$$= \frac{\quad}{1-w} + \ln|w-1| + C$$

PROBLEMS 14.4

In Problems 1–80, find the indefinite integrals.

1. $\int (x+5)^7\, dx$

2. $\int 15(x+2)^4\, dx$

3. $\int 2x(x^2+3)^5\, dx$

4. $\int (4x+3)(2x^2+3x+1)\, dx$

5. $\int (3y^2+6y)(y^3+3y^2+1)^{2/3}\, dy$

6. $\int (15t^2-6t+1)(5t^3-3t^2+t)^{17}\, dt$

7. $\int \frac{5}{(3x-1)^3}\, dx$

8. $\int \frac{4x}{(2x^2-7)^{10}}\, dx$

9. $\int \sqrt{7x+3}\, dx$

10. $\int \frac{1}{\sqrt{x-5}}\, dx$

11. $\int (7x-6)^4\, dx$

12. $\int x^2(3x^3+7)^3\, dx$

13. $\int u(5u^2-9)^{14}\, du$

14. $\int x\sqrt{3+5x^2}\, dx$

15. $\int 4x^4(27+x^5)^{1/3}\, dx$

16. $\int (4-5x)^9\, dx$

17. $\int 3e^{3x}\, dx$

18. $\int 5e^{3t+7}\, dt$

19. $\int (3t+1)e^{3t^2+2t+1}\, dt$

20. $\int -3w^2e^{-w^3}\, dw$

21. $\int xe^{7x^2}\, dx$

22. $\int x^3e^{4x^4}\, dx$

23. $\int 4e^{-3x}\, dx$

24. $\int 24x^5e^{-2x^6+7}\, dx$

25. $\int \frac{1}{x+5}\, dx$

26. $\int \frac{12x^2+4x+2}{x+x^2+2x^3}\, dx$

27. $\int \frac{3x^2+4x^3}{x^3+x^4}\, dx$

28. $\int \frac{6x^2-6x}{1-3x^2+2x^3}\, dx$

29. $\int \frac{8z}{(z^2-5)^7}\, dz$

30. $\int \frac{3}{(5v-1)^4}\, dv$

31. $\int \frac{4}{x}\,dx$

32. $\int \frac{3}{1+2y}\,dy$

33. $\int \frac{s^2}{s^3+5}\,ds$

34. $\int \frac{32x^3}{4x^4+9}\,dx$

35. $\int \frac{5}{4-2x}\,dx$

36. $\int \frac{7t}{5t^2-6}\,dt$

37. $\int \sqrt{5x}\,dx$

38. $\int \frac{1}{(3x)^6}\,dx$

39. $\int \frac{x}{\sqrt{ax^2+b}}\,dx$

40. $\int \frac{9}{1-3x}\,dx$

41. $\int 2y^3e^{y^4+1}\,dy$

42. $\int 2\sqrt{2x-1}\,dx$

43. $\int v^2e^{-2v^3+1}\,dv$

44. $\int \frac{x^2+x+1}{\sqrt[3]{x^3+\frac{3}{2}x^2+3x}}\,dx$

45. $\int (e^{-5x}+2e^x)\,dx$

46. $\int 4\sqrt[3]{y+1}\,dy$

47. $\int (8x+10)(7-2x^2-5x)^3\,dx$

48. $\int 2ye^{3y^2}\,dy$

49. $\int \frac{6x^2+8}{x^3+4x}\,dx$

50. $\int (e^x+2e^{-3x}-e^{5x})\,dx$

51. $\int \frac{16s-4}{3-2s+4s^2}\,ds$

52. $\int (6t^2+4t)(t^3+t^2+1)^6\,dt$

53. $\int x(2x^2+1)^{-1}\,dx$

54. $\int (45w^4+18w^2+12)(3w^5+2w^3+4)^{-4}\,dw$

55. $\int -(x^2-2x^5)(x^3-x^6)^{-10}\,dx$

56. $\int \frac{3}{5}(v-2)e^{2-4v+v^2}\,dv$

57. $\int (2x^3+x)(x^4+x^2)\,dx$

58. $\int (e^{3.1})^2\,dx$

59. $\int \frac{9+18x}{(5-x-x^2)^4}\,dx$

60. $\int (e^x-e^{-x})^2\,dx$

61. $\int x(2x+1)e^{4x^3+3x^2-4}\,dx$

62. $\int (u^3-ue^{6-3u^2})\,du$

63. $\int x\sqrt{(8-5x^2)^3}\,dx$

64. $\int e^{ax}\,dx$

65. $\int \left(\sqrt{2x}-\frac{1}{\sqrt{2x}}\right)dx$

66. $\int 3\frac{x^4}{e^{x^5}}\,dx$

67. $\int (x^2+1)^2\,dx$

68. $\int \left[x(x^2-16)^2-\frac{1}{2x+5}\right]dx$

69. $\int \left(\frac{x}{x^2+1}+\frac{x}{(x^2+1)^2}\right)dx$

70. $\int \left[\frac{3}{x-1}+\frac{1}{(x-1)^2}\right]dx$

71. $\int \left[\frac{2}{4x+1}-(4x^2-8x^5)(x^3-x^6)^{-8}\right]dx$

72. $\int (r^3+5)^2\,dr$

73. $\int \left[\sqrt{3x+1}-\frac{x}{x^2+3}\right]dx$

74. $\int \left(\frac{x}{7x^2+2}-\frac{x^2}{(x^3+2)^4}\right)dx$

75. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

76. $\int (e^5-3^e)\,dx$

77. $\int \frac{1+e^{2x}}{4e^x}\,dx$

78. $\int \frac{2}{t^2}\sqrt{\frac{1}{t}+9}\,dt$

79. $\int \frac{4x+3}{2x^2+3x}\ln(2x^2+3x)\,dx$

80. $\int \sqrt[3]{x}e^{\sqrt[3]{8x^4}}\,dx$

In Problems 81–84, find y subject to the given conditions.

81. $y' = (3-2x)^2;\quad y(0) = 1$

82. $y' = \frac{x}{x^2+6};\quad y(1) = 0$

83. $y'' = \frac{1}{x^2};\quad y'(-2) = 3,\ y(1) = 2$

84. $y'' = (x+1)^{1/2};\quad y'(8) = 19,\ y(24) = \frac{2572}{3}$

85. **Real Estate** The rate of change of the value of a house that cost \$350,000 to build can be modeled by $\frac{dV}{dt} = 8e^{0.05t}$, where t is the time in years since the house was built and V is the value (in thousands of dollars) of the house. Find $V(t)$.

86. **Life Span** If the rate of change of the expected life span l at birth of people born in the United States can be modeled by $\frac{dl}{dt} = \frac{12}{2t+50}$, where t is the number of years after 1940 and the expected life span was 63 years in 1940, find the expected life span for people born in 1998.

87. **Oxygen in Capillary** In a discussion of the diffusion of oxygen from capillaries,[5] concentric cylinders of radius r are used as a model for a capillary. The concentration C of oxygen in the capillary is given by

$$C = \int \left(\frac{Rr}{2K}+\frac{B_1}{r}\right)dr$$

where R is the constant rate at which oxygen diffuses from the capillary, and K and B_1 are constants. Find C. (Write the constant of integration as B_2.)

88. Find $f(2)$ if $f\left(\frac{1}{3}\right) = 2$ and $f'(x) = e^{3x+2} - 3x$.

Objective

To discuss techniques of handling more challenging integration problems, namely, by algebraic manipulation and by fitting the integrand to a familiar form. To integrate an exponential function with a base different from e and to find the consumption function, given the marginal propensity to consume.

14.5 Techniques of Integration

We turn now to some more difficult integration problems.

When integrating fractions, sometimes a preliminary division is needed to get familiar integration forms, as the next example shows.

[5] W. Simon, *Mathematical Techniques for Physiology and Medicine* (New York: Academic Press, Inc., 1972).

If we let $u = 3I$, then $du = 3\,dI = d(3I)$, and

$$C = \frac{3}{4}I - \left(\frac{1}{2}\right)\frac{1}{3}\int (3I)^{-1/2}\,d(3I)$$
$$= \frac{3}{4}I - \frac{1}{6}\frac{(3I)^{1/2}}{\frac{1}{2}} + K$$
$$C = \frac{3}{4}I - \frac{\sqrt{3I}}{3} + K$$

This is an example of an initial-value problem.

When $I = 12$, $C = 10$, so

$$10 = \frac{3}{4}(12) - \frac{\sqrt{3(12)}}{3} + K$$
$$10 = 9 - 2 + K$$

Thus, $K = 3$, and the consumption function is

$$C = \frac{3}{4}I - \frac{\sqrt{3I}}{3} + 3$$

Now Work Problem 61 ◁

PROBLEMS 14.5

In Problems 1–56, determine the indefinite integrals.

1. $\int \frac{2x^6 + 8x^4 - 4x}{2x^2}\,dx$
2. $\int \frac{9x^2 + 5}{3x}\,dx$
3. $\int (3x^2 + 2)\sqrt{2x^3 + 4x + 1}\,dx$
4. $\int \frac{x}{\sqrt[4]{x^2 + 1}}\,dx$
5. $\int \frac{3}{\sqrt{4 - 5x}}\,dx$
6. $\int \frac{2xe^{x^2}\,dx}{e^{x^2} - 2}$
7. $\int 4^{7x}\,dx$
8. $\int 5^t\,dt$
9. $\int 2x(7 - e^{x^2/4})\,dx$
10. $\int \frac{e^x + 1}{e^x}\,dx$
11. $\int \frac{6x^2 - 11x + 5}{3x - 1}\,dx$
12. $\int \frac{(3x + 2)(x - 4)}{x - 3}\,dx$
13. $\int \frac{5e^{2x}}{7e^{2x} + 4}\,dx$
14. $\int 6(e^{4-3x})^2\,dx$
15. $\int \frac{5e^{13/x}}{x^2}\,dx$
16. $\int \frac{2x^4 - 6x^3 + x - 2}{x - 2}\,dx$
17. $\int \frac{5x^3}{x^2 + 9}\,dx$
18. $\int \frac{5 - 4x^2}{3 + 2x}\,dx$
19. $\int \frac{(\sqrt{x} + 2)^2}{3\sqrt{x}}\,dx$
20. $\int \frac{5e^s}{1 + 3e^s}\,ds$
21. $\int \frac{5(x^{1/3} + 2)^4}{\sqrt[3]{x^2}}\,dx$
22. $\int \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}}\,dx$
23. $\int \frac{\ln x}{x}\,dx$
24. $\int \sqrt{t}(3 - t\sqrt{t})^{0.6}\,dt$
25. $\int \frac{r\sqrt{\ln(r^2 + 1)}}{r^2 + 1}\,dr$
26. $\int \frac{9x^5 - 6x^4 - ex^3}{7x^2}\,dx$
27. $\int \frac{3^{\ln x}}{x}\,dx$
28. $\int \frac{4}{x\ln(2x^2)}\,dx$
29. $\int x^2\sqrt{e^{x^3+1}}\,dx$
30. $\int \frac{ax + b}{cx + d}\,dx \quad c \neq 0$
31. $\int \frac{8}{(x + 3)\ln(x + 3)}\,dx$
32. $\int (e^{e^2} + x^e - 2x)\,dx$
33. $\int \frac{x^3 + x^2 - x - 3}{x^2 - 3}\,dx$
34. $\int \frac{4x\ln\sqrt{1 + x^2}}{1 + x^2}\,dx$
35. $\int \frac{12x^3\sqrt{\ln(x^4 + 1)^3}}{x^4 + 1}\,dx$
36. $\int 3(x^2 + 2)^{-1/2}xe^{\sqrt{x^2+2}}\,dx$
37. $\int \left(\frac{x^3 - 1}{\sqrt{x^4 - 4x}} - \ln 7\right)dx$
38. $\int \frac{x - x^{-2}}{x^2 + 2x^{-1}}\,dx$
39. $\int \frac{2x^4 - 8x^3 - 6x^2 + 4}{x^3}\,dx$
40. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$
41. $\int \frac{x}{x + 1}\,dx$
42. $\int \frac{2x}{(x^2 + 1)\ln(x^2 + 1)}\,dx$
43. $\int \frac{xe^{x^2}}{\sqrt{e^{x^2} + 2}}\,dx$
44. $\int \frac{5}{(3x + 1)[1 + \ln(3x + 1)]^2}\,dx$
45. $\int \frac{(e^{-x} + 5)^3}{e^x}\,dx$
46. $\int \left[\frac{1}{8x + 1} - \frac{1}{e^x(8 + e^{-x})^2}\right]dx$
47. $\int (x^3 + ex)\sqrt{x^2 + e}\,dx$
48. $\int 3^{x\ln x}(1 + \ln x)\,dx$ [*Hint:* $\frac{d}{dx}(x\ln x) = 1 + \ln x$]
49. $\int \sqrt{x}\sqrt{(8x)^{3/2} + 3}\,dx$
50. $\int \frac{7}{x(\ln x)^\pi}\,dx$
51. $\int \frac{\sqrt{s}}{e^{\sqrt{s^3}}}\,ds$
52. $\int \frac{\ln^3 x}{3x}\,dx$
53. $\int e^{\ln(x^2+1)}\,dx$
54. $\int dx$

55. $\displaystyle\int \frac{\ln\left(\frac{e^x}{x}\right)}{x}\,dx$

56. $\displaystyle\int e^{f(x)+\ln(f'(x))}\,dx$ assuming $f' > 0$

In Problems 57 and 58, dr/dq is a marginal-revenue function. Find the demand function.

57. $\dfrac{dr}{dq} = \dfrac{200}{(q+2)^2}$

58. $\dfrac{dr}{dq} = \dfrac{900}{(2q+3)^3}$

In Problems 59 and 60, dc/dq is a marginal-cost function. Find the total-cost function if fixed costs in each case are 2000.

59. $\dfrac{dc}{dq} = \dfrac{20}{q+5}$

60. $\dfrac{dc}{dq} = 4e^{0.005q}$

In Problems 61–63, dC/dI represents the marginal propensity to consume. Find the consumption function subject to the given condition.

61. $\dfrac{dC}{dI} = \dfrac{1}{\sqrt{I}}$; $C(9) = 8$

62. $\dfrac{dC}{dI} = \dfrac{1}{2} - \dfrac{1}{2\sqrt{2I}}$; $C(2) = \dfrac{3}{4}$

63. $\dfrac{dC}{dI} = \dfrac{3}{4} - \dfrac{1}{6\sqrt{I}}$; $C(25) = 23$

64. Cost Function The marginal-cost function for a manufacturer's product is given by

$$\frac{dc}{dq} = 10 - \frac{100}{q+10}$$

where c is the total cost in dollars when q units are produced. When 100 units are produced, the average cost is \$50 per unit. To the nearest dollar, determine the manufacturer's fixed cost.

65. Cost Function Suppose the marginal-cost function for a manufacturer's product is given by

$$\frac{dc}{dq} = \frac{100q^2 - 3998q + 60}{q^2 - 40q + 1}$$

where c is the total cost in dollars when q units are produced.

(a) Determine the marginal cost when 40 units are produced.
(b) If fixed costs are \$10,000, find the total cost of producing 40 units.
(c) Use the results of parts (a) and (b) and differentials to approximate the total cost of producing 42 units.

66. Cost Function The marginal-cost function for a manufacturer's product is given by

$$\frac{dc}{dq} = \frac{9}{10}\sqrt{q}\sqrt{0.04q^{3/4} + 4}$$

where c is the total cost in dollars when q units are produced. Fixed costs are \$360.

(a) Determine the marginal cost when 25 units are produced.
(b) Find the total cost of producing 25 units.
(c) Use the results of parts (a) and (b) and differentials to approximate the total cost of producing 23 units.

67. Value of Land It is estimated that t years from now the value V (in dollars) of an acre of land near the ghost town of Cherokee, California, will be increasing at the rate of $\dfrac{8t^3}{\sqrt{0.2t^4 + 8000}}$ dollars per year. If the land is currently worth \$500 per acre, how much will it be worth in 10 years? Express your answer to the nearest dollar.

68. Revenue Function The marginal-revenue function for a manufacturer's product is of the form

$$\frac{dr}{dq} = \frac{a}{e^q + b}$$

for constants a and b, where r is the total revenue received (in dollars) when q units are produced and sold. Find the demand function, and express it in the form $p = f(q)$. (*Hint:* Rewrite dr/dq by multiplying both numerator and denominator by e^{-q}.)

69. Savings A certain country's marginal propensity to save is given by

$$\frac{dS}{dI} = \frac{5}{(I+2)^2}$$

where S and I represent total national savings and income, respectively, and are measured in billions of dollars. If total national consumption is \$7.5 billion when total national income is \$8 billion, for what value(s) of I is total national savings equal to zero?

70. Consumption Function A certain country's marginal propensity to save is given by

$$\frac{dS}{dI} = \frac{2}{5} - \frac{1.6}{\sqrt[3]{2I^2}}$$

where S and I represent total national savings and income, respectively, and are measured in billions of dollars.

(a) Determine the marginal propensity to consume when total national income is \$16 billion.
(b) Determine the consumption function, given that savings are \$10 billion when total national income is \$54 billion.
(c) Use the result in part (b) to show that consumption is $\$\frac{82}{5} = 16.4$ billion when total national income is \$16 billion (a deficit situation).
(d) Use differentials and the results in parts (a) and (c) to approximate consumption when total national income is \$18 billion.

Objective

To motivate, by means of the concept of area, the definite integral as a limit of a special sum; to evaluate simple definite integrals by using a limiting process.

14.6 The Definite Integral

Figure 14.2 shows the region R bounded by the lines $y = f(x) = 2x$, $y = 0$ (the x-axis), and $x = 1$. The region is simply a right triangle. If b and h are the lengths of the base and the height, respectively, then, from geometry, the area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(1)(2) = 1$ square unit. (Henceforth, we will treat areas as pure numbers and write *square unit* only if it seems necessary for emphasis.) We will now find this area by another method, which, as we will see later, applies to more complex regions. This method involves the summation of areas of rectangles.

of subintervals. Then the program proceeds to display the

PROBLEMS 14.6

In Problems 1–4, sketch the region in the first quadrant that is bounded by the given curves. Approximate the area of the region by the indicated sum. Use the right-hand endpoint of each subinterval.

1. $f(x) = x + 1, y = 0, x = 0, x = 1; \quad S_4$

2. $f(x) = 3x, y = 0, x = 1; \quad S_5$

3. $f(x) = x^2, y = 0, x = 1; \quad S_4$

4. $f(x) = x^2 + 1, y = 0, x = 0, x = 1; \quad S_2$

In Problems 5 and 6, by dividing the indicated interval into n subintervals of equal length, find S_n for the given function. Use the right-hand endpoint of each subinterval. Do not find $\lim_{n\to\infty} S_n$.

5. $f(x) = 4x; \quad [0, 1]$

6. $f(x) = 2x + 1; \quad [0, 2]$

In Problems 7 and 8, (a) simplify S_n and (b) find $\lim_{n\to\infty} S_n$.

7. $S_n = \frac{1}{n}\left[\left(\frac{1}{n} + 1\right) + \left(\frac{2}{n} + 1\right) + \cdots + \left(\frac{n}{n} + 1\right)\right]$

8. $S_n = \frac{2}{n}\left[\left(\frac{2}{n}\right)^2 + \left(2 \cdot \frac{2}{n}\right)^2 + \cdots + \left(n \cdot \frac{2}{n}\right)^2\right]$

In Problems 9–14, sketch the region in the first quadrant that is bounded by the given curves. Determine the exact area of the region by considering the limit of S_n as $n \to \infty$. Use the right-hand endpoint of each subinterval.

9. Region as described in Problem 1

10. Region as described in Problem 2

11. Region as described in Problem 3

12. $y = x^2, y = 0, x = 1, x = 2$

13. $f(x) = 3x^2, y = 0, x = 1$

14. $f(x) = 9 - x^2, y = 0, x = 0$

In Problems 15–20, evaluate the given definite integral by taking the limit of S_n. Use the right-hand endpoint of each subinterval. Sketch the graph, over the given interval, of the function to be integrated.

15. $\int_1^3 5x\,dx$

16. $\int_0^a b\,dx$

17. $\int_0^3 -4x\,dx$

18. $\int_1^4 (2x + 1)\,dx$

19. $\int_0^1 (x^2 + x)\,dx$

20. $\int_1^2 (x + 2)\,dx$

21. Find $\frac{d}{dx}\left(\int_0^1 \sqrt{1 - x^2}\,dx\right)$ without the use of limits.

22. Find $\int_0^3 f(x)\,dx$ without the use of limits, where

$$f(x) = \begin{cases} 2 & \text{if } 0 \le x < 1 \\ 4 - 2x & \text{if } 1 \le x < 2 \\ 5x - 10 & \text{if } 2 \le x \le 3 \end{cases}$$