# An Application of Dirichlet Process Isotonic Regression to the Study of Radiation Effects in Spaceborne Microelectronics

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## Motivation & Background

- This work is concerned with the vulnerability of spaceborne microelectronics to single event upset (SEU), a change of state caused by ions or electromagnetic radiation (e.g. solar wind) striking a sensitive node.
- The number of upsets, $c$, depends on the linear energy transfer (LET), $\ell$, the limiting cross-section of interaction, $\sigma$, and the fluence, or the strength of the radiation field, $f$.
- The number of upsets is assumed to be monotonically increasing with LET at a given orbit.

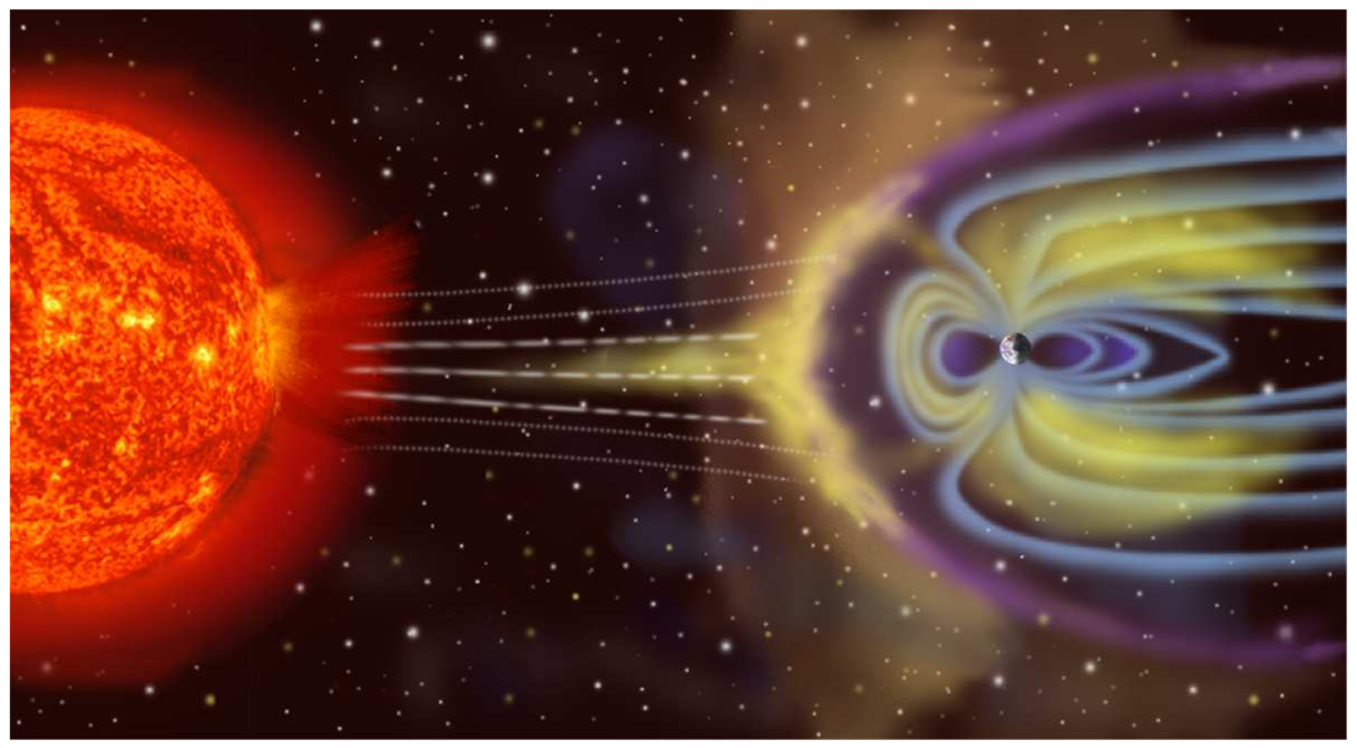

FIGURE 1: Artistic rendition of the Magnetosphere (image created by NASA).

- To measure the susceptibility of a semiconductor device to SEU, the device is exposed to high-energy particles in a particle accelerator.
- The key inferential objective in particle accelerator experiments is the cross-section vs. LET curve, $G(\ell)$.
- The standard practice in the nuclear physics literature is to assume a Weibull parametric form for $G(\ell)$ estimated with (weighted) least squares.
- The upset rate at a particular orbit is obtained by inputting the estimated value of cross-section to CREME96 (or Cosmic Ray Effects on Micro-Electronics), a widely-used code for modeling radiation environments to evaluate radiation effects in spacecraft

## Our Approach

- We work with a Poisson model for the upset counts and propose a semiparametric isotonic regression method for count responses.
- The approach is based on a Dirichlet Process (DP) prior for $G(\ell)$, which allows the data to drive the shape of the cross-section-LET relationship.
- We apply our methods to data obtained from three particle accelerator experiments corresponding to three different experimental scenarios.

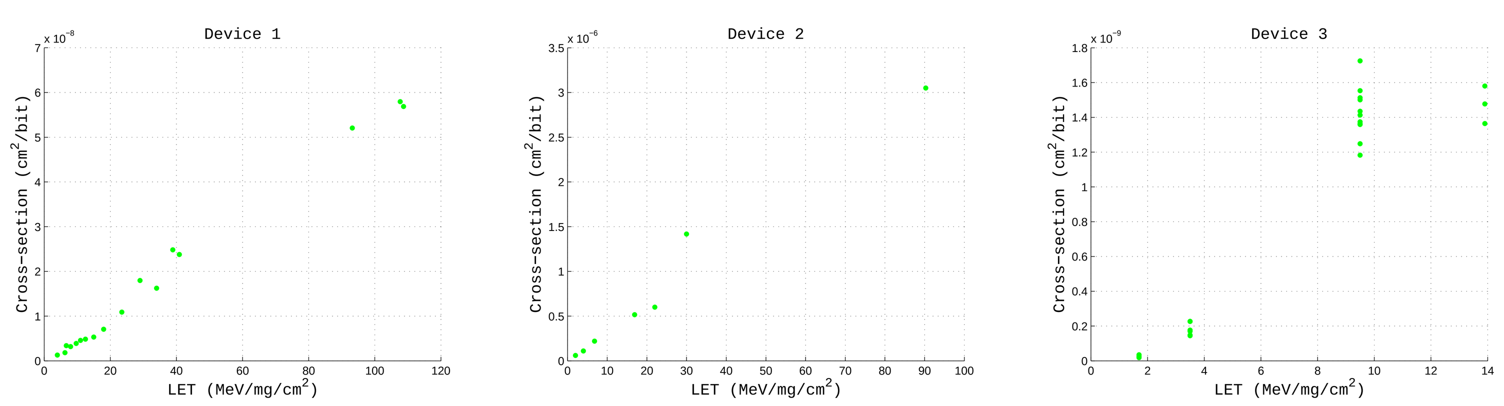


FIGURE 2: Experimental data for three devices. Observed cross-section equals to count/fluence.

## Bayesian Semi-Parametric Model

$$p(c_i|\mu_i) = \frac{\mu_i^{c_i}}{c_i!}\exp(-\mu_i), \quad \mu_i = f_i \sigma G(\ell_i)$$

$$G \sim DP\Big(\alpha, G_0(\ell_i; w, s)\Big)$$

$$G_0(\ell_i; w,s) = 1 - \exp\left(-\left(\frac{\ell_i}{w}\right)^s\right)$$

$$p(\sigma), p(\alpha), p(w), p(s) \overset{ind}{\sim} \text{Gamma}$$

- Devices 1 and 2 have distinct $\ell_i$ for each $c_i$. Let $\theta_i = G(\ell_i)$, $i = 1,\ldots,N$, $\boldsymbol{\theta} = (\theta_1,\ldots,\theta_N)$, and $\boldsymbol{\psi} = (w,s)$. Then, the joint posterior, $p(\boldsymbol{\theta},\sigma,\alpha,\boldsymbol{\psi}|\text{data})$, is proportional to

$$\sigma^{\sum_{i=1}^N c_i}\left(\prod_{i=1}^N \theta_i^{c_i}\right)\exp\left(-\sigma\sum_{i=1}^N f_i\theta_i\right) \times \frac{\Gamma(\alpha)}{\prod_{i=1}^{N+1}\Gamma(d_i(\boldsymbol{\psi}))}$$
$$\theta_1^{d_1(\boldsymbol{\psi})-1}(\theta_2-\theta_1)^{d_2(\boldsymbol{\psi})-1}\ldots(\theta_N-\theta_{N-1})^{d_N(\boldsymbol{\psi})-1}(1-\theta_N)^{d_{N+1}(\boldsymbol{\psi})-1}$$
$$\times\, p(\sigma)\times p(\alpha)\times p(s)\times p(w)$$

where $d_1(\boldsymbol{\psi}) = \alpha G_0(\ell_1;\boldsymbol{\psi})$, $d_i(\boldsymbol{\psi}) = \alpha\left(G_0(\ell_i;\boldsymbol{\psi}) - G_0(\ell_{i-1};\boldsymbol{\psi})\right)$, $i = 2,\ldots,N$, and $d_{N+1}(\boldsymbol{\psi}) = \alpha\left(1 - G_0(\ell_N;\boldsymbol{\psi})\right)$.

- Device 3 has repeated measurements for each $\ell_i$, which results in a different likelihood, but the joint posterior is obtained similarly.
- The posterior full conditional distribution of $\boldsymbol{\theta}$ is sampled through a carefully designed slice sampler. The posterior full conditional of $\sigma$ is given by a gamma distribution; $\alpha$ & the pair $(w,s)$ are sampled with two Metropolis-Hastings steps.

## Interpolation/Extrapolation

- Interpolation is based on the posterior predictive distribution $p\left(\tilde{\boldsymbol{\theta}}|\text{data}\right)$, where $\tilde{\boldsymbol{\theta}} = \left(\tilde{\theta}_1,\ldots,\tilde{\theta}_M\right)$ are the new cross-section values corresponding to unobserved LET values, $\tilde{\boldsymbol{\ell}} = \left(\tilde{\ell}_1,\ldots,\tilde{\ell}_M\right)$, such that $\ell_i \le \tilde{\ell}_1 \le \cdots \le \tilde{\ell}_M \le \ell_{i+1}$ for $i = 1,\ldots,N-1$.

$$p\left(\tilde{\boldsymbol{\theta}}|\text{data}\right) = \int p\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta},\sigma,\boldsymbol{\psi}|\text{data}\right) d\boldsymbol{\theta}\, d\sigma\, d\boldsymbol{\psi}$$

- $p\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{\theta}\right)$ is the density of:

$$\Big(\theta_i + (\theta_{i+1}-\theta_i)\omega_1\,,\ \theta_i + (\theta_{i+1}-\theta_i)\omega_2\,,\ \ldots\,,\ \theta_i + (\theta_{i+1}-\theta_i)\omega_M\Big),$$

with $(\omega_1,\omega_2,\ldots,\omega_M) \sim$ Ordered Dirichlet $\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_M,\tilde{d}_{M+1}\right)$, $\tilde{d}_1 = \alpha\left(G_0(\tilde{\ell}_1;\boldsymbol{\psi}) - G_0(\ell_i;\boldsymbol{\psi})\right)$, $\tilde{d}_m = \alpha\left(G_0(\tilde{\ell}_m;\boldsymbol{\psi}) - G_0(\tilde{\ell}_{m-1};\boldsymbol{\psi})\right)$, $m = 2,\ldots,M$, and $\tilde{d}_{M+1} = \alpha\left(G_0(\ell_{i+1};\boldsymbol{\psi}) - G_0(\tilde{\ell}_M;\boldsymbol{\psi})\right)$

- $p\left(\tilde{\boldsymbol{\theta}}|\text{data}\right)$ is then obtained using Monte Carlo integration.
- Extrapolation proceeds in an analogous fashion. Here, extrapolating beyond observed LET values is key because it could reveal important differences in the prediction of upset rates between the parametric and the semiparametric models.
- Distributions of upset rates at a particular orbit are obtained by inputting posterior samples of cross-section to CREME96.

## Results

The semiparametric model fits the data better but has wider probability intervals between observations and in the extrapolation region.

The choice of model impacts the predicted SEU. In general, the average SEU is smaller under the semiparametric model

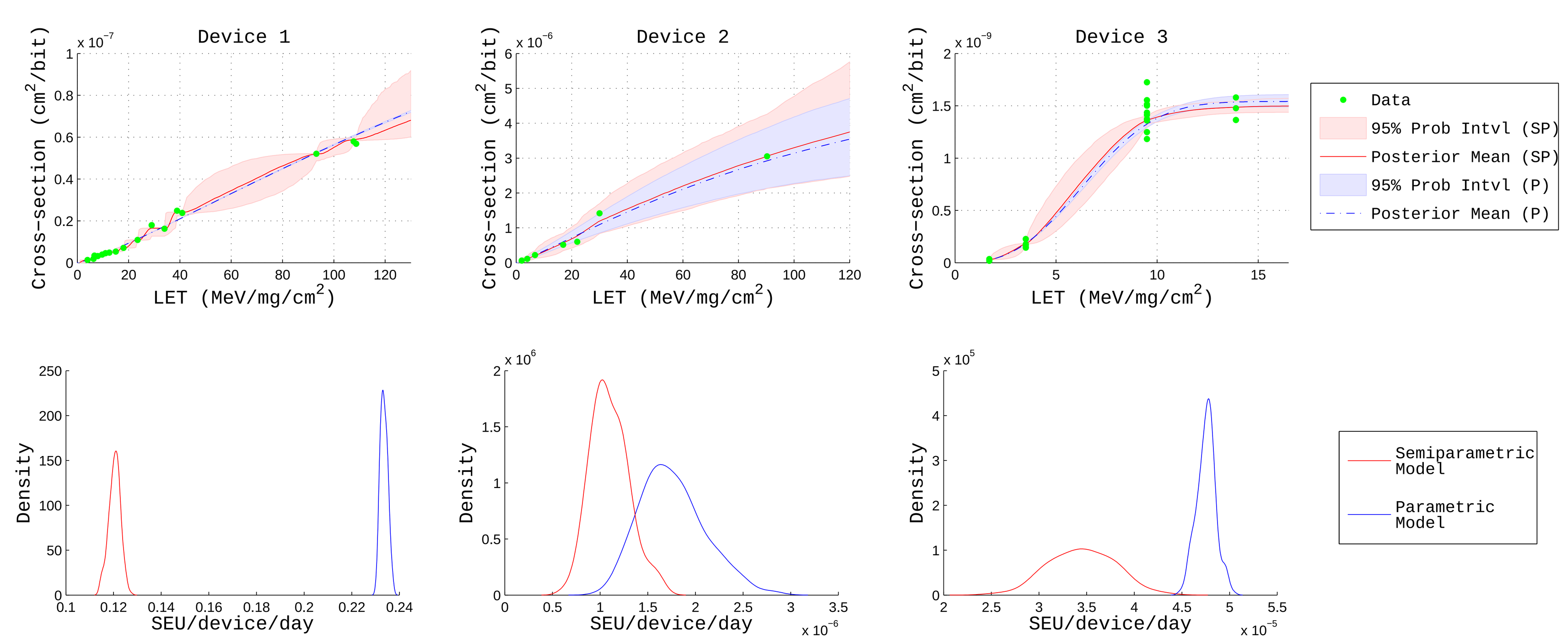


FIGURE 3: Upper panels: The posteriors of cross-section vs. LET curve under the semiparametric (SP) and parametric (P) models. Lower panels: The distribution of the SEU's under the semiparametric and parametric models

## Discussion

- We have presented a Bayesian semiparametric method for modeling the cross-section vs. LET curve, $G(\ell)$, for the prediction of distributions of SEU. Our method relaxes the parametric assumption of a Weibull fit and enables accurate quantification of uncertainty due to the functional form of $G(\ell)$.
- Fitting the model to the data from devices 1 & 2 is challenging, because only the left tail of the distribution associated with the cross-section-LET curve has been observed. This results in weak identifiability between $w$ and $\sigma$. We resolve this issue by placing a highly informative prior on $\sigma$.
- We are currently conducting a formal model comparison between the semiparametric and parametric models using a cross-validation posterior predictive criterion. In future work, we will extend the model to account for fluence uncertainty.