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Modeling of topology evolutions and implication on proactive routing overhead in MANETs ☆

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Abstract

We present a mathematical framework for quantifying the impact of node mobility on the overhead of proactive routing protocols in mobile ad hoc networks (MANETs). We focus on MANETs in which nodes move randomly. The analytical model we introduce models signaling overhead as a function of stability of topology, and characterizes the statistical distribution of topology evolutions. Although we could apply our analytical framework to any proactive routing scheme, we use the OLSR protocol as an example of our model, because it is a leading example of proactive routing for ad hoc networking. We corroborate the accuracy of the results obtained analytically by means of results obtained with discrete-event simulations using the same parameters adopted in the analytical model.

Keywords: Analytical models; Mobile ad hoc network; Topology evolution; Proactive routing; OLSR

1. Introduction

Mobility brings fundamental challenges to the design of protocol stacks for mobile mesh networks (MANETs). The mobility of nodes implies that the routing protocols of MANETs have to cope with frequent topology changes while attempting to produce correct routing tables. Proactive routing protocols, which are the focus of this paper, provide fast response to topology changes by continuously monitoring topology changes and disseminating the related information as needed over the network. However, the price they pay is the increase in signaling overhead as the topology changes increase, and this can further lead into smaller packet delivery ratios and longer delays. In the worst case, "broadcast-storms" [1] can result, congesting the entire network. Hence, it is essential to understand the intricate relations between routing overhead and topology changes for the design of routing protocols in MANETs.

Characterizing the impact of mobility on the performance of proactive routing protocols is a very complex problem. Consequently, the provision of such characterization has been limited to simulation-based approaches [2–6]. Few if any analytical studies have been pursued on this topic. Zhou et al. [7] gave an analytical view of routing overhead of reactive protocols, assuming static network (manhattan grid) with unreliable nodes and concludes the scalability of reactive protocols with localized traffic pattern. Topology changes resulting from node mobility was not considered in [7]. In [8], an information theoretic analysis is pursued to bound the memory requirement and overhead incurred by a hierarchical routing protocol for MANETs based on entropy rate of topology changes.

The previous work does provide a good understanding of the scalability properties of the signaling of routing schemes. However, to the best of our knowledge, there is no previous analytical work that establishes an analytical

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connection between routing overhead and topology changes due to mobility. Moreover, the past work has not even characterized topology changes as a function of node mobility, which is crucial to make the connection we seek.

In this paper, we provide the first analytical framework for the modeling of proactive routing overhead as a function of node mobility. In so doing, we model topology changes explicitly as a function of node mobility. Section 2 summarizes the network model used in our analysis and formulates the problem to be solved. Section 3 explains the general framework for the modeling of proactive routing overhead. Section 4 discusses properties of the topology of a MANET and factors that affect its stability. Section 5 explains our analytical model. Clearly, our results complement previous information theoretic analysis [8] by providing entropy rate and a model of topology changes.

Because of its practical importance, Section 6 applies our general framework to the analysis of the optimized link state routing protocol (OLSR) [9]. Our analysis of OLSR provides a better insight on its operation, and corroborates the effectiveness of our modeling framework. We compare our analytical results against *Qualnet* simulations based on scenarios assuming random node mobility. The results illustrate the accuracy of our analytical framework. Section 7 concludes this paper.

2. System model and problem statement

We consider a network operating in a square area, which is consistent with several prior analytical models [10–12]. The entire network is of size $L \times L$ and there are n nodes initially randomly deployed in such a "square network." Note that, although we consider a square network in the paper, our analysis can be extended to networks of any shape in a straightforward way.

Nodes are mobile and initially equally distributed over the network. The movement of each node is independent and unrestricted, i.e., the trajectories of nodes can lead to anywhere in the network. For node $i \in V = \{1, 2, \ldots, N\}$, let $\{T_i(t), t \geqslant 0\}$ be the random process representing its trajectory and take values in D, where D denotes the domain across which the given node moves. To simplify our modeling task, we make the following assumption on the trajectory processes.

Assumption 1 (*Stationarity*). Each of the trajectory processes $(T_i(t))$ is stationary, i.e., the spacial node distribution reaches its steady-state distribution irrespective of the initial location. The N trajectory processes are *jointly stationary*, i.e., the whole network eventually reaches the same steady state from any initial node placements, within which the statistical spatial nodes' distribution of the network remains the same over time.

The above assumption is quite fundamental in the sense that it lays the foundation for the modeling of node movement. Most existing models, (e.g., random direction mobility models [13–17], random waypoint mobility models [18,19] and random trip mobility model [20]) clearly satisfy our assumption. In other words, our assumption ensures that, on the long run, the network converges to its steady-state and the stationary spatial nodes' distribution can be used in the performance analysis of the network.

The availability of communication links (e.g., from node i to node j) is governed by the Signal-to-Interference-plus-Noise Ratio (SINR) protocol model as,

$$\frac{P_i(t) g_{ij}(t)}{N_0 + \sum_{k \in A_s(t), k \neq i} P_k(t) g_{kj}(t)} \geqslant \beta \tag{1}$$

where $P_i(t)$ denotes the transmitting power of node i at time t, $A_s(t)$ is the set of active nodes transmitting at time t, N_0 denotes the thermal noise and β is the minimum SINR for the receiver to successfully decode data packets. The channel gain from node k to node l at time t is represented by $g_{kl}(t)$, which captures path loss, fading and shadowing effects in the wireless environment. Eq. (1) simply states the physical requirement of the existence of a directional link from node i to node j at time t. Given that many routing algorithms require bi-directional links, we expect the SINR law to be satisfied for the reverse link, e.g., $j \rightarrow i$. We simply call a bi-directional link as a link throughout this paper.

The topology (or connectivity graph) $\mathcal{G}(t)$ of the network at time t can be obtained by replacing the available wireless links with lines connecting the corresponding node pairs. We use the terms topology and connectivity graph interchangeably.

Given the above terminology and assumptions, we seek answers to the following questions:

- Is there an analytical model to statistically characterize the distribution of topology changes in MANETs? If so, are we able to derive the associated parameters analytically?
- If there is such a model, are we able to apply the model to analyze the effect of mobility on the control overhead of proactive routing protocols? Or mathematically, could we find the function $\mathcal{F}$ that projects the control overhead $\mathcal{O}_d$ in MANETs given that we know the node mobility $\mathcal{V}$ and the control overhead $\mathcal{O}_s$ incurred by the protocol in a static topology?

$$\mathcal{F} : \mathcal{O}_s \times \mathcal{V} \rightarrow \mathcal{O}_d \tag{2}$$

3. Proactive routing overhead in dynamic graphs

A routing protocol operates on the connectivity graph (topology) $\mathcal{G}$ of a MANET. Let $\vec{\mathcal{G}} = \{\mathcal{G}_i\}$ be the set of all possible connectivity graphs of the MANET. In steady-state, the connectivity graph $\mathcal{G}(t)$ travels across all such graphs with a stable distribution vector $\vec{p} = \{p_i\}$ derived from the stationary spatial nodes' distribution.

A change that occurs in the connectivity of the MANET induces the transition from a connectivity graph of the

MANET to another connectivity graph. For simplicity, in the rest of this paper, we refer to the transition from one connectivity graph to another as a *topology evolution*.

If we look at the connectivity graph from the standpoint of single node, a topology evolution can be triggered by changes in its immediate neighborhood or by updates received from its neighbors. If we observe the protocol behavior at a typical active node k, we can derive from $\vec{\mathcal{G}}$ the set of all possible local connectivity graphs $\vec{\mathcal{G}^k} = \{\mathcal{G}_i^k\}$ with the corresponding distribution vector $\vec{p^k} = \{p_i^k\}$.

As Fig. 1 illustrates, we assume that when there is no change in topology, nodes periodically broadcast topology control (TC) messages at regular interval T_c. For this case, the average TC messages per active node in static scenarios $\mathcal{O}_s$ is simply

$$P(\mathcal{O}_s) = P(\mathcal{G}_i^k) = 1/T_c, \quad \forall i \tag{3}$$

If we assume that a topology change happens at time $t_i, KT_c < t_i \leqslant (K+1)T_c$, it induces the transition of the local connectivity graph from $\mathcal{G}_i^k$ to $\mathcal{G}_j^k$. The routing protocol reacts to the change by advancing the TC message broadcast at some time $t_i^*, KT_c < t_i^* \leqslant (K+1)T_c$, rather than broadcasting at the next planned time $(K+1)T_c$. The subsequent TC message broadcast will perform regularly with graph $\mathcal{G}_j^k$. In this case, compared to the static scenario where no change occurs, the increase $\gamma_i(t)$ in generated TC message associated with $\mathcal{G}_i^k$ can be computed as follows:

$$\gamma_i(t_i) = \frac{(K+1)}{t_i^*} \Big/ \frac{K+1}{(K+1)T_c} = \frac{\lceil t_i^*/T_c \rceil}{t_i^*/T_c} \tag{4}$$

where $\lceil \cdot \rceil$ is the ceiling operator.

The average increase γ_i in generated TC messages in the graph $\mathcal{G}_i^k$ can be computed as

$$\gamma_i = E_{t_i}\left(\frac{\lceil t_i^*/T_c \rceil}{t_i^*/T_c}\right) \tag{5}$$

Statistically, γ_i measures the normalized transition cost for $\mathcal{G}_i^k$ and t_i^* is determined by the t_i that captures the stability of the local topology $\mathcal{G}_i^k$. Summing over all possible topologies, we can estimate the average number of generated TC message per active node as

$$P = \sum_{\forall i} p_i^k P(\mathcal{G}_i^k) * \gamma_i \tag{6}$$

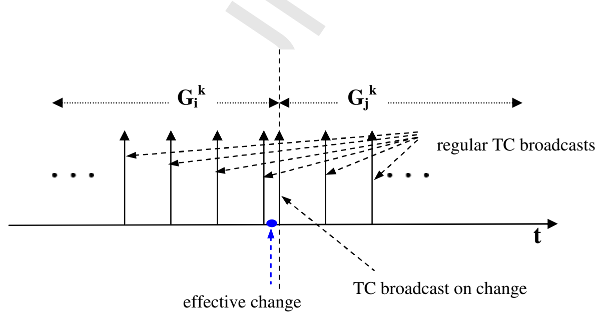

Fig. 1. Protocol behaviors with local connectivity graphs.

As we will see in Section 5, if we are only concerned with nodal mobility and given that nodes are moving randomly and independently of one another, we could assume that link changes arrive independently and $\{t_i\}$ are of identical statistical distributions, being a renewal process. We have then

$$P = \gamma \times \sum_{\forall i} p_i^k P(\mathcal{G}_i^k) \tag{7}$$

$$\gamma = E\left(\frac{\lceil \zeta^*/T_c \rceil}{\zeta^*/T_c}\right) \tag{8}$$

where ζ^* is decided on ζ and ζ is the observed stability of the local connectivity graph per active node. γ is the *penalty factor* that measures the cost in graph transitions for an active node and as we will see later, it is a function of nodal mobility and stability of the local connectivity graph. Furthermore, a closer look at Eq. (8) shows that the increased traffic overhead can be estimated from the average performance of static graphs, which is exactly the right term in the equation.

In a homogeneous network, every node in the network operates in a similar way. Therefore, we can expect similar results on the whole network. Hence, we propose a model that estimates the control traffic overhead from the knowledge of the mean overhead $\mathcal{O}_s$ that occurs in static scenarios. Mathematically, we can write it as the tentative answer for the question raised in Section 2 as:

We could have a function $\mathcal{F}$ that projects the control overhead $P(\mathcal{O}_d)$ in MANETs with the knowledge of mobility $\mathcal{V}$ and control overhead $P(\mathcal{O}_s)$ of protocol at static scenarios. And the function can be written as,

$$\mathcal{F} : P(\mathcal{O}_d) = \gamma(\mathcal{V}) * P(\mathcal{O}_s) \tag{9}$$

However, we need to know the distribution of topology evolutions (t_i in Eq. (4) for the computation of mobility effect on proactive routing overhead. To obtain such a model, we will first discuss factors that affect the stability of topology and then propose analytical model for topology evolution.

4. Topology: factors for changes

4.1. Setup

Due to node mobility and the surrounding parallel transmissions, links between nodes are set up and broken dynamically. We introduce a $\{0,1\}$-valued on–off process $f_{ij}(t), t \geqslant 0$ to model such link changes as $f_{ij}(t) = 1$ (or $f_{ij}(t) = 0$) if the unidirectional link from node i to node j, is available (or unavailable) at time $t \geqslant 0$. Clearly, we have $f_{ij}(t) = f_{ji}(t)$ because we only consider bi-directional links.

If we map every active (on) link to an edge in a graph with N vertices where each vertex stands for a node in V, we can obtain the time-varying graph (topology) $\mathcal{G}(t)$ with a time-varying set $E(t)$ of edges as

$$E(t) := \{\{i,j\} \in V \times V, i \neq j; f_{ij}(t) = 1\} \tag{10}$$

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It should be noted that $\mathcal{G}(t)$ is the connectivity graph of the network, which is an *undirected* graph, given that we consider bi-directional links. Let E be the complete set of possible links in the graph, i.e.,

$$E := \{\{i,j\} \in V \times V, i \neq j\} \tag{11}$$

The complementary set $E^c(t)$ of $E(t)$ can be computed as

$$E^c(t) = E - E(t) \tag{12}$$

Each link change, such as new link formation or breakage of existing links, results in a change in the connectivity graph and could further result in a protocol event in the network to distribute such change. Let τ be the moment that the connectivity graph $\mathcal{G}(t)$ changes at time $t+\tau$ from its last change at time t. Clearly, τ is the random variable describing the duration of stability of the connectivity graph $\mathcal{G}(t)$. In general, there are two different scenarios responsible for changes of $\mathcal{G}(t)$. One is the creation or arrival of new link. Let τ_o be the random variable capturing the time duration of such new link arrivals or addition of new edges in $\mathcal{G}(t)$. Similarly, we have another random variable τ_f characterizing the breakage of existing links or deletions of edges in $\mathcal{G}(t)$. We will have

$$\tau = \min\{\tau_o, \tau_f\} \tag{13}$$

Our objective is first to identify the factors that affect the stability τ of the connectivity graph $\mathcal{G}(t)$ and then find the analytical model that characterizes the statistical distribution of τ.

4.2. Factors in connectivity graph

It is apparent from Eq. (1) that the availability of links depends on the wireless environment (captured in channel gain $g_{kl}(t)$) and also on the traffic and MAC schemes, which together decide the active set of transmitting nodes $A_s(t)$. If we do not explicitly model the shadowing effect and short-term channel variations such as channel fading between nodes, it is reasonable to assume that the channel gain can be computed according to the exponential attenuation model, that is,

$$g = r^{-\alpha} \tag{14}$$

where r denotes the Euclidean distance between two communicating nodes and α is the exponential attenuation coefficient, normally ranging from 2 to 5 with various wireless environments.

By introducing a dynamic and sometimes intractable active set $A_s(t)$, the involvement of traffic and MAC schemes significantly complicates the problem with a dynamic varying interference term. We call such a term *environmental mobility*, which results from surrounding traffics and parallel transmissions.

When the MAC protocol schedules transmissions perfectly, multiple access interference is negligible compared to the noise and can be considered zero, i.e., no environmental mobility. In such case, the deciding factors for link availability lies in the transmission power and radio propagation loss and it can be expressed as

$$\frac{P_i(t)g_{ij}(t)}{N_0} \geqslant \beta \quad \text{and} \quad \frac{P_j(t)g_{ji}(t)}{N_0} \geqslant \beta \tag{15}$$

If all nodes transmit with a uniform power, given Eq. (14), the link between two nodes becomes available as soon as they are within communication range of each other, i.e., their Euclidean distance is smaller than the maximum radio coverage R for a transmitting node. Under these assumptions, the availability of links is purely a function of the relative distances between nodes, which in turn are determined by nodal mobility.

Thus far, we have identified two factors affecting the connectivity graph, *environmental mobility* and *nodal mobility*. However, the defining feature of MANETs is *nodal mobility*, which is a natural result from nodal movements. Accordingly, given that no analytical models exist for topology evolutions resulting from *nodal mobility* in MANETs, this is the focus of the model we describe next.

5. Modeling nodal mobility

Nodal motion changes the distances among nodes, and therefore results in the dynamic establishment and termination of links. Compared to the SINR law in Eq. (1), links defined by Eq. (15) are longer and exist for the maximum possible duration of link availability if only the effects of mobility are considered. In practice, the offered traffic and the scheduling of packets provided by the MAC protocol renders a smaller utilization of links. Hence, the link utilization under a real MAC protocol is smaller than the one predicted by Eq. (15).

For each link in set $E(t)$, let $T^o_{ij}(t)$ denote the *residual* lifetime of the link after time t, i.e., $T^o_{ij}(t)$ is the amount of the time that elapses from time t until link is unavailable. Correspondingly, for each link in set $E^c(t)$, $T^f_{ij}(t)$ be the *residual* silence time of link after time t, i.e., $T^f_{ij}(t)$ is the amount of time elapsed from time t until a link is available. Due to the underlying stationarity implied from the joint stationarity of trajectory processes, it suffices to consider only the case $t = 0$ and we can simply drop the time parameter t. Hence, $T^o_{ij} = T^o_{ij}(t)$. Clearly, we have

$$\tau_o = \min\{T^o_{ij} \text{ of link } \{i,j\}, \forall \{i,j\} \in E(t)\} \tag{16}$$

$$\tau_f = \min\{T^f_{ij} \text{ of link } \{i,j\}, \forall \{i,j\} \in E^c(t)\} \tag{17}$$

For each link $\{i,j\}$, the associated link availability process $f_{ij}(t)$, where $t \geqslant 0$, is simply an on–off process with successive up and down states with associated time durations, denoted by random variables $f_{ij}(k)$; $k = 1, 2, \ldots$ and $f_{ji}(k)$; $k = 1, 2, \ldots$, respectively. Such a processes can also be obtained from nodes' relative trajectories. When only nodal mobility is considered as the variable of interest, according to Eq. (15), a link between nodes i and j in V is available at time $t \geqslant 0$ if and only if their distance is smaller than R. As a result, the link availability is given by

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$$f_{ij}(t) := 1[\|T_i(t) - T_j(t)\| \leqslant R]; t \geqslant 0, \tag{18}$$

where $\|\cdot\|$ denotes the Euclidean operator to compute the distance.

Let $Z(t) = \sum_{\forall\{i,j\}} f_{ij}(t)$ and it is clear that $Z(t)$ is a renewal process comprised from a total number of $|E|$ on–off link availability processes, where $\|\cdot\|$ is the cardinality operator. Clearly, τ describes the refreshing interval, τ_o specifies the interval between upward renewals and τ_f denotes the interval between downward renewals of the renewal process $Z(t)$. By applying the well-known results from renewal processes and independent on–off processes in equilibrium [21], we have following theorem on τ.

Theorem 1. [Stability Model] *When sets $E(t)$ and $E^c(t)$ involve a sufficient number of links and all such links are assumed to be independent, the distribution of τ_o and τ_f can be approximated by the exponential distribution with parameter λ_o and λ_f. And the distribution of stability τ of the connectivity graph is also exponentially distributed with parameter $\lambda = \lambda_o + \lambda_f$. Therefore,*

$$P(\tau_o \leqslant t) = 1 - e^{-\lambda_o t} \tag{19}$$

$$P(\tau_f \leqslant t) = 1 - e^{-\lambda_f t} \tag{20}$$

$$P(\tau \leqslant t) = 1 - e^{\lambda t} = 1 - e^{-(\lambda_o + \lambda_f)t} \tag{21}$$

The above result is also known as Palm's theorem [21]. It states that the distribution of a superposition of N_r i.i.d. random variables converges to the exponential distribution as N_r approaches infinity. This result can be generalized to incorporate cases of independent but non-homogeneous motions, where some nodes may follow different mobility models from others.

The independence assumption for links, and the application of Palm's theorem, can be questioned in MANETs, because of the broadcast nature of their links. However, if the movement of nodes satisfies some *mixing conditions* known as *m-dependence* [22], the statement in Theorem 1 still holds. Such relaxed conditions introduce a form of asymptotic independence as the hop distance between links increases, while allowing dependence in neighborhoods. Specifically, *m-dependence* means that the correlation between links decreases as the hop distance between links increases and links can be assumed to be independent when the hop distance between links is greater than a given value m. Fortunately, most mobility models used to study MANETs fall in this category (e.g., the random waypoint mobility model, random direction mobility model and random trip mobility model) and our results can be applied to a wide-variety of scenarios.

5.1. Relations between λ_o and λ_f

We have observed that the new link formation process and link breakage process can be approximated by Poisson process with parameters λ_f and λ_o, respectively. For the new link formation process (or the link breakage process), λ_f (or λ_o) characterizes the average number of new link arrivals (or link breakages). Let us consider a time window T that is sufficiently large. The number of new link arrivals N_a and link breakages N_b within the time window can be approximated by

$$N_a = \lambda_f \, * T \tag{22}$$

$$N_b = \lambda_o \, * T \tag{23}$$

For a network with a finite number of nodes that is observed for an infinite length of time, the difference of the number of new link arrivals and link breakages can be denoted by

$$\lim_{T\to\infty}(N_a - N_b) = \lim_{T\to\infty} T * (\lambda_f - \lambda_o). \tag{24}$$

Clearly, the only choice is

$$\lambda_f = \lambda_o. \tag{25}$$

This indicates that, on the long run, the new link arrival process should be balanced off by the link breakage process. Otherwise, it contradicts the fact that the network only involves a finite number of nodes.

5.2. Analytical evaluation of λ_f or λ_o

If we know the parameter for the link breakage or link creation process, we can infer the other one. The link breakage process is characterized by the distribution of residual link life time, a direct evaluation of which requires exact knowledge of the underlying mobility characteristics. However, we can make general statements on the underlying new link formation process, resorting to the exponential modeling with parameter λ_l of point-to-point link formation in [23], as described in Appendix A.

For a particular connectivity graph $\mathcal{G}_i$ with associated sets E_i and E_i^c, there is a total number of $|E_i^c|$ potential point-to-point links that can be created. Because the time distribution of new link formation can be modeled as exponentially distributed with parameter λ_l, the stability for this particular connectivity graph can be measured with parameter

$$\lambda_f(\mathcal{G}_i) = |E_i^c| * \lambda_l \tag{26}$$

When a network is running in steady-state and inferring from the joint stationarity assumption of underlying trajectory processes, $\mathcal{G}(t)$ is a stationary and ergodic process that will experience all possible connectivity graphs with an associated probability vector derived from the steady-state nodes' distribution. By averaging all possible graphs, we can compute the parameter λ_f as

$$\lambda_f = E(|E_i^c|) * \lambda_l \tag{27}$$

where $E(\cdot)$ stands for expected value.

A general model of MANETs in steady-state exists and is known as a *random geometric graph* [24]. This model has been widely adopted in analytical works of MANETs and considered as an improvement over the model of *random graph* in static networks. Using the model of *random geometric graph*, we can compute λ_f as

$$\lambda_f = \overline{N}_f * \lambda_l \tag{28}$$

where $\overline{N}_f$ is the average number of potential link pairs and it can be computed as

$$\overline{N}_f = \frac{N*(N-1)}{2} * \left(1 - \frac{\pi R^2}{L^2}\right) \tag{29}$$

We thus arrive to the following theorem on the distribution of the stability τ of the connectivity graph.

Theorem 2 (Analytical Stability Model). *The distribution of stability τ of the connectivity graph in MANETs can be approximated as exponentially distributed with parameter λ and the parameter λ is given by*

$$\lambda = N*(N-1)*\left(1 - \frac{\pi R^2}{L^2}\right) \underbrace{*\, 2E[V_*]R \int_0^L \int_0^L \pi^2(x,y)\mathrm{d}x\mathrm{d}y}_{\lambda_l} \tag{30}$$

where $\pi(x,y)$ denotes the steady-state spatial nodes' distribution and $E[V_*]$ is the average relative velocity.

5.3. Model validations

We validated our analytical model of the stability of topologies by comparing its results against simulations. In the scenario used for comparison, there are a total of 100 nodes randomly placed for each $1000\,\mathrm{m} \times 1000\,\mathrm{m}$ square cell. Each node has the same transmit power and the radio transmission range considered is 250 m, that is the nominal coverage of IEEE 802.11 PHY layer. Four different speeds $\{5\,\mathrm{m/s}, 10\,\mathrm{m/s}, 15\,\mathrm{m/s}, 20\,\mathrm{m/s}\}$ are simulated for both the random waypoint mobility model (RWMM) and random direction mobility model (RDMM). Nodes are randomly activated to randomly choose destination node for data transmission. The traffic of activated nodes are supplied from a CBR source with a packet rate 0.5 p/s.

Figs. 2 and 3 present the results on complementary cumulative distribution function (CCDF) of the distribution of topology evolutions for RWMM and RDMM, respectively. It can be observed that for both cases, the exponential distribution model match pretty well with the simulation results and the analytical evaluation of the parameter also exhibits quite good approximation to the simulations.

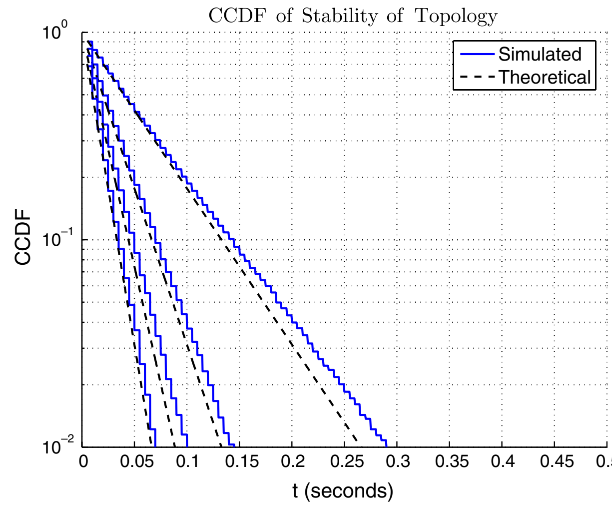

Fig. 2. Distribution of stability of topologies: RWMM, $R = 250$ m.

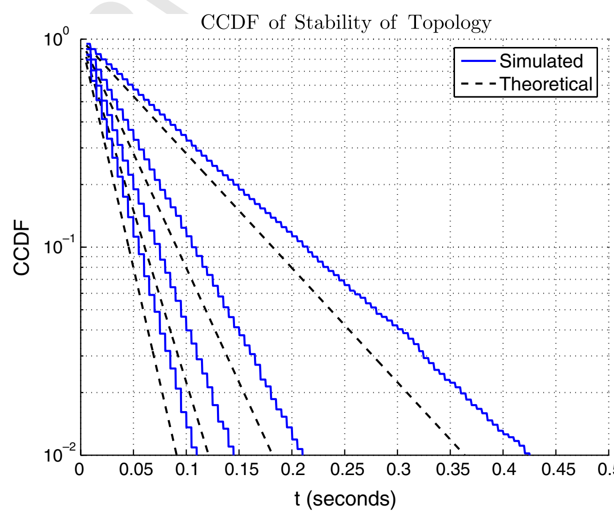

Fig. 3. Distribution of stability of topologies: RDMM, $R = 250$ m.

6. Analyzing control traffic overhead in OLSR

From the previous sections, we already know that the distribution of stability of the connectivity graph can be approximated as exponentially distributed with parameter λ given in Theorem 2. We apply our model to project the control traffic overhead of the OLSR protocol.

6.1. Brief overview of OLSR

In OLSR, nodes periodically send out HELLO messages to keep track of their neighbors. A HELLO message contains the one-hop neighbors of a node and status of adjacent links. Upon receiving and analyzing HELLO messages, nodes can compute their multipoint relays (MPR). The MPR set of a node is a subset of its neighbor nodes that are connected (i.e., cover) all their two-hop neighbors. The node making the selection of MPRs is called *MPR selector*. Every node could have multiple nodes to select itself as a MPR node, i.e., have multiple MPR selectors. Topology control (TC) messages are generated periodically by nodes with non-empty sets of *MPR selectors* to disseminate {MPR selector, MPR} link information to the whole network. In case of nodes detecting changes in the set of *MPR selector*, TC message could be initiated earlier than the regular interval to respond to the change. Node keep track of the TC messages and use such link information for path selection and traffic routing.

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The purpose of using MPRs in OLSR is to reduce the flooding of broadcast packets. For every node, its TC packets are retransmitted only by its MPR neighbor nodes and thus results in a saving of duplicate transmissions but still maintains satisfactory packet delivery. Clearly, the smaller the MPR set is, the more saving in the protocol.

A link breakage in OLSR is detected when a node fails to receive several consecutive HELLO messages from one of its neighbor node. A link addition is detected when a node starts to receive HELLO messages from a node not in its current one-hop neighbor set. Every change in the two-hop neighborhood link set will result in a protocol event of the node reacting to the change by recomputing its MPR set and could further result in MPR set. Therefore, it could lead to earlier TC message broadcast and the increase in the control traffic.

6.2. Parameterizing the MPR selection algorithm

By employing MPRs in OLSR, link changes need not result in a protocol event. However, the changes that happen at *critical links* (i.e., {MPR selector, MPR} pairs) surely trigger a protocol event. For the reason, we need to find a parameter that characterizes the performance of the MPR selection algorithm in OLSR, and further utilize it to derive the distribution of the connectivity graph. Before proceeding with choosing the appropriate performance metric, we need to first review the MPR selection algorithm. The MPR selection algorithm works as follows:

(1) Select the node within the set of one-hop neighbor nodes as MPR node, if among the two-hop neighbor nodes, there are one or more than one nodes that are only covered by the node.
(2) Choose a one-hop neighbor node as MPR node, if it covers the most of remaining two-hop neighbor nodes that are not covered by nodes in the MPR set. Repeat the step until all two-hop neighbor nodes are covered by the MPR set.

The MPR selection algorithm is a greedy algorithm and its performance varies depending on the graphs on which it operates. Its heuristic nature, edge effects, and its graph-dependent performance significantly complicates the modeling problem and prevents an analytical modeling (if feasible) of the algorithm. For this reason, the parameter that we are looking for should reflect the statistical performance of the MPR algorithm and an evaluation of such parameter could be obtained by statistical evaluation with random geometric graph model.

A natural choice of the parameter should be the performance metric that answers the questions how much savings the MPR selection algorithm brings in reducing the duplicate flooding packet. Let us define $Neighbor\{i\}$ as the set of one-hop neighbor nodes and let $MPR\{i\}$ be the MPR set for node i. It is obvious that, $MPR\{i\} \subseteq Neighbor\{i\}$. Then the one-hop saving β_i from MPR selection can be evaluated as

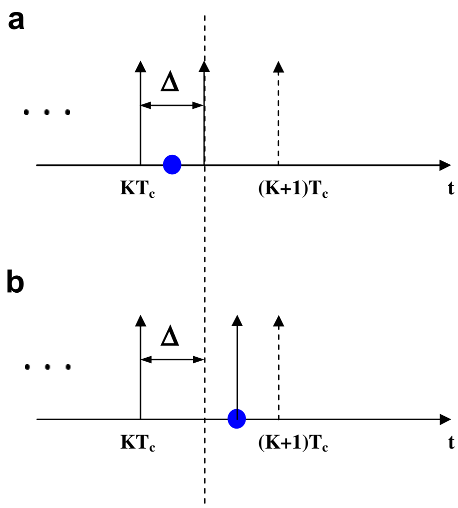

Fig. 4. Graphical illustration on change response.

$$\beta_i = \frac{|MPR\{i\}|}{|Neighbor\{i\}|} \quad (31)$$

Clearly, $0 < \beta_i \leqslant 1$. Eventually, we define a parameter β termed as *broadcast efficiency* to characterize the statistical performance of MPR selection algorithm. And it can be obtained through the statistical averaging over all possible nodes and graphs of the one-hop saving computed in Eq. (31).

$$\beta = E_{\mathcal{G},i}(\beta_i), 0 < \beta \leqslant 1 \quad (32)$$

The smaller β is, the more saving the MPR algorithm brings. β is also a statistical measure of the percentage of critical links ({MPR selector, MPR} pairs) out of total links in OLSR. From Section 5, we can infer that the distribution of link breakages of such links can also be approximated as exponentially distributed with parameter $\lambda_c = \beta * \lambda_o$.[1]

6.3. Computation of penalty factor

The only remaining problem is to compute γ as a function of nodal mobility or the stability ζ of the local connectivity graph. First, we need to look at how ζ^* is determined from ζ, i.e., to understand how OLSR reacts to an effective change. Effective change means that the node detect a change in the set of MPR selectors, since OLSR operates on the sub-graph from critical links.

Fig. 4 illustrates how OLSR reacts to an effective change. Suppose that a change arrives at $KT_c < \zeta \leqslant (K+1)T_c$, then the next scheduled TC message is advanced to be broadcasted at time ζ^*, the choice of which depends on when the change actually happened. If $KT_c < \zeta \leqslant KT_c + \Delta$, then the TC message will be broad-

[1] It can be derived from the fact that parameters of exponential distribution of topology evolutions are linearly proportional to the number of links evaluated and β denotes the percentage of the number of MPR links out of total links.

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casted at $\zeta^* = KT_c + \Delta$. For other cases $KT_c + \Delta < \zeta \leqslant (K+1)T_c$, TC message will be broadcasted immediately ($\zeta^* = \zeta$) when change is detected. The purpose of having Δ in OLSR is to avoid the case in which changes arrive too often and result in too much flooding from broadcasting TC messages. By aggregating such changes during Δ period in one TC message, the protocol can limit the maximum TC message broadcast rate but still achieve satisfactory performance. Summarizing the above analysis, one has

$$\zeta^* = \begin{cases} KT_c + \Delta, & KT_c < \zeta \leqslant KT_c + \Delta \\ \zeta, & KT_c + \Delta < \zeta \leqslant (K+1)T_c \end{cases} \qquad (33)$$

An effective change is the change that results in a change in the set of MPR selectors. Such changes depend on the stability of the local connectivity graph. Any changes in the local connectivity graph could lead to a recomputation of MPR set and further results in an effective change. We have the following itemized discussions on changes,

- A new link is detected in the local connectivity graph of node k. It will result in a MPR set recomputation of neighbors within two-hop distance of the new link. Such link may or may not lead to a change in MPR selectors of node k.
- A link breakage is detected in the local connectivity graph but not in the critical links of node k. For such cases, it still leads to a recomputation of MPR set but not necessarily affect the operation of node k.
- A link breakage in critical links of node k is detected and as a result, node k will detect a change in the set of MPR selectors. Such change is surely an effective change on node k and node k needs to react to the change by earlier TC message broadcast.

Due to the heuristic characteristic of MPR selection algorithm, an analysis of the first two scenarios could be significantly complicated (if feasible at all). Taking a conservative approach, we only consider the last scenario, where link breakage is detected in critical links. Because we know that the stability of overall critical links can be approximated by an exponential distribution with parameter λ_c, we can approximate the single node stability ζ of critical links as also exponentially distributed with parameter $\lambda_s = N{*}\lambda_c$. Note that such approximation becomes closer as node density increases, i.e., nodes associated with more critical links.

We can then compute the penalty factor γ as a function mobility $\mathcal{V}$ as

$$\gamma(\mathcal{V}) = E\left(\frac{\lceil \zeta^*/T_c \rceil}{\zeta^*/T_c}\right) = f(\lambda_s) \qquad (34)$$

where $f(\cdot)$ denotes mapping function and can be numerically computed after knowing the parameter λ_s of ζ (or ζ^*). It is also worthy of noting that the penalty factor is a direct function of local connectivity graph and suggests that the stability of connectivity graph can greatly affect the protocol performance.

6.4. Simulation results

In the simulation, the area of the network is a $1000\ \text{m} \times 1000\ \text{m}$ square cell. Each node has the same transmit power and the radio transmission range considered is 250 m. The number of nodes changes in the set $\{40, 60, 80, 100\}$ to simulate various node densities. The implementation of OLSR is the default implementation in *Qualnet 3.9.5*. Nodes are randomly activated to randomly choose destination node for data transmission. The traffic of activated nodes are supplied from a CBR source with a packet rate 0.5 p/s. And the movement follows the random waypoint model as the default setting in *Qualnet*. The maximum speeds considered are $\{0\ \text{m/s}, 5\ \text{m/s}, 10\ \text{m/s}, 15\ \text{m/s}, 20\ \text{m/s}\}$, ranging from static topologies, pedestrian speed to normal vehicle speed. The MAC layer is set as the 802.11 MAC. Overall, we simulate a total of 20 different network configurations. For each configuration, 50 simulations with random generated seeds are conducted to capture the statistical performance.

To study the effect of nodal mobility, we modified the *Qualnet* simulator to eliminate packet losses due to collisions in the channel. We call this case *perfectMAC*. Figs. 5–8 demonstrate the performance of the analytical model versus simulated performance when *nodal mobility* is the only performance factor. It can be observed that the analytical model provides a very good estimate compared to the simulations. Because we take a conservative approach in Section 6.3, the analytical model usually underestimates the overhead. As expected, the difference between the model and simulations decreases as node density increases, as critical links become more dominance in the local connectivity graph or link changes at non-critical links brings less effect on the sub-graph from critical links.

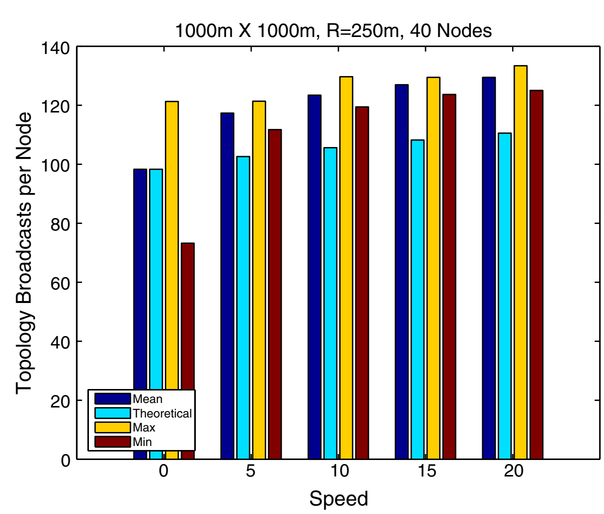

Fig. 5. perfectMac: N40.

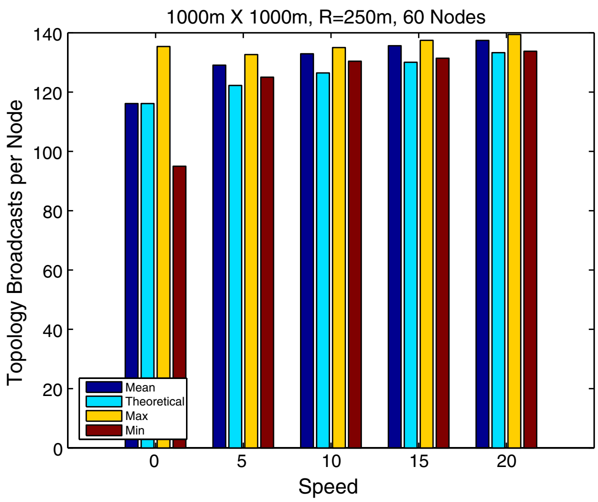

Fig. 6. perfectMac: N60.

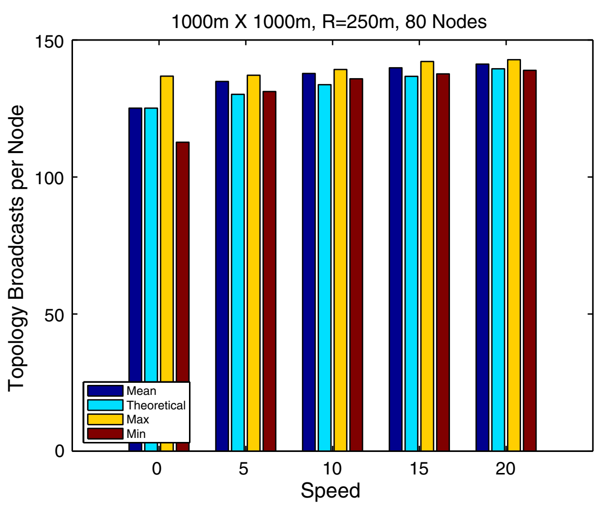

Fig. 7. perfectMac: N80.

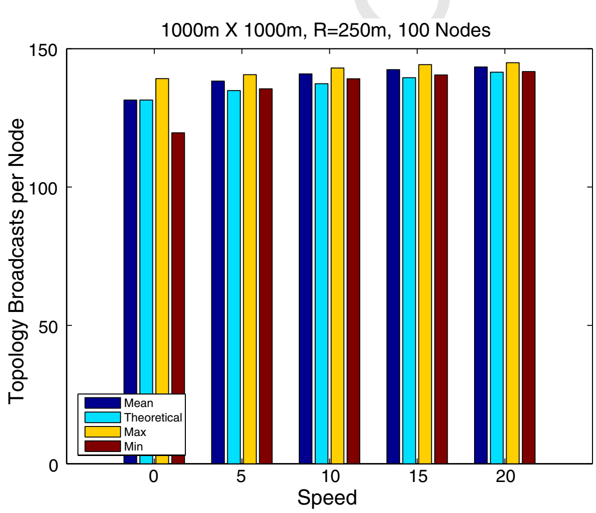

Fig. 8. perfectMac: N100.

To evaluate the model in practical scenarios, we used the original setting of *Qaulnet* in interference computation. In this case, the real 802.11 MAC works under collisions and back-offs. The simulation results are then illustrated in Figs. 9–12. In general, the model still provides a good approximation; however, the difference between the model and simulations are more pronounced due to additional effect from *environmental mobility*. Overall, we believe that our model provides satisfactory performance in estimating the routing overhead and brings deeper insight on how mobility affect the routing overhead.

7. Conclusion

We evaluated analytically the interdependence between routing overhead and the stability of the network topology by characterizing the statistical distribution of topology evolutions. The stability of topology can be modeled as exponentially distributed with a parameter computed from

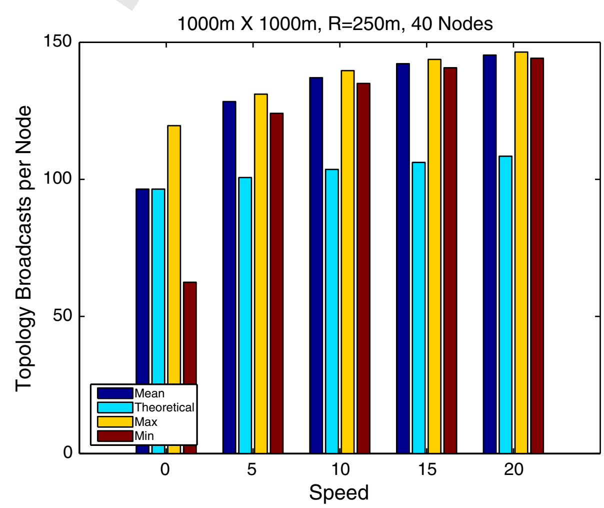

Fig. 9. Real Mac: N40.

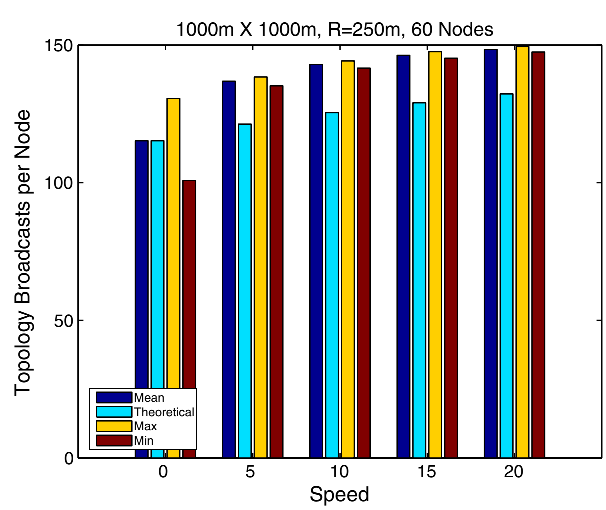

Fig. 10. Real Mac: N60.

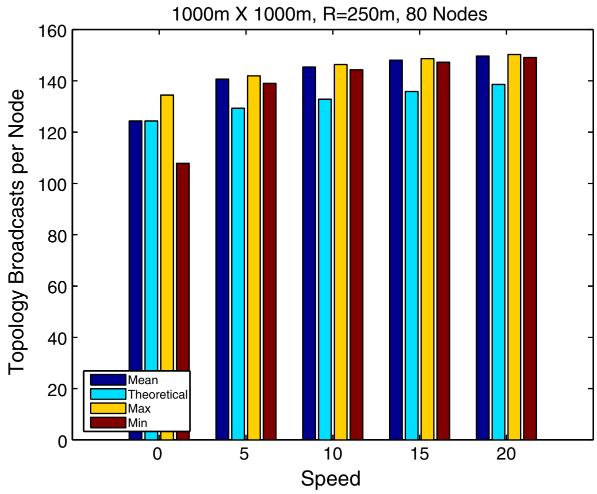

Fig. 11. Real Mac: N80.

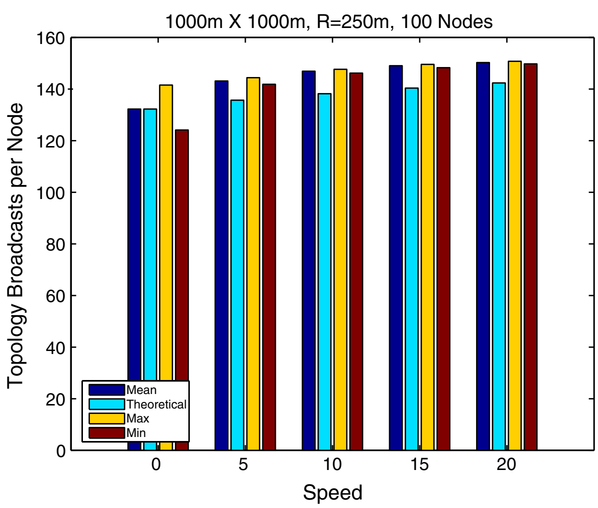

Fig. 12. Real Mac: N100.

network configurations. Utilizing the proposed model, the routing overhead of OLSR was analyzed and the results showed that the proposed model gives good estimate of routing overhead and meanwhile provides good insight on how nodal mobility affects the routing overhead.

Appendix A

Theorem 3. *Let two nodes move independently of each other in a square of size $L \times L$ with speeds V_1 and V_2. Let $E[V_*]$ be the average relative speed between the two nodes, and let $\pi(x,y)$ be the distribution of the node location in steady-state. If the transmission range $R \ll L$ and the location of a node at time t is independent of its location at time $t + \Delta_t$, for some small Δ_t, then the distribution F of new link arrivals for the two node is approximately exponentially distributed with parameter λ_l, where λ_l is given by*

$$\lambda_l \approx 2E[V_*]R \int_0^L \int_0^L \pi^2(x,y)\mathrm{d}x\mathrm{d}y. \tag{35}$$

The average time for the new link arrival is

$$E[F] = \frac{1}{\lambda_l} = \frac{1}{2E[V_*]R \int_0^L \int_0^L \pi^2(x,y)\mathrm{d}x\mathrm{d}y} \tag{36}$$

In particular, for random direction mobility model and random waypoint mobility model, it has following corollary.

Corollary 1. *The distribution of new link arrival between two nodes for the random direction mobility model for $R \ll L$ is approximately exponentially (λ_{RD}) distributed, where λ_{RD} is*

$$\lambda_{RD} \approx= \frac{2E[V_*]R}{L^2} \tag{37}$$

The expected time for the new link arrival is given by

$$E[F_{RD}] \approx \frac{L^2}{2E[V_*]R}. \tag{38}$$

Likewise, for the random waypoint mobility model we have

$$\lambda_{RW} \approx \frac{2\omega E[V_*]R}{L^2}, \tag{39}$$

$$E[F_{RW}] \approx \frac{L^2}{2\omega E[V_*]R}, \tag{40}$$

where ω is the waypoint constant.

It is worthy of noting that such a point-to-point exponential modeling of new link formation has also been extended to MANETs with restricted mobility [25].

References

[1] S.-Y. Ni, Y.-C. Tseng, Y.-S. Chen, J.-P. Sheu, The broadcast storm problem in a mobile ad hoc network, in: ACM Mobicom 1999, pp. 151–162, New York, 1999.
[2] J. Broch, D.A. Maltz, D.B. Johnson, Y. Hu, J. Jetcheva, A performance comparison of multi-hop wireless ad hoc network routing protocols, in: Mobicom'98, pp. 85–97, Dallas, 1998.
[3] P. Johansson, T. Larsson, N. Hedman, B. Mielczarek, M. Degermark, Scenario-based performance analysis of routing protocols for mobile ad-hoc networks, in: Mobicom'99, pp. 195–206, Washington, August 1999.
[4] F. Bai, N. Sadagopan, A. Helmy, Important: a framework to systematically analyze the impact of mobility on performance of routing protocols for adhoc networks, in: IEEE INFOCOM, San Francisco, 2003.
[5] F. Bai, G. Bhaskara, A. Helmy, Building the blocks of protocol design and analysis: challenges and lessons learned from case studies on mobile ad hoc routing and micro-mobility protocols, ACM SIGCOMM Computer Communication Review 34 (3) (2004) 57–70.
[6] T.H. Clausen, G. Hansen, L. Christensen, G. Behrmann, The optimized link state routing protocol, evaluation through experiments and simulation, in: IEEE Symposium on Wireless Personal Mobile Communications, 2001.
[7] N. Zhou, H. Wu, A.A. Abouzeid, Reactive routing overhead in networks with unreliable nodes, in: Mobicom'03, San Diego, September 2003.
[8] N. Zhou, A.A. Abouzeid, Routing in ad hoc networks: a theoretical framework with practical implications, in: INFOCOM 2005, pp. 1240–1251, Miami, 2005.

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[9] T. Clausen, P. Jacquet, Optimized link state routing protocol (OLSR), IETF RFC 3626 (2003).

[10] M. Grossglauser, D. Tse, Mobility increases the capacity of ad-hoc wireless network, in: Proceedings of Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies, vol. 3, pp. 1360–1369, April 2001.

[11] M. Grossglauser, D. Tse, Mobility increases the capacity of adhoc wireless networks, IEEE/ACM Transactions on Networking 10 (4) (2002) 477–486.

[12] A.E. Gammal, J. Mammen, B. Prabhaker, D. Shah, Throughput-delay trade-off in wireless networks, in: Infocom, vol. 1, pp. 464–475, 2004.

[13] S. Jiang, D. He, J. Rao, A prediction-based link availability estimation for mobile ad hoc networks, in: IEEE Infocom, pp. 1745–1752, April 2001.

[14] S. Jiang, D. He, J. Rao, A prediction-based link availability estimation for routing metrics in manets, IEEE/ACM Transactions on Networking 13 (6) (2005) 1302–1312.

[15] A.B. McDonald, T.F. Znati, A mobility-based framework for adaptive clustering in wireless ad hoc networks, IEEE Journal on Selected Areas in Communications 17 (8) (1999) 1466–1487.

[16] C. Bettstetter, Mobility modeling in wireless network: categorization, smooth movement and border effects, ACM Mobile Computing and Communication Review 5 (2001) 55–67.

[17] R. Guerin, Channel occupancy time distribution in a cellular ratio system, IEEE Transactions on Vehicular Technology VT-35 (3) (1987) 89–99.

[18] W. Navid, T. Camp, Stationary distributions for the random waypoint model, IEEE Transactions on Mobile Computing 3 (1) (2004) 99–108.

[19] J. Yoon, M. Liu, B. Noble, Sound mobility models, in: ACM Mobicom 2003, San Diego, 2003.

[20] J.-Y. Le Boudec, M. Vojnovic, Perfect simulation and stationarity of a class of mobility models, in: INFOCOM 2005, Miami, 2005.

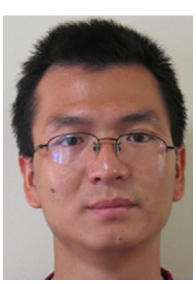
[21] D.P. Heyman, M.J. Sobel, Stochastic Models in Operations Research, vol. 1, McGraw-Hill, New York, 1982.

[22] Y. Han, R.J. La, Path selection in mobile ad-hoc networks and distribution of path duration, in: IEEE Infocom, Barcelona, April 2006.

[23] R. Groenevelt, Stochastic models in mobile ad hoc networks, PhD thesis, INRIA, Sophia Antipolis, France, 2005.

[24] M. Penrose, Random Geometric Graphs, Oxford University Press, Oxford, 2003.

[25] X. Wu, H.R. Sadjadpour, J.J. Garcia-Luna-Aceves, Link lifetime as a function of node mobility in manets with restricted mobility: modeling and applications, in: WiOpt 2007, April 2007.



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