

## Critical points and Optimization

1. Find the critical points of the functions below.

a.  $f(x, y) = 3x^2 - 12xy + 19y^2 - 2x - 4y + 5.$

b.  $g(s, t) = s^3 + 3t^2 + 12st + 2.$

c.  $h(u, v) = u^3 + v^3 - 3u^2 - 3v + 5.$

2. Use the second derivative test to classify the critical values of the functions in the previous problem.

3. ACME Widgets produces two competing products, type A widgets and type B widgets. The joint demand functions for these products are

$$Q_A = 100 - 3P_A + 2P_B \quad \text{and} \quad Q_B = 60 + 2P_A - 2P_B$$

and ACME's cost function is

$$C = 20Q_A + 30Q_B + 1200.$$

Find the prices that ACME should charge to maximize their profit, the corresponding output levels and the max profit. Justify your claim that the prices you found yield the absolute maximum profit.

4. An electronics retailer has determined that the number  $N$  of laptops she can sell per week is

$$N = \frac{9x}{4+x} + \frac{20y}{5+y}$$

where  $x$  is her weekly expenditure on radio advertising and  $y$  is her weekly expenditure on internet advertising, both measured in \$100s. Her weekly profit is \$400 per sale, less the cost of advertising.

Find the amount of money that the retailer should spend on radio and internet advertising, respectively, to maximize her weekly profit. Verify that the point you found yields a relative maximum value. What is the maximum profit?