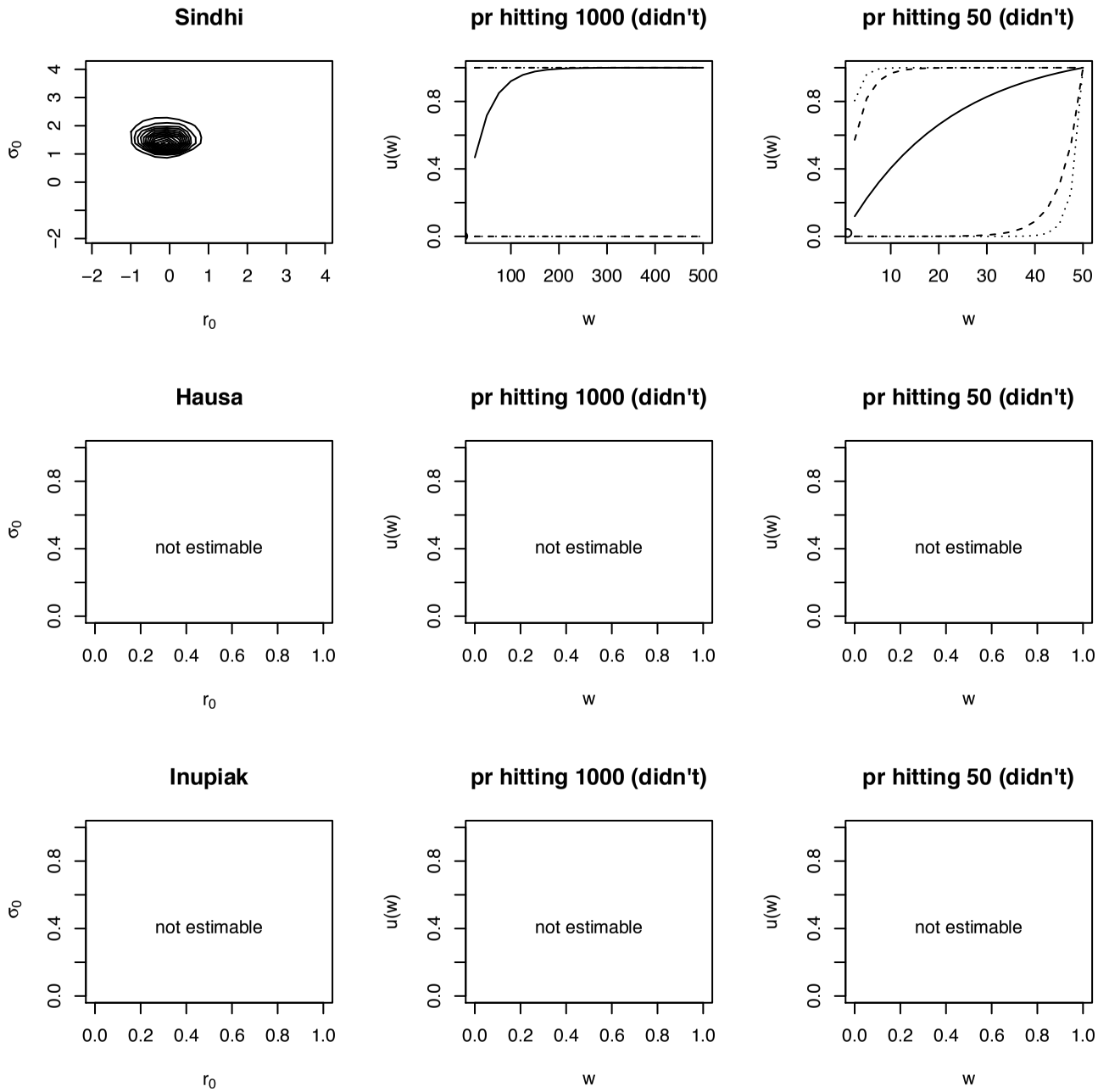

Min Dong
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Zhuang
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Tahitian
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w

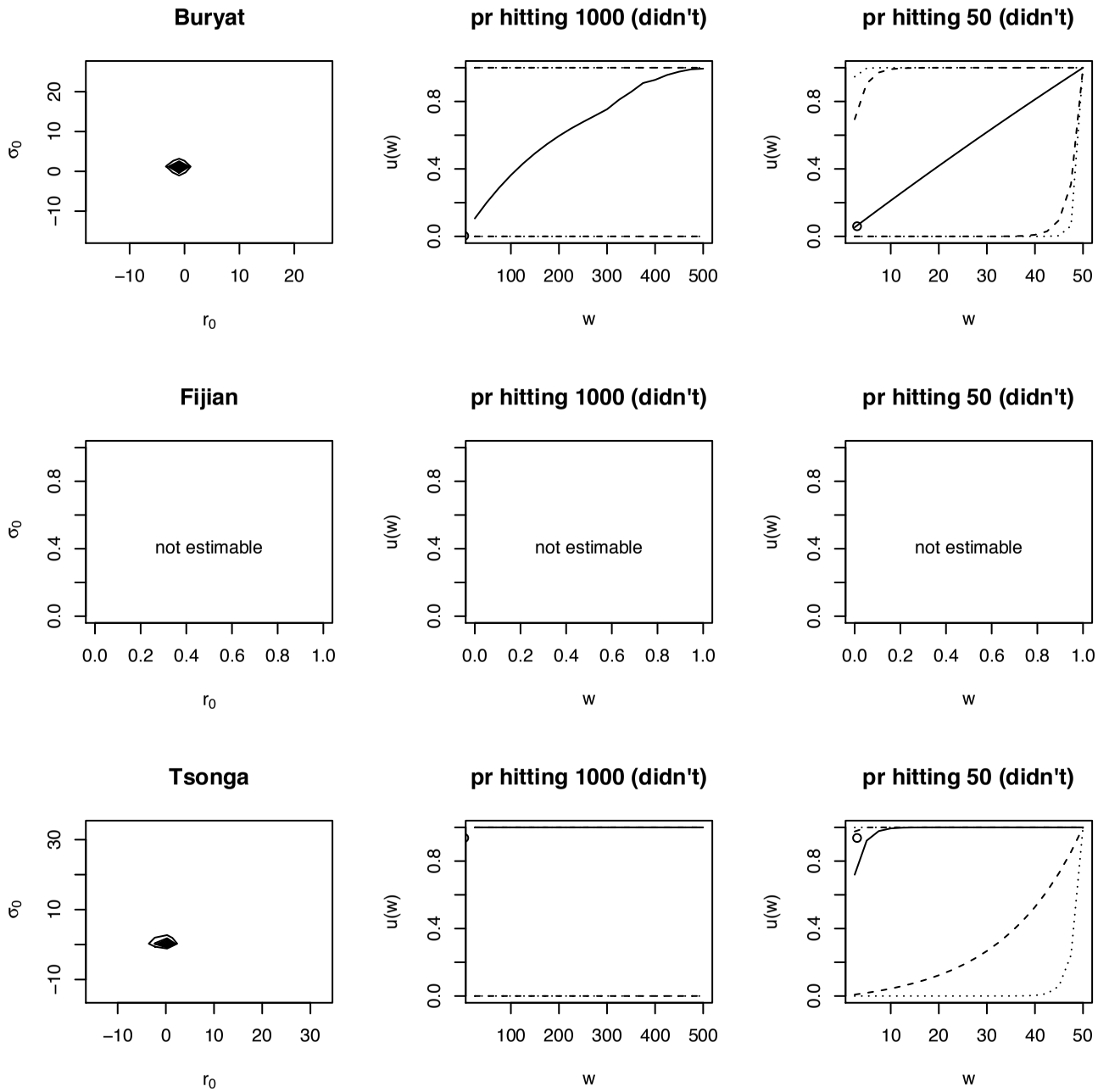

Lak
not estimable
σ0
r0
pr hitting 1000 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w
Pontic
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w
Kongo
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w

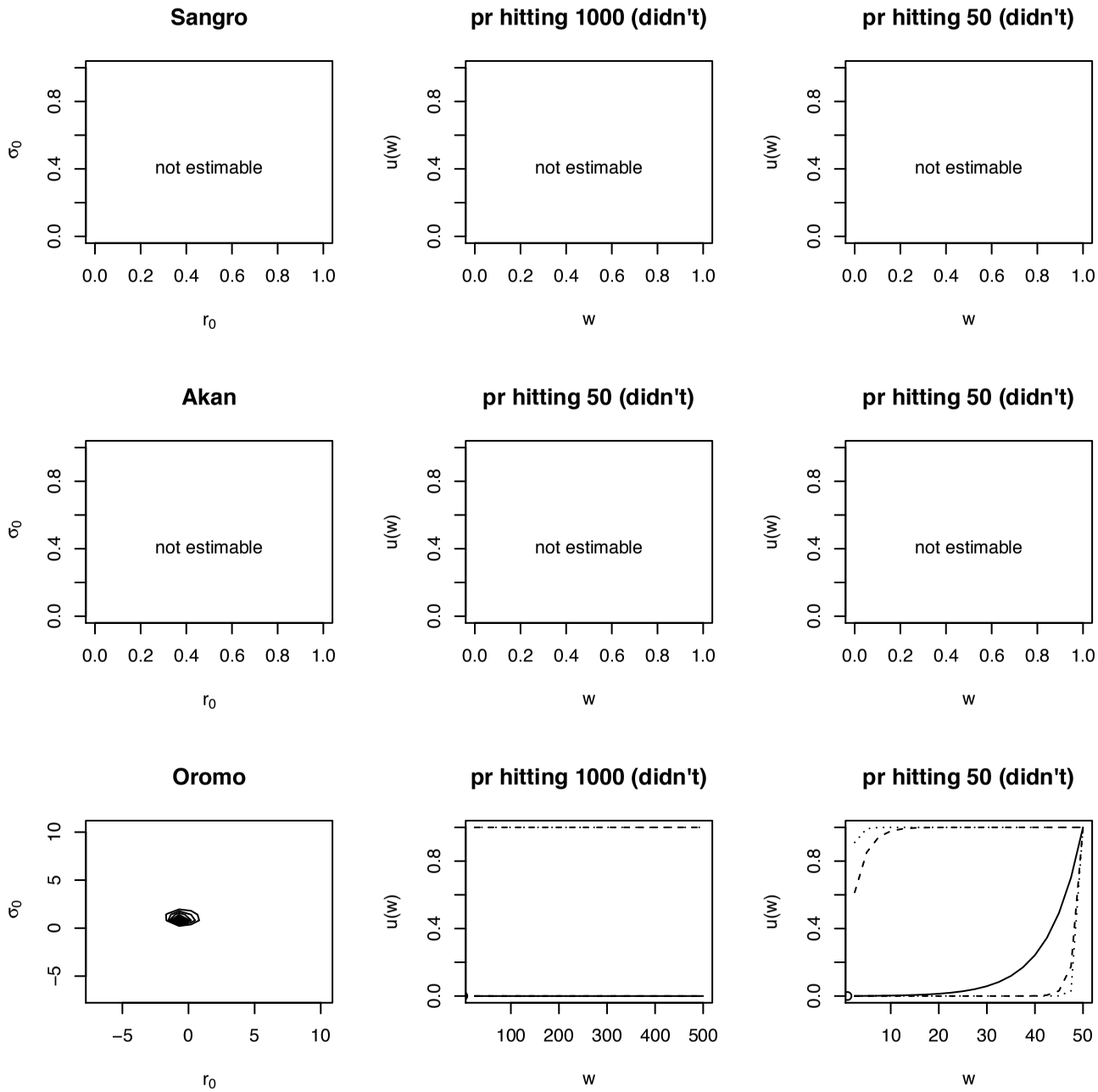

Chamorro
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Bislama
pr hitting 50 (didn't)
pr hitting 50 (didn't)
Siswati
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

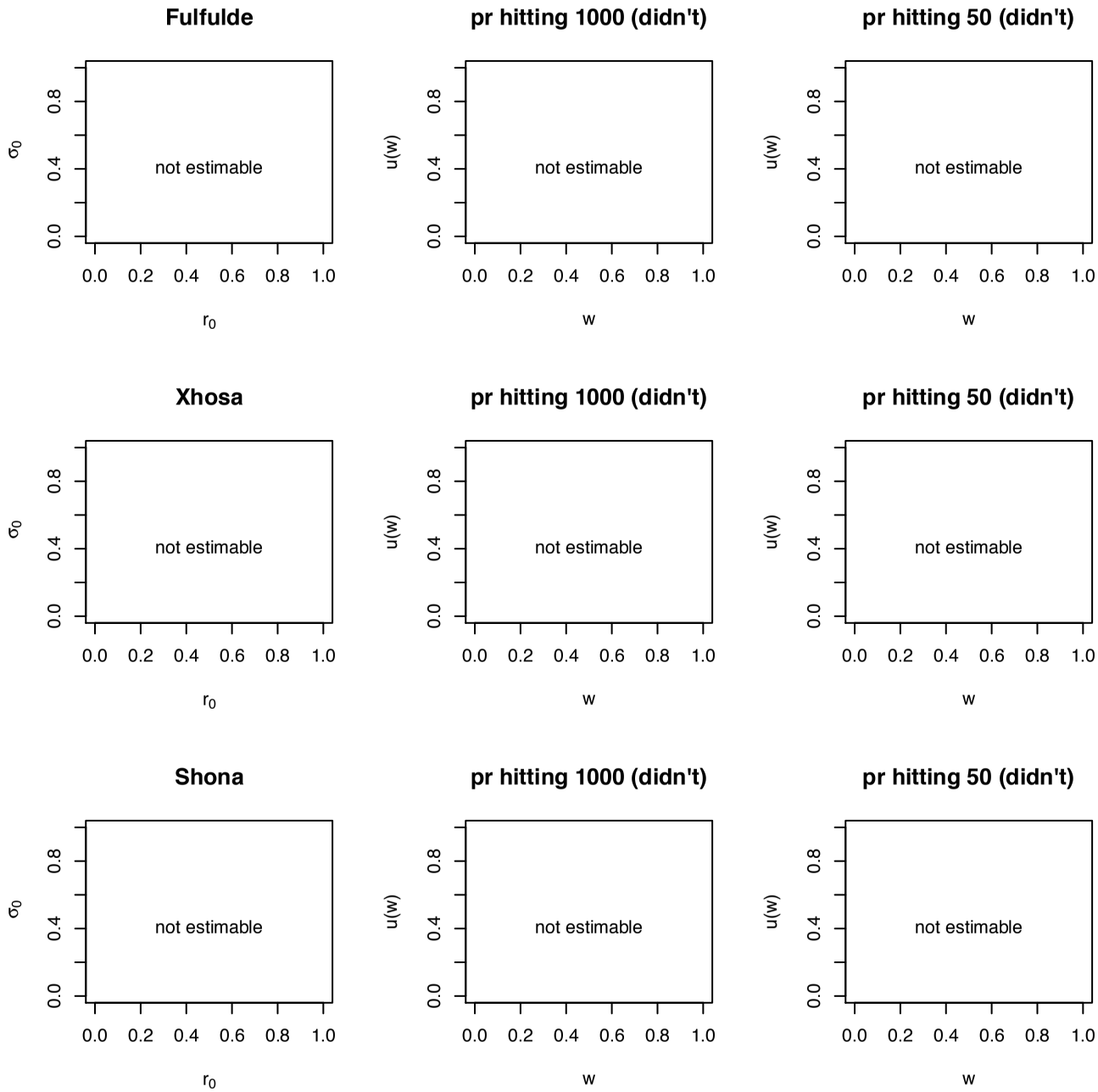

Sesotho
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Kashmiri
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Cree
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

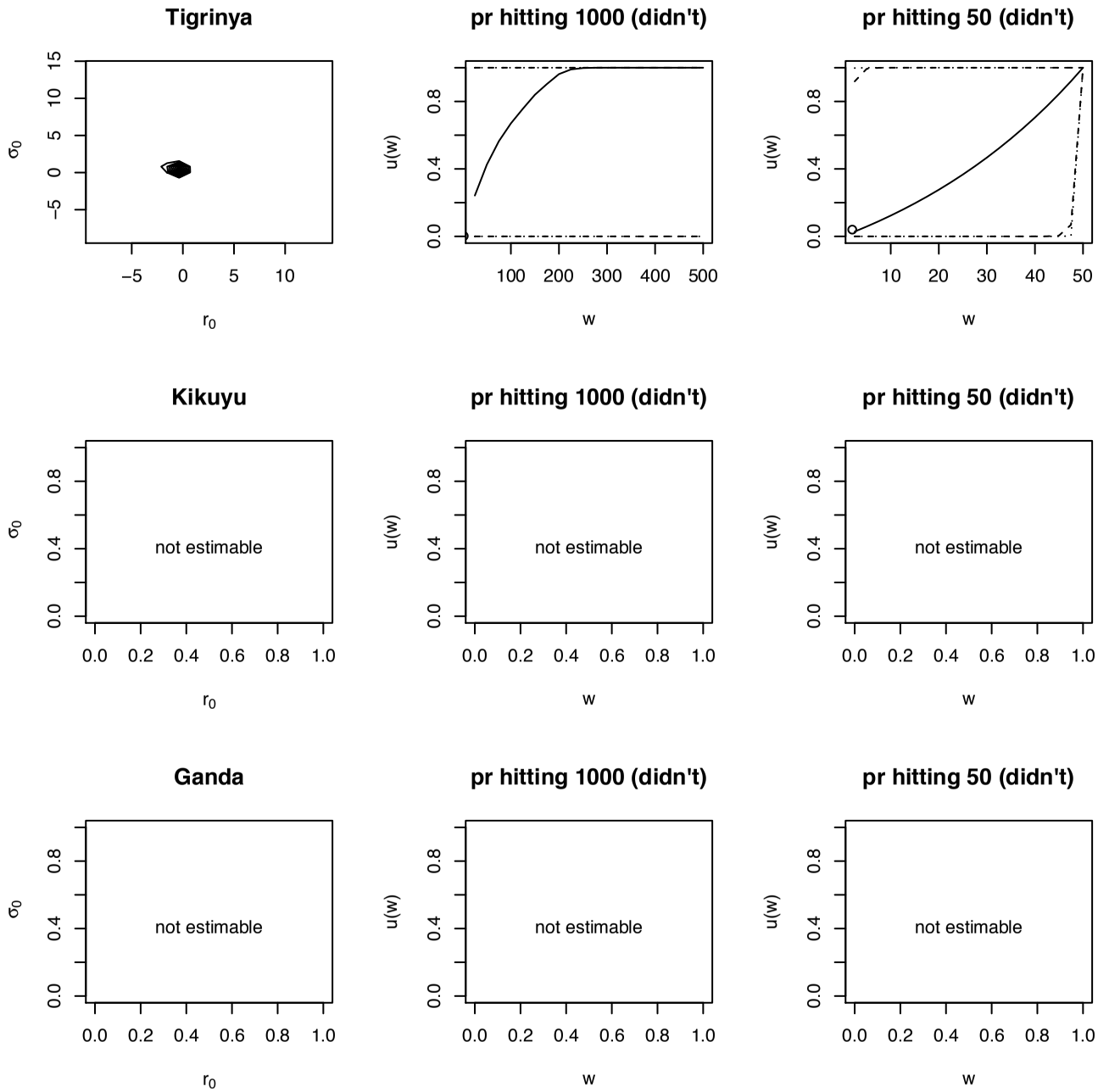

Samoan
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w
Dzongkha
Gothic

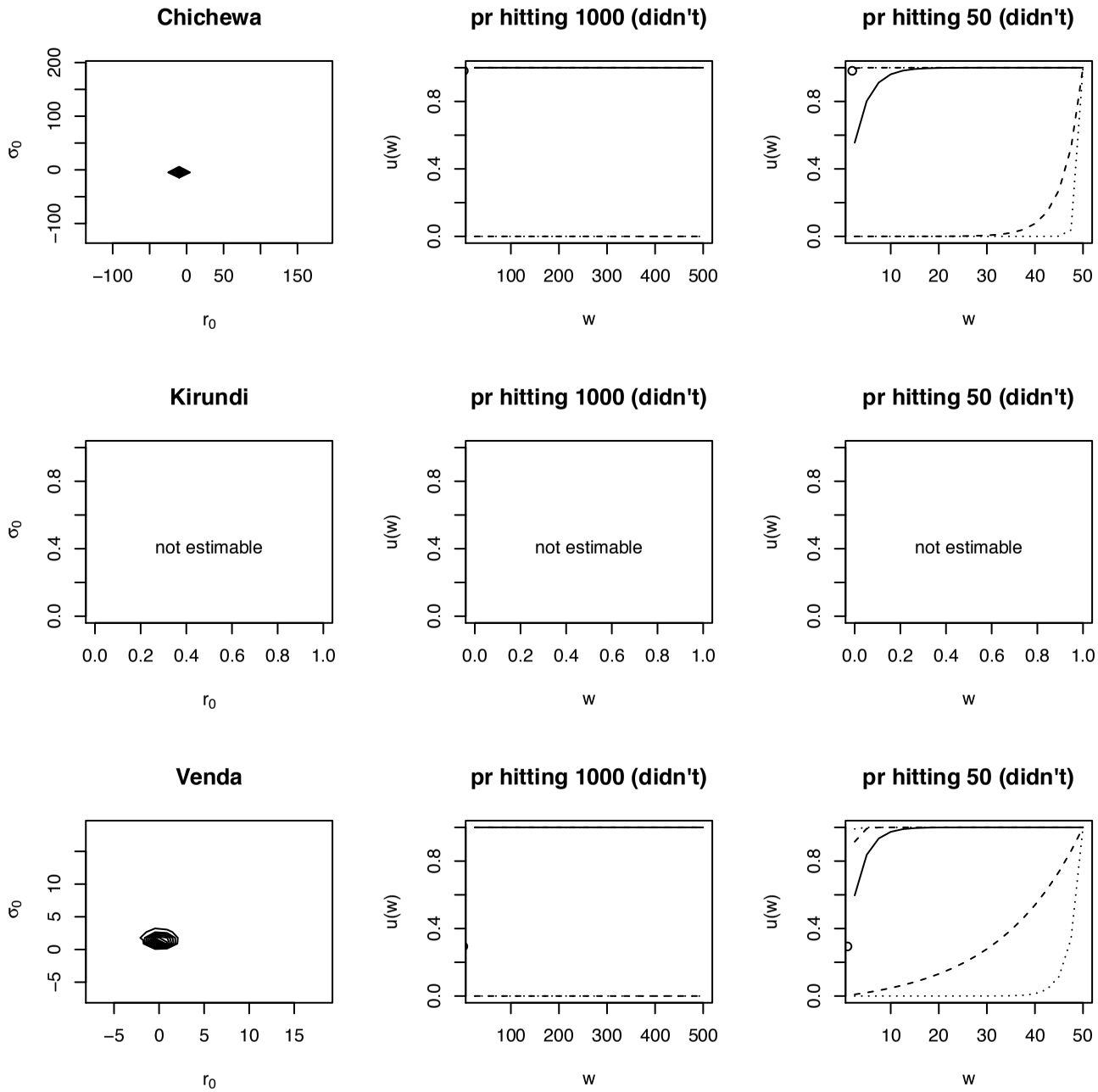

Sindhi
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Hausa
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Inupiak
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
not estimable
not estimable
not estimable
not estimable
not estimable
σ0
r0
u(w)
w

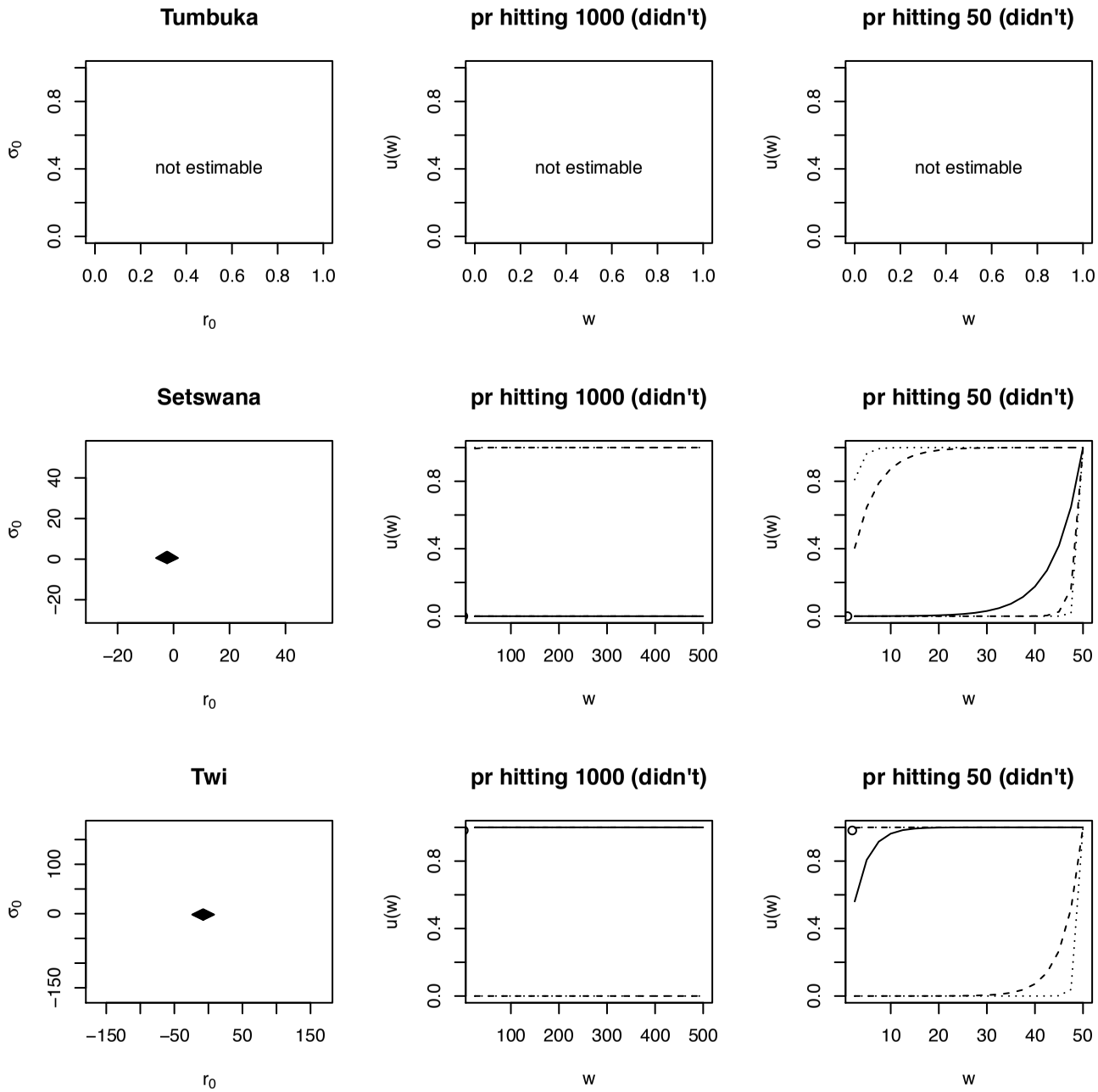

Buryat
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Fijian
not estimable
not estimable
not estimable
Tsonga
σ0
r0
u(w)
w

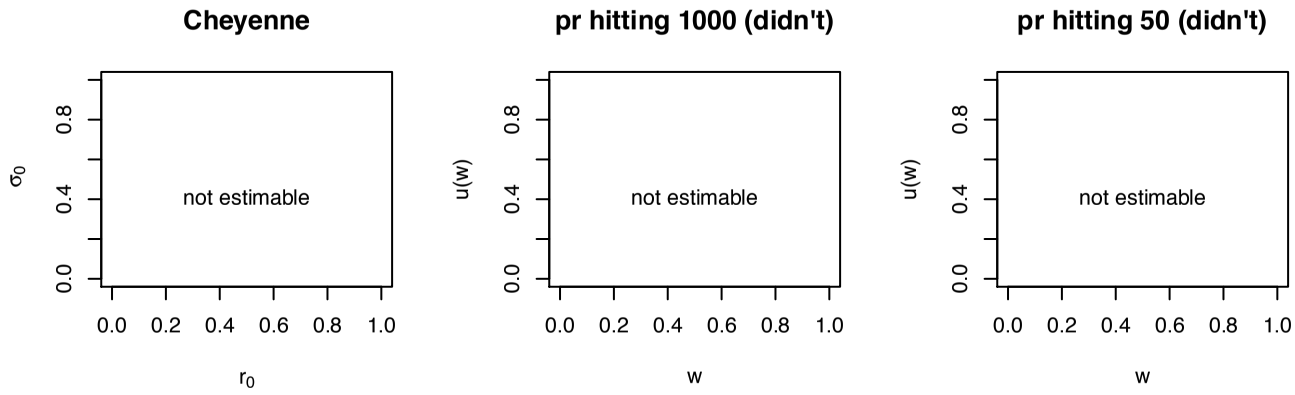

Sangro
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Akan
pr hitting 50 (didn't)
pr hitting 50 (didn't)
Oromo
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w


Fulfulde
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Xhosa
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Shona
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w
0.0
0.4
0.8
0.2
0.6
1.0


Tigrinya
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Kikuyu
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Ganda
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
not estimable
σ0
r0
u(w)
w


Chichewa
pr hitting 1000 (didn't)
pr hitting 50 (didn't)
Kirundi
not estimable
not estimable
not estimable
Venda
σ0
r0
u(w)
w


Tumbuka
not estimable
σ0
r0
pr hitting 1000 (didn't)
not estimable
u(w)
w
pr hitting 50 (didn't)
not estimable
u(w)
w
Setswana
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w
Twi
σ0
r0
pr hitting 1000 (didn't)
u(w)
w
pr hitting 50 (didn't)
u(w)
w


Cheyenne
not estimable
σ_0
r_0
0.0
0.4
0.8
0.0 0.2 0.4 0.6 0.8 1.0
pr hitting 1000 (didn't)
not estimable
u(w)
w
0.0
0.4
0.8
0.0 0.2 0.4 0.6 0.8 1.0
pr hitting 50 (didn't)
not estimable
u(w)
w
0.0
0.4
0.8
0.0 0.2 0.4 0.6 0.8 1.0