

4.1

- a.** $T(n) = 2T(n/2) + n^3 = \Theta(n^3)$. This is a divide-and-conquer recurrence with $a = 2$, $b = 2$, $f(n) = n^3$, and $n^{\log_b a} = n^{\log_2 2} = n$. Since $n^3 = \Omega(n^{\log_2 2 + 2})$ and $a/b^k = 2/2^3 = 1/4 < 1$, case 3 of the master theorem applies, and $T(n) = \Theta(n^3)$.
- c.** $T(n) = 16T(n/4) + n^2 = \Theta(n^2 \lg n)$. This is another divide-and-conquer recurrence with $a = 16$, $b = 4$, $f(n) = n^2$, and $n^{\log_b a} = n^{\log_4 16} = n^2$. Since $n^2 = \Theta(n^{\log_4 16})$, case 2 of the master theorem applies, and $T(n) = \Theta(n^2 \lg n)$.
- e.** $T(n) = 7T(n/2) + n^2 = O(n^{\lg 7})$. This is a divide-and-conquer recurrence with $a = 7$, $b = 2$, $f(n) = n^2$, and $n^{\log_b a} = n^{\log_2 7}$. Since $2 < \lg 7 < 3$, we have that $n^2 = O(n^{\log_2 7 - \epsilon})$ for some constant $\epsilon > 0$. Thus, case 1 of the master theorem applies, and $T(n) = \Theta(n^{\lg 7})$.

4.4

a. $T(n) = 3T(n/2) + n \lg n$

We have $f(n) = n \lg n$ and $n^{\log_b a} = n^{\lg 3} \approx n^{1.585}$. Since $n \lg n = O(n^{\lg 3 - \epsilon})$ for any $0 < \epsilon \leq 0.58$, by case 1 of the master theorem, we have $T(n) = \Theta(n^{\lg 3})$.

c. $T(n) = 4T(n/2) + n^2 \sqrt{n}$

We have $f(n) = n^2 \sqrt{n} = n^{5/2}$ and $n^{\log_b a} = n^{\log_2 4} = n^{\lg 2}$. Since $n^{5/2} = \Omega(n^{\lg 2 + 3/2})$, we look at the regularity condition in case 3 of the master theorem. We have $af(n/b) = 4(n/2)^2 \sqrt{n/2} = n^{5/2} / \sqrt{2} \leq cn^{5/2}$ for $1/\sqrt{2} \leq c < 1$. Case 3 applies, and we have $T(n) = \Theta(n^2 \sqrt{n})$.