REFERENCES

1. G. Birkhoff and S. MacLane, *A Brief Survey of Modern Algebra*, 2nd edition, Macmillan Publishing Co., New York, NY, 1965.
2. J. A. Gallian, *On the Converse of Lagrange's Theorem*, this MAGAZINE, **63** (1993), 23.
3. J. A. Gallian, *Contemporary Abstract Algebra*, 3rd edition, D. C. Heath and Company, Lexington, MA, 1994.
4. I. N. Herstein, *Abstract Algebra*, 2nd edition, Macmillan Publishing Co., New York, NY, 1990.

A Persian Folk Method of Figuring Interest

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I recently learned a very quick and effective way of estimating monthly payments on a loan. My father showed me the method, having learned it himself from my grandfather, who was a merchant in nineteenth century Iran. While its origins remain a mystery, the method is still in use among merchants all around Iran, and perhaps elsewhere.

My father used the formula:

$$\text{Monthly payment} = \frac{1}{\text{Number of months}}(\text{Principal} + \text{Interest});$$

he calculated the interest as

$$\text{Interest} = \tfrac{1}{2}\,\text{Principal} \times \text{Number of years} \times \text{Annual interest rate}.$$

The *exact* formula, assuming interest accrued monthly, can be found in any basic finance textbook:

$$C = \frac{r(1+r)^N P}{(1+r)^N - 1}, \tag{1}$$

where C is the (exact) monthly payment, r is the monthly interest rate ($1/12$ the annual interest rate), N is the total number of months, and P is the principal. With this notation, the folk formula becomes

$$C_f = \frac{1}{N}\left(P + \tfrac{1}{2}PNr\right). \tag{2}$$

In many cases, C_f is a surprisingly good approximation to C. As an example, for a 4-year auto loan of \$10,000 at an annual rate of 7% compounded monthly, the exact formula gives monthly payments of \$239.46 while the folk estimate gives \$237.50.

To see why the approximation works, we regard C as a function of r, with all other quantities held fixed. (The singularity in (1) at $r=0$ can be cancelled out.) A straightforward calculation shows that the first order Maclaurin polynomial for $C(r)$ has the form

$$C(r) \approx \frac{1}{N}\left(P + \frac{1}{2}P(N+1)r\right), \tag{3}$$

which closely resembles the definition of C_f. For a fixed P, when r is sufficiently small and N sufficiently large, the difference between (2) and (3) is small.