

# Table of Selected Integrals

## Rational Forms Containing $(a + bu)$

1.  $\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
2.  $\int \frac{du}{a + bu} = \frac{1}{b} \ln |a + bu| + C$
3.  $\int \frac{u du}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln |a + bu| + C$
4.  $\int \frac{u^2 du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln |a + bu| + C$
5.  $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$
6.  $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
7.  $\int \frac{u du}{(a + bu)^2} = \frac{1}{b^2} \left( \ln |a + bu| + \frac{a}{a + bu} \right) + C$
8.  $\int \frac{u^2 du}{(a + bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a + bu)} - \frac{2a}{b^3} \ln |a + bu| + C$
9.  $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} + \frac{1}{a^2} \ln \left| \frac{u}{a + bu} \right| + C$
10.  $\int \frac{du}{u^2(a + bu)^2} = -\frac{a + 2bu}{a^2 u(a + bu)} + \frac{2b}{a^3} \ln \left| \frac{a + bu}{u} \right| + C$
11.  $\int \frac{du}{(a + bu)(c + ku)} = \frac{1}{bc - ak} \ln \left| \frac{a + bu}{c + ku} \right| + C$
12.  $\int \frac{u du}{(a + bu)(c + ku)} = \frac{1}{bc - ak} \left[ \frac{c}{k} \ln |c + ku| - \frac{a}{b} \ln |a + bu| \right] + C$

## Forms Containing $\sqrt{a + bu}$

13.  $\int u\sqrt{a + bu} du = \frac{2(3bu - 2a)(a + bu)^{3/2}}{15b^2} + C$
14.  $\int u^2\sqrt{a + bu} du = \frac{2(8a^2 - 12abu + 15b^2u^2)(a + bu)^{3/2}}{105b^3} + C$
15.  $\int \frac{u du}{\sqrt{a + bu}} = \frac{2(bu - 2a)\sqrt{a + bu}}{3b^2} + C$
16.  $\int \frac{u^2 du}{\sqrt{a + bu}} = \frac{2(3b^2u^2 - 4abu + 8a^2)\sqrt{a + bu}}{15b^3} + C$

$$17. \int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C, \quad a > 0$$

$$18. \int \frac{\sqrt{a+bu} du}{u} = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

### Forms Containing $\sqrt{a^2 - u^2}$

$$19. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$$

$$20. \int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$21. \int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2u} + C$$

$$22. \int \frac{\sqrt{a^2 - u^2} du}{u} = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C, \quad a > 0$$

### Forms Containing $\sqrt{u^2 \pm a^2}$

$$23. \int \sqrt{u^2 \pm a^2} du = \frac{1}{2} \left( u\sqrt{u^2 \pm a^2} \pm a^2 \ln |u + \sqrt{u^2 \pm a^2}| \right) + C$$

$$24. \int u^2 \sqrt{u^2 \pm a^2} du = \frac{u}{8} (2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$25. \int \frac{\sqrt{u^2 + a^2} du}{u} = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$26. \int \frac{\sqrt{u^2 \pm a^2} du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$27. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$28. \int \frac{du}{u\sqrt{u^2 \pm a^2}} = \frac{1}{a} \ln \left| \frac{\sqrt{u^2 \pm a^2} - a}{u} \right| + C$$

$$29. \int \frac{u^2 du}{\sqrt{u^2 \pm a^2}} = \frac{1}{2} \left( u\sqrt{u^2 \pm a^2} \mp a^2 \ln |u + \sqrt{u^2 \pm a^2}| \right) + C$$

$$30. \int \frac{du}{u^2\sqrt{u^2 \pm a^2}} = -\frac{\pm\sqrt{u^2 \pm a^2}}{a^2u} + C$$

$$31. \int (u^2 \pm a^2)^{3/2} du = \frac{u}{8} (2u^2 \pm 5a^2) \sqrt{u^2 \pm a^2} + \frac{3a^4}{8} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$32. \int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$$

$$33. \int \frac{u^2 du}{(u^2 \pm a^2)^{3/2}} = \frac{-u}{\sqrt{u^2 \pm a^2}} + \ln |u + \sqrt{u^2 \pm a^2}| + C$$

### Rational Forms Containing $a^2 - u^2$ and $u^2 - a^2$

$$34. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

$$35. \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

## Exponential and Logarithmic Forms

$$36. \int e^u du = e^u + C$$

$$37. \int a^u du = \frac{a^u}{\ln a} + C, \quad a > 0, a \neq 1$$

$$38. \int ue^{au} du = \frac{e^{au}}{a^2}(au - 1) + C$$

$$39. \int u^n e^{au} du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$40. \int \frac{e^{au} du}{u^n} = -\frac{e^{au}}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{e^{au} du}{u^{n-1}}, \quad n \neq 1$$

$$41. \int \ln u du = u \ln u - u + C$$

$$42. \int u^n \ln u du = \frac{u^{n+1} \ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \quad n \neq -1$$

$$43. \int u^n \ln^m u du = \frac{u^{n+1}}{n+1} \ln^m u - \frac{m}{n+1} \int u^n \ln^{m-1} u du, \quad m, n \neq -1$$

$$44. \int \frac{du}{u \ln u} = \ln \left| \ln u \right| + C$$

$$45. \int \frac{du}{a + be^{cu}} = \frac{1}{ac} \left( cu - \ln \left| a + be^{cu} \right| \right) + C$$

## Miscellaneous Forms

$$46. \int \sqrt{\frac{a+u}{b+u}} du = \sqrt{(a+u)(b+u)} + (a-b) \ln(\sqrt{a+u} + \sqrt{b+u}) + C$$

$$47. \int \frac{du}{\sqrt{(a+u)(b+u)}} = \ln \left| \frac{a+b}{2} + u + \sqrt{(a+u)(b+u)} \right| + C$$

$$48. \int \sqrt{a+bu+cu^2} du = \frac{2cu+b}{4c} \sqrt{a+bu+cu^2} - \frac{b^2-4ac}{8c^{3/2}} \ln \left| 2cu+b+2\sqrt{c}\sqrt{a+bu+cu^2} \right| + C, \quad c > 0$$