Hence,

$$\frac{dp}{dq} = \frac{1}{\frac{dq}{dp}} = -\frac{\sqrt{2500 - p^2}}{p}$$

Now Work Problem 27 ◁

PROBLEMS 14.1

In Problems 1–10, find the differential of the function in terms of x and dx.

1. $y = ax + b$
2. $y = 2$
3. $f(x) = \sqrt{x^4 - 9}$
4. $f(x) = (4x^2 - 5x + 2)^3$
5. $u = \dfrac{1}{x^2}$
6. $u = \sqrt{x}$
7. $p = \ln(x^2 + 7)$
8. $p = e^{x^3+2x-5}$
9. $y = (9x + 3)e^{2x^2+3}$
10. $y = \ln\sqrt{x^2 + 12}$

In Problems 11–16, find Δy and dy for the given values of x and dx.

11. $y = ax + b$; for any x and any dx
12. $y = 5x^2; x = -1, dx = -0.02$
13. $y = 2x^2 + 5x - 7; x = -2, dx = 0.1$
14. $y = (3x + 2)^2; x = -1, dx = -0.03$
15. $y = \sqrt{32 - x^2}; x = 4, dx = -0.05$ Round your answer to three decimal places.
16. $y = \ln x; x = 1, dx = 0.01$
17. Let $f(x) = \dfrac{x + 5}{x + 1}$.
    (a) Evaluate $f'(1)$.
    (b) Use differentials to estimate the value of $f(1.1)$.
18. Let $f(x) = x^{3x}$.
    (a) Evaluate $f'(1)$.
    (b) Use differentials to estimate the value of $f(0.98)$.

$= \int (x - x^{-2})\,dx$

and so on.

numerator by the

$$\int \frac{x^3 - 1}{x^2}\,dx = \int \left(\frac{x^3}{x^2} - \frac{1}{x^2}\right) dx = \int (x - x^{-2})\,dx$$

$$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + C = \frac{x^2}{2} + \frac{1}{x} + C$$

Now Work Problem 49 ◁

PROBLEMS 14.2

In Problems 1–52, find the indefinite integrals.

1. $\int 7\,dx$
2. $\int \frac{1}{x}\,dx$
3. $\int x^8\,dx$
4. $\int 5x^{24}\,dx$
5. $\int 5x^{-7}\,dx$
6. $\int \frac{z^{-3}}{3}\,dz$
7. $\int \frac{5}{x^7}\,dx$
8. $\int \frac{7}{x^4\,dx}$
9. $\int \frac{1}{t^{7/4}}\,dt$
10. $\int \frac{7}{2x^{9/4}}\,dx$
11. $\int (4 + t)\,dt$
12. $\int (7r^5 + 4r^2 + 1)\,dr$
13. $\int (y^5 - 5y)\,dy$
14. $\int (5 - 2w - 6w^2)\,dw$
15. $\int (3t^2 - 4t + 5)\,dt$
16. $\int (1 + t^2 + t^4 + t^6)\,dt$

17. $\int (\sqrt{2} + e)\,dx$

18. $\int (5 - 2^{-1})\,dx$

19. $\int \left(\frac{x}{7} - \frac{3}{4}x^4\right)\,dx$

20. $\int \left(\frac{2x^2}{7} - \frac{8}{3}x^4\right)\,dx$

21. $\int \pi e^x\,dx$

22. $\int (e^x + 3x^2 + 2x)\,dx$

23. $\int (x^{8.3} - 9x^6 + 3x^{-4} + x^{-3})\,dx$

24. $\int (0.7y^3 + 10 + 2y^{-3})\,dy$

25. $\int \frac{-2\sqrt{x}}{3}\,dx$

26. $\int dz$

27. $\int \frac{5}{3\sqrt[3]{x^2}}\,dx$

28. $\int \frac{-4}{(3x)^3}\,dx$

29. $\int \left(\frac{x^3}{3} - \frac{3}{x^3}\right)\,dx$

30. $\int \left(\frac{1}{2x^3} - \frac{1}{x^4}\right)\,dx$

31. $\int \left(\frac{3w^2}{2} - \frac{2}{3w^2}\right)\,dw$

32. $\int 7e^{-s}\,ds$

33. $\int \frac{3u - 4}{5}\,du$

34. $\int \frac{1}{12}\left(\frac{1}{3}e^x\right)\,dx$

35. $\int (u^e + e^u)\,du$

36. $\int \left(3y^3 - 2y^2 + \frac{e^y}{6}\right)\,dy$

37. $\int \left(\frac{3}{\sqrt{x}} - 12\sqrt[3]{x}\right)\,dx$

38. $\int 0\,dt$

39. $\int \left(-\frac{\sqrt[3]{x^2}}{5} - \frac{7}{2\sqrt{x}} + 6x\right)\,dx$

40. $\int \left(\sqrt[3]{u} + \frac{1}{\sqrt{u}}\right)\,du$

41. $\int (x^2 + 5)(x - 3)\,dx$

42. $\int x^3(x^2 + 5x + 2)\,dx$

43. $\int \sqrt{x}(x + 3)\,dx$

44. $\int (z + 2)^2\,dz$

45. $\int (3u + 2)^3\,du$

46. $\int \left(\frac{2}{\sqrt[5]{x}} - 1\right)^2\,dx$

47. $\int x^{-2}(3x^4 + 4x^2 - 5)\,dx$

48. $\int (6e^u - u^3(\sqrt{u} + 1))\,du$

49. $\int \frac{z^4 + 10z^3}{2z^2}\,dz$

50. $\int \frac{x^4 - 5x^2 + 2x}{5x^2}\,dx$

51. $\int \frac{e^x + e^{2x}}{e^x}\,dx$

52. $\int \frac{(x^2 + 1)^3}{x}\,dx$

53. If $F(x)$ and $G(x)$ are such that $F'(x) = G'(x)$, is it true that $F(x) - G(x)$ must be zero?

54. **(a)** Find a function F such that $\int F(x)\,dx = xe^x + C$.
(b) Is there only one function F satisfying the equation given in part (a), or are there many such functions?

55. Find $\int \frac{d}{dx}\left(\frac{1}{\sqrt{x^2 + 1}}\right)\,dx$.

PROBLEMS 14.3

In Problems 1 and 2, find y subject to the given conditions.

1. $dy/dx = 3x - 4; \quad y(-1) = \frac{13}{2}$
2. $dy/dx = x^2 - x; \quad y(3) = \frac{19}{2}$

In Problems 3 and 4, if y satisfies the given conditions, find y(x) for the given value of x.

3. $y' = \dfrac{9}{8\sqrt{x}}, y(16) = 10; \quad x = 9$
4. $y' = -x^2 + 2x, \ y(2) = 1; \quad x = 1$

In Problems 5–8, find y subject to the given conditions.

5. $y'' = -3x^2 + 4x; \quad y'(1) = 2, \ y(1) = 3$
6. $y'' = x + 1; \quad y'(0) = 0, \ y(0) = 5$
7. $y''' = 2x; \quad y''(-1) = 3, \ y'(3) = 10, \ y(0) = 13$
8. $y''' = 2e^{-x} + 3; \quad y''(0) = 7, \ y'(0) = 5, \ y(0) = 1$

In Problems 9–12, dr/dq is a marginal-revenue function. Find the demand function.

9. $dr/dq = 0.7$
10. $dr/dq = 10 - \dfrac{1}{16}q$
11. $dr/dq = 275 - q - 0.3q^2$
12. $dr/dq = 5{,}000 - 3(2q + 2q^3)$

In Problems 13–16, dc/dq is a marginal-cost function and fixed costs are indicated in braces. For Problems 13 and 14, find the total-cost function. For Problems 15 and 16, find the total cost for the indicated value of q.

13. $dc/dq = 2.47; \quad \{159\}$
14. $dc/dq = 2q + 75; \quad \{2000\}$
15. $dc/dq = 0.08q^2 - 1.6q + 6.5; \quad \{8000\}; \quad q = 25$
16. $dc/dq = 0.000204q^2 - 0.046q + 6; \quad \{15{,}000\}; \quad q = 200$

17. **Diet for Rats** A group of biologists studied the nutritional effects on rats that were fed a diet containing 10% protein.[2] The protein consisted of yeast and corn flour.

Over a period of time, the group found that the (approximate) rate of change of the average weight gain G (in grams) of a rat with respect to the percentage P of yeast in the protein mix was

$$\frac{dG}{dP} = -\frac{P}{25} + 2 \qquad 0 \le P \le 100$$

If $G = 38$ when $P = 10$, find G.

18. **Winter Moth** A study of the winter moth was made in Nova Scotia.[3] The prepupae of the moth fall onto the ground from host trees. It was found that the (approximate) rate at which prepupal density y (the number of prepupae per square foot of soil) changes with respect to distance x (in feet) from the base of a host tree is

$$\frac{dy}{dx} = -1.5 - x \quad 1 \le x \le 9$$

If $y = 59.6$ when $x = 1$, find y.

19. **Fluid Flow** In the study of the flow of fluid in a tube of constant radius R, such as blood flow in portions of the body, one can think of the tube as consisting of concentric tubes of radius r, where $0 \le r \le R$. The velocity v of the fluid is a function of r and is given by[4]

$$v = \int -\frac{(P_1 - P_2)r}{2l\eta}\,dr$$

where P_1 and P_2 are pressures at the ends of the tube, η (a Greek letter read "eta") is fluid viscosity, and l is the length of the tube. If $v = 0$ when $r = R$, show that

$$v = \frac{(P_1 - P_2)(R^2 - r^2)}{4l\eta}$$

20. **Elasticity of Demand** The sole producer of a product has determined that the marginal-revenue function is

$$\frac{dr}{dq} = 100 - 3q^2$$

Determine the point elasticity of demand for the product when $q = 5$. (*Hint:* First find the demand function.)

21. **Average Cost** A manufacturer has determined that the marginal-cost function is

$$\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$$

where q is the number of units produced. If marginal cost is \$27.50 when $q = 50$ and fixed costs are \$5000, what is the *average* cost of producing 100 units?

22. If $f''(x) = 30x^4 + 12x$ and $f'(1) = 10$, evaluate

$$f(965.335245) - f(-965.335245)$$

14.4 More Integration Formulas

Objective

To learn and apply the formulas for $\int u^a\,du$, $\int e^u\,du$, and $\int \frac{1}{u}\,du$.

Power Rule for Integration

The formula

$$\int x^a\,dx = \frac{x^{a+1}}{n+1} + C \qquad \text{if } a \neq -1$$

[2]Adapted from R. Bressani, "The Use of Yeast in Human Foods," in *Single-Cell Protein*, eds. R. I. Mateles and S. R. Tannenbaum (Cambridge, MA: MIT Press, 1968).
[3]Adapted from D. G. Embree, "The Population Dynamics of the Winter Moth in Nova Scotia, 1954–1962," *Memoirs of the Entomological Society of Canada*, no. 46 (1965).
[4]R. W. Stacy et al., *Essentials of Biological and Medical Physics* (New York: McGraw-Hill, 1955).