UNIVERSITY OF CALIFORNIA
SANTA CRUZ

# THE CAPACITY OF WIRELESS AD HOC NETWORKS WITH HETEROGENEOUS PROPERTIES

A thesis submitted in partial satisfaction of the
requirements for the degree of

MASTER OF SCIENCE

in

ELECTRICAL ENGINEERING

by

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September 2010

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# Abstract

We study the scaling laws for wireless ad hoc networks in which the distribution of $n$ nodes in the network is homogeneous but the traffic they carry is heterogeneous. More specifically, we consider the case in which a given node is the data-gathering sink for $k$ sources sending different information to it, while the rest of the $s = n - k$ nodes participate in unicast sessions with random destinations chosen uniformly. We first study this type of network under both protocol and physical model but with a bandwidth assumption which is that the bandwidth of each node is proportional to the traffic of the corresponding cell or cluster.

Then we release the bandwidth assumption and present a *separation theorem* for heterogeneous traffic showing that the optimum order aggregate throughput can be attained in a wireless network in which traffic classes are distributed uniformly by endowing each node with multiple radios, each operating in a separate orthogonal channel, and by allocating a radio per node to each traffic class. Based on this theorem, we show how this order capacity can be attained for the unicast and data-gathering traffic classes by both extending cooperative communication scheme [1] under information theoretical model and pure "straight line" non-cooperative routing scheme [2] under protocol model

respectively.

# Chapter 1

# Introduction

## 1.1 Overview of Capacity of Wireless Ad-hoc Networks

The scaling laws of wireless networks with homogeneous traffic and uniform node distribution have been extensively studied in the literature. Gupta and Kumar [2] evaluated the capacity of wireless networks with uniform traffic and showed that the capacity scales as $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$ according to the protocol model[1]. This result was achieved by considering no cooperation among nodes with simple point-to-point communication. Xie and Kumar [3], [4] subsequently investigated the information-theoretic capacity of wireless networks with cooperation among different nodes in the extended network. The achievable capacity with cooperation for dense networks was studied by Özgür et al. in [1] who showed this capacity is $\Theta\left(n^{1-\varepsilon}\right)$, where $\varepsilon$ is any small positive constant.

[1]Given two functions $f$ and $g$, we say that: 1) $f(n) = O\left(g(n)\right)$ if there exists a constant $c$ and integer $N$ such that $f(n) \leq cg(n)$ for $n > N$. 2) $f(n) = o\left(g(n)\right)$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$. 3) $f(n) = \Omega\left(g(n)\right)$ if $g(n) = O\left(f(n)\right)$. 4) $f(n) = \omega\left(g(n)\right)$ if $g(n) = o\left(f(n)\right)$. 5) $f(n) = \Theta\left(g(n)\right)$ if $f(n) = O\left(g(n)\right)$ and $g(n) = O\left(f(n)\right)$.

Only a handful of prior works investigate heterogeneous traffic in the network. Keshavarz-Haddad et al. [5] introduced the concept of transmission arena. Based on that definition, they presented a method to compute the upper bound of the capacity for different traffic patterns and different topologies of the network. However, they did not provide closed-form scaling laws for the network capacity. Toumpis [6] investigated the throughput capacity when there are $s$ sources and $s^{\varepsilon}$ destinations in the network, where $0 < \varepsilon < 1$. Liu et al. [7] extended this result by relaxing the constraint on the number of sources and destinations. While these results [6, 7] address asymmetric traffic, the results apply to the case of a single type of traffic pattern in the network. Moreover, none of this works apply cooperative communication schemes.

## 1.2 The Motivation of This Work

In practice, we always see the ad hoc networks with more than one type of traffic. For example, in a military network, some solders may have unicast transmissions between them and some solders may want to contact their higher authority at the meantime. Thus, we need to study this type of network model. In this thesis, we consider heterogeneous traffic in the network supporting different types of traffic patterns. In particular, we consider two types of traffic classes, namely, data-gathering traffic in which one of the nodes in the network acts as a sink with many sources transmitting different information to that node, and the rest of the nodes in the network participate in unicast traffic flows. The distribution of nodes in the network is still uniform. To the best of our knowledge, this heterogeneous traffic model has not been studied in the literature, except our own work. There is some prior work addressing data gathering [8], [9], [10] as the only type of traffic in a network. Rodoplu et al. [11], [12] computed the network capacity for data gathering and unicast flows separately by utilizing game theory and defining a new capacity concept named core capacity.

This work first present the capacity of this type of network under both protocol model and physical model but with a bandwidth assumption which is the bandwidth of each node is proportional to the traffic of the corresponding cell or cluster. Then we release the bandwidth assumption and introduce a new approach to support heterogeneous traffic efficiently in a wireless network by dividing the available bandwidth into multiple channels separated in frequency and allocated dynamically to specific traffic classes consisting of the aggregation of one or more flows. We present a *separation theorem* showing that, in a multiple-channel multiple-radio (MR-MC) wireless network, the optimum order achievable throughput can be attained in a wireless network in which traffic classes are distributed uniformly by allocating a radio per node to each traffic class. Based on this theorem, we first extend the *Three-Phase* approach first introduced by Özgür et. al. [1] to accomodate different traffic classes. We demonstrate that the maximum per-node capacity of $\Theta(\log(n))$ for unicast traffic can be attained in some regions. This capacity was provided as an upper bound originally by Özgür et al [1] without providing any specific communication scheme. Then, due to the complexity of the *Three Phase* schemes, we provide the upper bound and achievable lower bound of the throughput by only using pure routing scheme without any cooperation between nodes under protocol model again.

In each chapter of this thesis, for convenience, we will reuse the same subscripts of the same letter in other chapters to represent different constant value, but we will state each constant value clearly.

## 1.3 The Organization of This Thesis

The thesis is organized as follows. In Chapter 2, we will present the wireless model that are used in literatures and in this thesis. Then we introduce capacity result under protocol model and the bandwidth assumption in Chapter 3. Later on, the capacity result under physical model will be presented in Chapter 4. In Chapter 5, we study the same network model but under the information theoretical model and release the bandwidth assumption. We also introduce the *Separation Theorem* in this chapter. Then we study the network under protocol model again but releasing the bandwidth assumption in Chapter 6. In Chapter 7, we conclude the thesis and give the future work.

# Chapter 2

# Wireless Network Model

In this chapter, we will present different network models used in literatures. Then in the rest of this thesis, we derive the throughput in the network based on these models.

First, we will introduce the network models which are most common in literatures. This classification corresponds to the distribution of the nodes in the network.

## 2.1 Arbitrary Network Model

The arbitrary network means that the nodes in the network are arbitrarily distributed in the network and each node can choose an arbitrary range or power level for any transmission. According to the condition of successful transmission, we have three models defined below, which are protocol model, physical model and information theoretical model.

**Definition 2.1** Protocol Model: *Suppose node $X_i$ transmits over the $m$-th sub channel to a node $X_j$. Then this transmission at rate $W_m$ bits/sec is assumed to be successfully received by node $X_j$ if $|X_k - X_j| \geq (1+\Delta)|X_i - X_j|$, for every other node $X_k$ simultaneously transmitting over the same sub-channel.*

**Definition 2.2** Physical Model: *Let $\{X_k; k \in \mathcal{T}(t)\}$ be the subset of nodes simultaneously transmitting at some time instant over a certain subchannel $m$. Let $P_k$ be the power level chosen by node $X_k$, for $k \in \mathcal{T}(t)$. Then the transmission between node $X_i$, $i \in T(t)$ and $X_{R(i)}$ is successful if*

$$\frac{\frac{P_i}{|Xi - X_{i(R)}|^\alpha}}{N + \sum_{k \in \mathcal{T}, k \neq i} \frac{P_k}{|X_k - X_{k(R)}|^\alpha}} \geq \beta. \tag{2.1}$$

*where $N$ is the ambient noise power level.*

Both the protocol and physicals model are a simplification of the successful transmission condition. The actual amount of information that can be transmitted through the network should be derived from information theory, which is referred as the information theoretical model.

## 2.2 Random Network Model

For the random network, we mainly consider that the $n$ nodes are uniformly and independently distributed in the network. Similar to the arbitrary network model, we still have the three models defined below.

**Definition 2.3** Protocol Model: *All nodes use a common transmission range $r(n)$ for all their communication. The network area is assumed to be a unit square area. Node $X_i$ can successfully transmit to node $X_j$ if for any node $X_k, k \neq i$, that transmits at the same time as $X_i$, it is true that $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$.*

**Definition 2.4** Physical Model: *Let $\{X_k; k \in \mathcal{N}\}$ be the subset of nodes simultaneously transmitting at some time instant over a certain subchannel. All nodes in this subchannel choose a common power level $P$ for all their transmissions. For each subchannel, the noise power is $N$. A node can transmit over several subchannels. A transmission from a node $X_i$, $i \in \mathcal{N}$, is successfully received by a node $X_{i(R)}$ if*

$$\frac{\frac{P}{|Xi - X_{i(R)}|^{\alpha}}}{N + \sum_{k \in \mathcal{N}, k \neq i} \frac{P}{|X_k - X_{k(R)}|^{\alpha}}} \geq \beta. \tag{2.2}$$

for every subchannel.

The information theoretical model is also referred as the actual capacity which can be achieved by using information theory.

## 2.3 Other Network Model

There are some other network models according to the different distribution of the nodes. For example, in [13], the nodes are assumed to be Poisson distribution. Moreover, in [14], the author assumed the node distribution of the network is a shot-noise Cox process.

There are typically two models which are referred as dense network model and extended network model. In the dense network model, the area is fixed and the node density tends to infinity, which is typically assumed for protocol and physical models. Meanwhile, in the extended network model, the node density is fixed and the area increases with the number of nodes in the network, which is typically used in information theoretical model.

According to different traffic types of the network, the network model can be referred as the network with homogeneous traffic and the network with heterogeneous traffic which is mainly considered in this thesis.

## 2.4 The Wireless Network Model Used in This Work

In this work, we mainly consider the the category of dense random network which is the combination of dense network and random network. The traffic type in this work is assumed to be heterogeneous traffic which means that there are two types of traffic in the network. One is unicast traffic [2]. The other is data-gathering traffic [10], [7], [9]. This means that part of the nodes in the network perform unicast traffic and the rest of the nodes have data-gathering traffic. To calculate the aggregate throughput in the network. All of the protocol model, physical model and information theoretical model will be used. In the following chapters, we will explain the details of the definition for each type of traffic.

# Chapter 3

# The Capacity of Wireless Ad-hoc Networks with Heterogeneous Traffic under Protocol Model: Part 1

In this chapter, we will give the capacity of wireless network with heterogeneous traffic by using the protocol model shown in *Definition* 2.3. First, we will give the network model and assumptions of this chapter. Then, the lower bound of the capacity will be calculated. Third, we will provide the computation of the upper bound of the capacity. In the end, we will discuss and conclude this work. This work is presented as [15].

## 3.1 Wireless Network Model

We consider a network with nodes uniformly distributed in a dense network, where the area of the network is a constant unit square. We assume heterogeneous traffic for the network, such that a single node (called the access point) is the destination for $k$ sources in the network. For the rest of the $n-k$ nodes in the network, we assume random and uniformly distributed source-destination pairs. Therefore, the source-destination pair selection for unicast communications is similar to that used by Gupta and Kumar [2] for the rest of $n-k$ nodes in the network. network model is shown in Fig. 3.1.

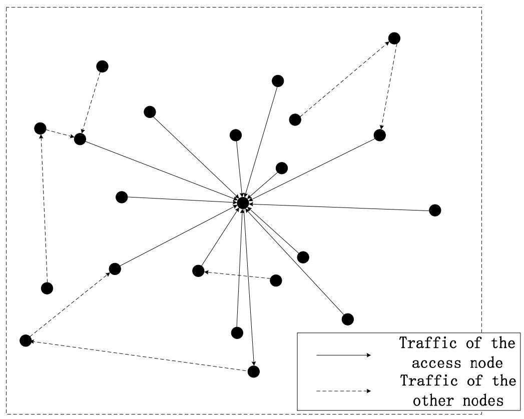


Figure 3.1: The Network Model

The transmission range is assumed to be the same for all the nodes and the communication between nodes is point-to-point. A successful communication between two nodes is modeled according to the protocol model which is shown in *Definition* 2.3.

The definitions of feasible throughput and order throughput capacity are shown below [2].

**Definition 3.1** *Feasible Throughput:*

*A throughput of $\lambda_i(n)$ bits per second is said to be feasible for the $i^{th}$ source-destination pair if there is a common transmission range $r(n)$, and a scheme to schedule transmissions and there are routes between source and destination, such that source $i$ can transmit to its destination at such rate successfully. For heterogeneous traffic, the feasible throughput is defined for each source-destination pair.*

**Definition 3.2** *Order of Throughput Capacity: The total throughput capacity is said to be of order $\Theta(f(n))$ bits per second if there exist a constant $c$ and $c'$ such that*

$$\begin{aligned} &\lim_{n\to\infty} \Pr(\lambda(n) = \sum_{i=1}^{n} \lambda_i(n) = cf(n) \ \textit{is feasible}) = 1; \ \textit{and} \\ &\liminf_{n\to\infty} \Pr(\lambda(n) = \sum_{i=1}^{n} \lambda_i(n) = c'f(n) \ \textit{is feasible}) < 1. \end{aligned} \tag{3.1}$$

## 3.2 The Lower Bound of the Capacity

We need to emphasize that there are two types of traffic in our model. One traffic is associated to the $k$ sources transmitting packets to the access node and the other traffic stems from the rest of $n-k$ nodes in the network with unicast communications. Therefore, we need to define the routing protocol and scheduling under this traffic model.

### 3.2.1 The Routing Scheme and the Scheduling Protocol

The selection of sources for the access node $i$ is based on the technique described in [16]. We randomly and uniformly select $k$ locations in the network and choose the closest nodes to these $k$ locations as sources for the access node. The routing trajectory is a straight line $L_i$ from access node to these $k$ locations. Then the packets traverse from each source to destination in a multi-hop fashion passing through all the cells that cross $L_i$. For the rest of $j$ nodes with unicast traffic where $1 \leq j \leq n-k$, both selections of source-destination pairs and routing is similar to the above technique.

For the scheduling scheme, we utilize a TDMA scheme similar to [16] with some modifications to take into account the heterogeneity of the traffic.

### 3.2.2 The traffic caused by access node

Let us define a traffic from node $i$ to node $j$ as commodity [12]. Clearly, the number of commodities for access node is $k$ which is also equivalent to the number of lines (paths) passing through the cell that contains the access node. For simplicity of the analysis, we assume that the access node is located at the center of the network. Now we compute the number of commodities for a cell that has a distance of $x$ from the access node. From Fig. 3.2 and by choosing $X_iC = \sqrt{2}$, the area of triangle is

$$S_{X_iAB} = \sqrt{2}\frac{\sqrt{2}d_n}{\sqrt{(x+d_n)^2 - d_n^2}} < \frac{2d_n}{x}, \tag{3.2}$$

where $d_n = C_1\sqrt{\frac{\log n}{n}}$ is selected to guarantee the connectivity between adjacent cells in the network [2] and $C_1$ is a constant factor.

**Theorem 3.3** *For any cell with a distance of $x_j$ from the access node, the upper bound*

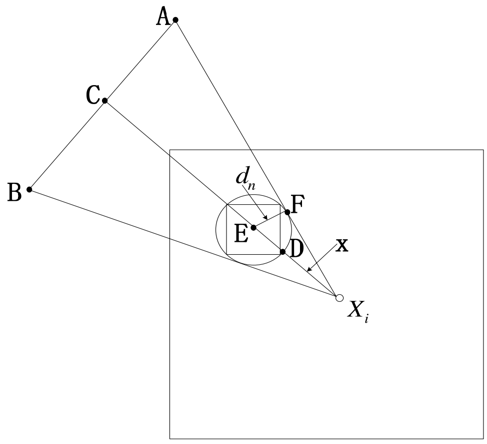


Figure 3.2: A geometric description of traffic by the access node in the network. $X_i$ is the access node and $X_iAB$ is a triangle whose altitude $X_iC$ is $\sqrt{2}$. $D$ is the intersecting point between $X_iC$ and the circle centered at $E$ with radius $EF = d_n$. The length of $DX_i$ is $x$.

*for the number of commodities caused by the traffic from the access node is*

$$N_{x_j} < 2\frac{d_n}{x_j}k \tag{3.3}$$

*when* $k = \Omega(\sqrt{\frac{n}{\log n}})$.

**proof 3.4** *The average number of lines passing through the cell (*$\mathrm{E}[N_{x_j}]$*) whose distance from access node i is* $x_j$ *is less than* $2\frac{d_n}{x_j}k$ *since k source nodes are uniformly distributed in the network. Utilizing the Chernoff bound [17], we have*

$$\begin{aligned} &\Pr\left(N_{x_j} - \mathrm{E}[N_{x_j}] > \delta\,\mathrm{E}[N_{x_j}]\right) \\ &< \exp\left[-((1+\delta)\log(1+\delta) - \delta)\,\mathrm{E}(N_{x_j})\right] \end{aligned} \tag{3.4}$$

*and*

$$\Pr\left(N_{x_j} - \mathrm{E}[N_{x_j}] < -\delta\,\mathrm{E}[N_{x_j}]\right) < \exp\left[-\frac{\delta^2}{2}\,\mathrm{E}[N_{x_j}]\right] \tag{3.5}$$

*where* $0 < \delta < 1$. *Combining the results and considering* $\mathrm{E}[N_{x_j}] < \frac{2d_n}{x_j}k$, *we obtain*

$$\begin{aligned}&\Pr\left(|N_{x_j} - \mathrm{E}[N_{x_j}]| > \delta\,\mathrm{E}[N_{x_j}]\right) < \\ &\exp\left[-((1+\delta)\log(1+\delta)-\delta)\frac{2d_n}{x_j}k\right] + \exp\left[-\frac{\delta^2}{2}\frac{2d_n}{x_j}k\right].\end{aligned} \tag{3.6}$$

*Thus, the probability that the values of the random variables* $N_{x_j}$ *for all* $j$ *can simultaneously be arbitrarily close to* $\mathrm{E}[N_{x_j}]$ *is given by*

$$\begin{aligned}&\Pr\left[\bigcap_j |N_{x_j} - \mathrm{E}[N_{x_j}]| < \delta\,\mathrm{E}[N_{x_j}]\right] \\ &= 1 - \Pr\left[\bigcup_j |N_{x_j} - \mathrm{E}[N_{x_j}]| > \delta\,\mathrm{E}[N_{x_j}]\right] \\ &\geq 1 - \sum_j \Pr\left[|N_{x_j} - \mathrm{E}[N_{x_j}]| > \delta\,\mathrm{E}[N_{x_j}]\right] \\ &> 1 - \sum_j \left(\exp\left[-((1+\delta)\log(1+\delta)-\delta)\frac{2d_n}{x_j}k\right]\right. \\ &\left. + \exp\left[-\frac{\delta^2}{2}\frac{2d_n}{x_j}k\right]\right).\end{aligned} \tag{3.7}$$

*Denote that if* $k = \Omega(\sqrt{\frac{n}{\log n}})$ *and* $d_n = \Theta(\sqrt{\frac{\log n}{n}})$, *then this probability tends to 1 when* $n \to \infty$.

### 3.2.3 The traffic caused by unicast communications

In this section, we derive the number of lines passing through each cell because of unicast traffic in the network. Since the unicast traffic is distributed uniformly in the network, this value is the same for all the cells in the network.

**Lemma 3.5** *For any cell $S$, the maximum number of lines intersecting this cell caused by unicast traffic is given by*

$$\Pr(\textit{Maximum number of lines } L_i \textit{ passing through} S \leq C_2(n-k)\sqrt{\frac{\log n}{n}}) \to 1,$$

*when $n-k \neq$ constant.*

**proof 3.6** *Our proof is similar to that of [16] except that we account for $n-k$ unicast pairs in the network. The probability that the destination node $j$ is $x$ away from the source node is $C_3\pi(x+d_n)$ [16] where $C_3$ is a constant. Thus, the probability $p$ that there is a line passing through the cell $S$ which is with distance $x$ from $j$ is*

$$\begin{aligned} \Pr(L_i \textit{ intersects } S) = p < \int_{d_n}^{\sqrt{2}} \left(\frac{2d_n}{x} \bigvee 1\right) k \cdot C_3 \\ \times \pi(x+d_n)dx \leq C_4\sqrt{\frac{\log n}{n}} \end{aligned} \tag{3.8}$$

*where $C_4$ is a constant value. Each of $n-k$ nodes randomly and uniformly selects any* *other node in the network as destination. Define i.i.d. random variable* $I_i$ *as*

$$I_i = \begin{cases} 1 & \text{If } L_i \text{ intersect } S \\ 0, & \text{Otherwise} \end{cases} \tag{3.9}$$

*where* $i = 1, 2, \cdots, n-k$. *It is clear from Eq. (3.8) that* $\Pr(I_i = 1) = p < C_4\sqrt{\frac{\log n}{n}}$. *Denote* $Z_n = \sum_{i=1}^{n-k} I_i$ *as the number of lines passing through the cell* $S$. *Thus for positive values of* $a$ *and* $m$ *and using Chernoff Bound, we have*

$$\Pr(Z_n > m) \leq \frac{\mathrm{E}\, e^{aZ_n}}{e^{am}}. \tag{3.10}$$

*Further, it can be shown that [17]*

$$\begin{aligned} \mathrm{E}\, e^{aZ_n} &= (1 + (e^a - 1)p)^{n-k} \\ &\leq \exp((n-k)(e^a - 1)p) \\ &\leq \exp(C_4(n-k)(e^a - 1)\sqrt{\frac{\log n}{n}}) \end{aligned} \tag{3.11}$$

*Let's define* $m = C_2(n-k)\sqrt{\frac{\log n}{n}}$, *then Eq. (3.10) becomes*

$$\begin{aligned} &\Pr(Z_n > C_2(n-k)\sqrt{\frac{\log n}{n}}) \\ &\leq \exp((n-k)\sqrt{\frac{\log n}{n}}(C_4(e^a - 1) - C_2 a)). \end{aligned} \tag{3.12}$$

*If we select $C_2$ such that $C_2 a - C_4(e^a - 1) = \epsilon > 0$, then*

$$\Pr(Z_n > C_2(n-k)\sqrt{\frac{\log n}{n}}) \leq \exp\left(-\epsilon(n-k)\sqrt{\frac{\log n}{n}}\right). \tag{3.13}$$

*If the area for each cell is defined as $s_n^2 = \Theta(\frac{\log n}{n})$, then by utilizing the union bound we arrive at*

$$\begin{aligned}
&\Pr(\textit{Some cells have more than } (n-k)\sqrt{\frac{\log n}{n}} \textit{ lines}) \\
&\leq \sum_{\textit{all the cells}} \Pr(Z_n > (n-k)\sqrt{\frac{\log n}{n}}) \\
&\leq \frac{1}{s_n^2}\exp(-\epsilon(n-k)\sqrt{\frac{\log n}{n}}) \\
&= \frac{n}{2C_1 \log n}\exp(-\epsilon(n-k)\sqrt{\frac{\log n}{n}}).
\end{aligned} \tag{3.14}$$

*This probability goes to zero as $n$ tends to infinity as long as $n - k \neq$ constant.*

### 3.2.4 The Lower Bound of the Capacity

#### 3.2.4.1 Case of $n - k \neq$ constant

From the previous two sections, we deduce that the number of lines passing through a cell with distance $x$ from the access node is upper bounded as $\frac{2d_n k}{x} + C_2(n-k)\sqrt{\frac{\log n}{n}}$ and for the cell that contains the access node is $k + C_2(n-k)\sqrt{\frac{\log n}{n}}$. In the traditional analysis of capacity with homogeneous traffic, the inverse of traffic for a cell using a TDMA scheme provides the throughput capacity. Given that this value varies for different cells in heterogeneous traffic, we assign a bandwidth to each cell that is proportional to the number of lines passing through the cell. This assignment is based on the fact that each link in the network has the same bandwidth (similar to the approach by Gupta and Kumar) but more allocation of bandwidth is given to a cell with higher traffic. Clearly, our results demonstrate that the cell that contains the access node has the highest traffic. If we divide the network into layers of cells starting from the access point as shown in Fig. 3.3, the traffic for cells in each layer is the same order. Let's assume the traffic for each layer is $T_i$ where $i = 1, ..., \Theta(\sqrt{\frac{n}{\log n}})$. Then our bandwidth requirement for each layer is given by

$$\frac{W_o}{T_o} = \frac{W_1}{T_1} = ... = \frac{W_{\Theta(\sqrt{\frac{n}{\log n}})}}{T_{\Theta(\sqrt{\frac{n}{\log n}})}} = c(n). \tag{3.15}$$

Note that $W_o = W_{\max}, T_o = T_{\max}$ and $c(n)$ is a pre-determined function of $n$. This assumption basically means that more bandwidth is provided to a cell with higher traffic.

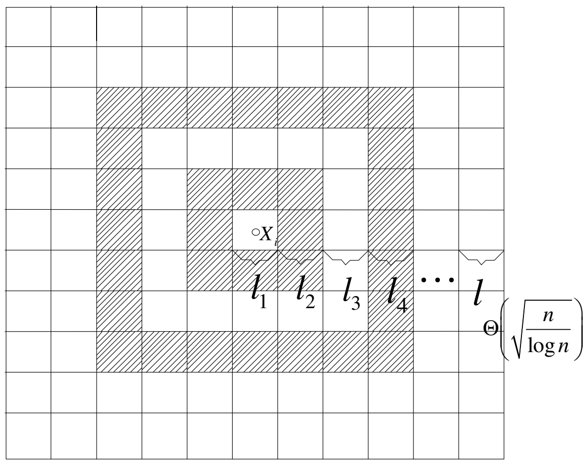


Figure 3.3: The layers around $X_i$

The average number of nodes in each cell is proportional to $\Theta(\log n)$, then the lower bound capacity is

$$
\begin{aligned}
C_{\text{lower}} &= \frac{1}{MW_{\max}} \left( \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} \frac{8lW_l}{T_l} + \frac{W_0}{T_0} \right) \cdot \Theta(\log n), \\
&= \frac{1}{MW_{\max}} \left( \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} 8lc(n) + c(n) \right) \cdot \Theta(\log n), \\
&= \frac{1}{MW_{\max}} \cdot \Theta\left( \frac{n}{\log n} + \sqrt{\frac{n}{\log n}} \right) \cdot \Theta(\log n) \cdot c(n), \\
&= \Omega(\frac{c(n)n}{W_{\max}}) = \Omega(\frac{n}{T_{\max}}),
\end{aligned}
\tag{3.16}
$$

where $M$ is the TDMA parameter that is required to separate cells in order to satisfy the protocol model.

Note that the capacity defined in this paper is the total capacity since the traffic for each node is different and per node capacity may not be meaningful.

#### 3.2.4.2 Case of $n - k =$ constant

Under this condition, clearly all the traffic is contributed by the access node and since each source is sending different packet to the access node, the achievable capacity is $\Omega(1)$ by allowing one source at the time to transmit its packet to the access node.

Combining the above results, we state the following theorem for the achievable lower bound.

**Theorem 3.7** *The achievable lower bound for a heterogeneous traffic with maximum number of traffic of $T_{max}$ for a cell can be given as follows.*

$$C_{lower} = \begin{cases} \Omega(\frac{n}{T_{max}}) & when\ n-k \neq C_5 \\ \Omega(1) & when\ n-k = C_5 \end{cases} \tag{3.17}$$

Note that Theorem 3.1 is proved only for $k = \Omega(\sqrt{\frac{n}{\log n}})$. When $k = O(\sqrt{\frac{n}{\log n}})$, the value of $T_{\max}$ is less than that of $k = \Omega(\sqrt{\frac{n}{\log n}})$. Hence, the maximum value of $T_{\max}$ for $k = \Omega(\sqrt{\frac{n}{\log n}})$ can be utilized for all values of $k$ for computation of the lower bound capacity.

## 3.3 The Upper Bound of the Capacity

We first compute the capacity for the case when $n-k \neq$ constant. The capacity can be defined as

$$\begin{aligned} C_{\text{upper}} &= \frac{\text{the sum of capacity for all cells}}{\text{the average\#of hops for source-destination pairs}} \\ &\times \frac{1}{\text{max. bandwidth expansion} \times \text{TDMA parameter}}. \end{aligned}$$

First, we consider the case when $k = \Omega\left(\sqrt{\frac{n}{\log n}}\right)$. It is easy to show that $x \geq (2l-1)\frac{\sqrt{2}d_n}{2}$ where $l$ varies from a constant value up to $\Theta(\sqrt{\frac{n}{\log n}})$ depending on the location of cell from the access node. From this lower bound for $x$, we can derive the upper bound for

$T_l$.

$$T_l < \begin{cases} \dfrac{2d_n k}{(2l-1)\frac{\sqrt{2}d_n}{2}} + C_2(n-k)\sqrt{\dfrac{\log n}{n}} & l \neq 0 \\ k + C_2(n-k)\sqrt{\dfrac{\log n}{n}} & l = 0 \end{cases} \tag{3.18}$$

Then the capacity can be derived as

$$
\begin{aligned}
C_{\text{upper}} =& \frac{1}{MW_{\max}} \cdot \left( \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} \frac{8lW_l}{\frac{L-o(1)}{r(n)}} + \frac{W_0}{\frac{L-o(1)}{r(n)}} \right) \\
\stackrel{a}{\leq}& \frac{1}{W_{\max}M(L-o(1))} r(n)c(n) \times \\
& \left[ \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} 8l \left( \frac{2d_n k}{(2l-1)\frac{\sqrt{2}d_n}{2}} + C_2(n-k)\sqrt{\frac{\log n}{n}} \right) \right. \\
& \left. + \left( k + C_2(n-k)\sqrt{\frac{\log n}{n}} \right) \right] \\
\leq& \frac{1}{W_{\max}M(L-o(1))} r(n)c(n) \left( 2\sqrt{2}k \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} \frac{8l}{2l-1} \right. \\
& \left. + k + C_2(n-k)\sqrt{\frac{\log n}{n}} \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} (8l+1) \right) \\
=& \frac{1}{W_{\max}M(L-o(1))} r(n)c(n) \left( 2\sqrt{2}k \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} \left(4 \right. \right. \\
& \left. + \frac{4}{2l-1} \right) + k + \\
& \left. C_2(n-k)\sqrt{\frac{\log n}{n}} \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} (8l+1) \right) \\
=& \frac{1}{W_{\max}M(L-o(1))} r(n)c(n) \left( 2\sqrt{2}k\Theta \left( \sqrt{\frac{n}{\log n}} \right. \right. \\
& \left. + \log \left( \sqrt{\frac{n}{\log n}} \right) \right) + k \\
& \left. + C_2(n-k)\sqrt{\frac{\log n}{n}} \Theta \left( \frac{n}{\log n} \right) \right) \\
=& \frac{1}{W_{\max}M(L-o(1))} r(n)c(n) \left( 2\sqrt{2}k\Theta \left( \sqrt{\frac{n}{\log n}} \right) \right. \\
& \left. + C_2(n-k)\Theta \left( \sqrt{\frac{n}{\log n}} \right) \right)
\end{aligned}
$$

$$
\begin{aligned}
&\stackrel{b}{=}\frac{1}{W_{\max}M(L-o(1))}c(n)\Theta(\sqrt{\frac{\log n}{n}}) \\
&\left(2\sqrt{2}k\Theta\left(\sqrt{\frac{n}{\log n}}\right)+C_2(n-k)\Theta\left(\sqrt{\frac{n}{\log n}}\right)\right) \\
&=\frac{1}{W_{\max}M(L-o(1))}c(n)\left(2\sqrt{2}k+C_2(n-k)\right) \\
&=\Theta(\frac{c(n)n}{W_{\max}})=\Theta(\frac{n}{T_{\max}}) \qquad (3.19)
\end{aligned}
$$

where $L-o(1)=\Theta(1)$ in this derivation is the average length of each unicast or the average length over all distances between $k$ sources and the access node, (a) is derived by replacing $W_l = T_l c(n)$, and (b) is derived by replacing $r(n)$ with $\Theta(\sqrt{\frac{\log n}{n}})$.

Second, we consider the case when $k = O\left(\sqrt{\frac{n}{\log n}}\right)$. We know from [2] that the number of lines crossing a cell for n source-destination pair is $\Theta(\sqrt{n\log n})$ and we have at most $k$ traffic for many-to-one traffic for access node. Therefore, it is clear that $T_l \le T_{\max} = \Theta(\sqrt{n\log n}) + k$. Now following similar procedure, we can derive the capacity as

$$
\begin{aligned}
C_{\text{upper}} &= \frac{1}{MW_{\max}}\left(\sum_{l=1}^{\Theta\left(\sqrt{\frac{n}{\log n}}\right)} \frac{8lW_l}{\frac{L-o(1)}{r(n)}} + \frac{W_0}{\frac{L-o(1)}{r(n)}}\right) \\
&\leq \frac{1}{W_{\max}M(L-o(1))} r(n)c(n)\times \\
&\left(T_{\max}\sum_{l=1}^{\Theta\left(\sqrt{\frac{n}{\log n}}\right)} 8l + T_0\right), \\
&= \frac{1}{W_{\max}M(L-o(1))} r(n)c(n)\times \\
&\left(T_{\max}\Theta\left(\frac{n}{\log n}\right) + T_{\max}\right), \\
&\overset{a}{=} \frac{1}{W_{\max}M(L-o(1))} r(n)c(n)\times \\
&\left(\Theta\left(\sqrt{n\log n}+k\right)\Theta\left(\frac{n}{\log n}\right)\right), \\
&\overset{b}{=} \frac{1}{W_{\max}M(L-o(1))}\Theta\left(\sqrt{\frac{\log n}{n}}\right)c(n)\times \\
&\left(\Theta\left(\sqrt{n\log n}+k\right)\Theta\left(\frac{n}{\log n}\right)\right), \\
&\overset{c}{\leq} \frac{1}{W_{\max}M(L-o(1))}\Theta\left(\sqrt{\frac{\log n}{n}}\right)c(n)\times \\
&\left(\Theta\left(\sqrt{n\log n}+\sqrt{\frac{n}{\log n}}\right)\Theta\left(\frac{n}{\log n}\right)\right) \\
&= \frac{1}{T_{\max}M(L-o(1))}\Theta\left(n+\frac{n}{\log n}\right) \\
&= \Theta\left(\frac{n}{T_{\max}}\right)
\end{aligned} \tag{3.20}
$$

(a) is derived by replacing $T_{\max}$ with its maximum value, (b) is computed by replacing

$r(n)$ with $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and (c) is obtained by replacing $k$ with its maximum value.

The case of $n-k$ =constant is straightforward since we can at most have one data sent to the access node when all the communications is dominated by the access node.

Finally, from the analysis of the lower and upper bounds, we derive the following tight bound for the capacity this network.

**Theorem 3.8** *In a random ad hoc network, under the heterogeneous traffic pattern with one node performing as the destination for $k$ source nodes and $n-k$ nodes have unicast communications, the overall capacity is*

$$C = \begin{cases} \Theta\left(\sqrt{\dfrac{n}{\log n}}\right), & k = O(\sqrt{n \log n}) \\ \Theta\left(\dfrac{n}{k}\right), & k = \Omega(\sqrt{n \log n}) \\ \Theta(1). & \textit{when } n-k = C_5 \end{cases} \tag{3.21}$$

**proof 3.9** *We have shown that the lower and upper bounds capacity of the network is $\Theta(\frac{n}{T_{max}})$. Further, it is clear that the maximum traffic is always inside the cell with access node, i.e., $T_{max} = k + C_2(n-k)\sqrt{\frac{\log n}{n}}$. It is easy to show that when $k = O(\sqrt{n \log n})$, then $T_{max} = \Theta(\sqrt{n \log n})$ and for $k = \Omega(\sqrt{n \log n})$, we have $T_{max} = k$. The proof is immediate by combining these results.*

## 3.4 Discussion

Fig. 3.4 shows the throughput capacity of a wireless network obtained from (3.21) as a function of the number of sources for the access node. As the number of the sources for this access node $k$ increases from 1 to $\Theta(\sqrt{n \log n})$, the capacity of the network is $\Theta(\sqrt{\frac{n}{\log n}})$ which is the well known result computed by Gupta and Kumar for homogeneous traffic model. We call this region as *Homogeneous Traffic* region. It is clear that the capacity of the network in this region is dominated by the uniform unicast traffic. Once the value of $k$ passes this threshold of $\Theta(\sqrt{n \log n})$, the capacity of the network is $\Theta(\frac{n}{k})$ which is smaller than the capacity of the *Homogeneous Traffic* region. The capacity of the network is dominated by the access node which is the bottleneck in the network and we call this capacity region as *Heterogeneous Traffic* region. This result implies that for the cells near the access node, we should assign more resources (bandwidth or time) to guarantee the data rate for each traffic. Finally if the number of sources for the access node is such that $n - k = C_5$, then the capacity is $\Theta(1)$ which is the same as broadcast transport capacity [10]. Since the number of sources is relatively large in this case, we call this capacity region as *All to One Traffic* region. We can see that almost all of the nodes have traffic for the access node, thus, for the extreme case that all the nodes have traffic to the access node, at each time, only one node can transmit.

Furthermore, the capacity we calculated is a normalized capacity by the maximum bandwidth. We can see without this normalization, the capacity of the network is

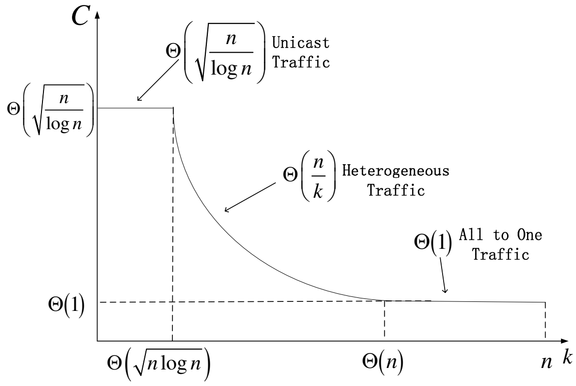


Figure 3.4: The capacity result

$nc(n)$ which is not related to $k$ (see Eqs. (16) and (19)). However, to achieve the same capacity for all nodes and for different values of $k$, we need to allocate more bandwidth to the more congested areas of the network. Fig. 3.5 demonstrates that in the *Homogenous Traffic* region, the maximum bandwidth needed is not related to $k$. However, in the *Heterogenous Traffic* region, the bandwidth grows linearly with $k$, which is the price for keeping the overall capacity the same. Finally, in the *All to One Traffic* region, the order of the maximum bandwidth does not change.

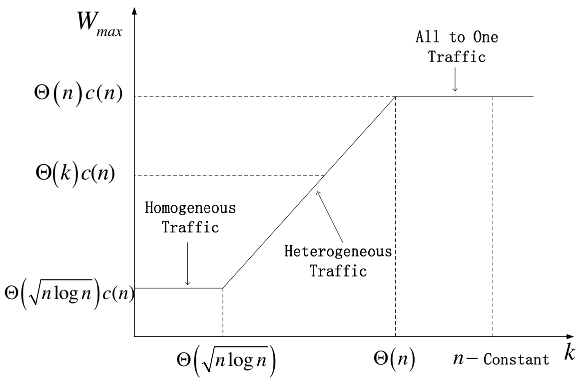


Figure 3.5: The maximum bandwidth required corresponding to different $k$

## 3.5 Conclusion

This chapter presented the first closed-form scaling laws for the capacity of wireless ad hoc networks with heterogeneous traffic. More specifically, we assumed an access node with $k$ sources choosing this node as destination and the rest of nodes in the network, having unicast communications. It was shown that the capacity of such heterogeneous network is $\Theta(\frac{n}{T_{\max}})$. Equivalently, our derivations reveal that, when $n-k \neq$ constant, then the capacity is equal to $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$ for $k = O(\sqrt{n \log n})$ and equal to $\Theta\left(\frac{n}{k}\right)$ for $k = \Omega(\sqrt{n \log n})$. Furthermore, when $n-k =$ constant, then the capacity is $\Theta(1)$. The results demonstrate that, as it should be expected, the capacity of a heterogeneous network is dominated by the maximum traffic (congestion) in any area of the network.

Notice that, in this work, we have an important assumption that the bandwidth of each node can be proportional to the traffic in that node or the corresponding cell. This may cause a significant waste of the bandwidth. In Chapter 6, we will present the result without this assumption.

# Chapter 4

# The Capacity of Wireless Ad-hoc Networks with Heterogeneous Traffic under Physical Model

In this chapter, we will give the capacity result in the wireless network with the same traffic type as in Chapter 3 but using physical model which is originally defined in *Definition* 2.4. However, according to the specific configuration of the network, we need to modify this definition. In this chapter, the wireless network model will be presented in Section 4.1. Then we will compute the upper bound and lower bound of the capacity respectively in Section 4.2 and 4.3.2. Finally, we give the discussion and conclusion of this work in Section 4.4 and 4.5. This work is presented in [18].

## 4.1 Wireless Network Model

Nodes are uniformly distributed in a dense network where the area of the network is a constant unit square. The heterogeneous traffic consists of data gathering traffic in which a single node (called the access node) is the destination for $k$ sources in the network. For the rest of the $n-k$ nodes in the network, we assume random and uniformly distributed source-destination pairs. Therefore, the source-destination pair selection for unicast communications is similar to that used by Gupta and Kumar [2]. This network model is shown in Figure 3.1.

The transmission range is the same for all the nodes and the communication between nodes is point-to-point. A successful communication between two nodes is modeled according to the physical model, which is modified according to our network configuration and defined below.

**Definition 4.1** *Physical Model: Let $\{X_k; k \in \mathcal{K}\}$ be the subset of nodes simultaneously transmitting at the same time over a certain subchannel. All nodes in this subchannel choose a common power level $P$ for all their transmissions. For each subchannel, the noise power is $N$. A node can transmit over several subchannels. A transmission from a node $X_i$, $i \in \mathcal{K}$, is successfully received by a node $X_{i(R)}$ if*

$$\frac{\frac{P}{|X_i - X_{i(R)}|^\alpha}}{N + \sum_{k \in \mathcal{K}, k \neq i} \frac{P}{|X_k - X_{k(R)}|^\alpha}} \geq \beta. \tag{4.1}$$

*for every subchannel.*

One important assumption of our analysis which is the same as that in Chapter 3 is that the bandwidth for each traffic is assumed to be the same, which means that the bandwidth for each node is proportional to its traffic. In another word, the fairness for each flow is guaranteed. Let's define bandwidth $W_i$ and traffic $T_i$ for cell $i$, then

$$\frac{W_i}{T_i} = c(n), \tag{4.2}$$

where $c(n)$ is a pre-determined function of $n$.

## 4.2 The Upper Bound of the Capacity

In this section, we compute the upper bound of the capacity. From [2] and [13], we know that the minimum transmission range under the physical model is $\Theta\left(\frac{1}{\sqrt{n}}\right)$. Therefore, the maximum number of hops for each source-destination pair is $\frac{L-o(1)}{r(n)} = \Theta(\sqrt{n})$. Note that there are at most $n$ source-destination pairs in the network. Thus, the total traffic is

$$\sum_l T_l = \Theta(n\sqrt{n}). \tag{4.3}$$

We know that each transmission consumes a disk of radius $\Theta(r(n))$ and these disks are disjoint. Note that all the traffic are carried by these disjoint disks and the bandwidth distributed to each cell is proportional to the traffic. Therefore, the upper bound of the capacity is given by

$$
\begin{aligned}
C_{\text{upper}} &= \frac{\text{the sum of traffic for all nodes}}{\text{the average number of hops for source-destination pairs}} \\
&\times \frac{1}{\text{maximum bandwidth expansion}}, \\
&= \frac{1}{W_{\max}} \cdot \frac{\sum_{l=1}^{n} W_l}{\frac{L-o(1)}{r(n)}} = \frac{1}{W_{\max}} \cdot \frac{\sum_{l=1}^{n} T_l c(n)}{\frac{L-o(1)}{r(n)}}, \\
&= \frac{1}{T_{\max} c(n)} \cdot \frac{\Theta(n\sqrt{n}c(n))}{\frac{L-o(1)}{\frac{1}{\sqrt{n}}}} = \Theta\left(\frac{n}{T_{\max}}\right),
\end{aligned} \tag{4.4}
$$

where $W_{\max}$ is the maximum bandwidth and $T_{\max}$ is the maximum traffic for a cell in the network.

## 4.3 The Lower Bound of the Capacity

For the lower bound of the capacity we need to emphasize that there are two types of traffic in our model. One traffic is associated to the $k$ sources transmitting packets to the access node and the other traffic stems from the rest of $n-k$ nodes in the network with unicast communications. Therefore, the routing protocol and scheduling are defined under this traffic model.

### 4.3.1 The Routing Scheme and the Scheduling Protocol

The selection of sources for the access node $i$ is based on the technique described in [16]. We randomly and uniformly select $k$ locations in the network and choose the closest nodes to these $k$ locations as sources for the access node. The routing trajectory is a straight line $L_i$ from access node to these $k$ locations. Then the packets traverse from each source to destination in a multi-hop fashion passing through all the cells that cross $L_i$. The side length of each cell $d_n$ is selected as $\Theta(r(n)) = C_1\sqrt{\frac{\log n}{n}}$ [2], where $C_1$ is a positive constant. For the rest of $j$ nodes with unicast traffic where $1 \leq j \leq n-k$, both selections of source-destination pairs and routing is similar to the above technique.

For the scheduling scheme, we utilize a TDMA scheme similar to [16] with some modifications to take into account the heterogeneity of the traffic.

### 4.3.2 The Lower Bound of the Capacity

For the lower bound of the capacity, we will introduce a specific network structure which divides the network into square cells. To guarantee the connectivity in the network, the side length of each cell is chosen as $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$. We will show that the lower bound of the capacity is still $\Theta\left(\frac{n}{T_{\max}}\right)$.

#### 4.3.2.1 Case of $n-k = \Omega\left(\sqrt{n\log n}\right)$

From [15], it can be deduced that the number of lines passing through a cell with distance $x$ from the access node is upper bounded as

$$T_l < \begin{cases} \dfrac{2d_n k}{(2l-1)\frac{\sqrt{2}d_n}{2}} + C_2(n-k)\sqrt{\dfrac{\log n}{n}} & l \neq 0 \\ k + C_2(n-k)\sqrt{\dfrac{\log n}{n}} & l = 0 \end{cases} \tag{4.5}$$

In the traditional analysis of capacity with homogeneous traffic, the inverse of traffic for a cell using a TDMA scheme provides the throughput capacity. Given that this value varies for different cells in heterogeneous traffic, as mentioned before, we assign a bandwidth to the cell that is proportional to the number of lines passing through a cell. This assignment is based on the fact that each link in the network has the same bandwidth (similar to the approach by Gupta and Kumar) but more allocation of bandwidth is given to a cell with higher traffic. Clearly, our results demonstrate that the cell that contains the access node has the highest traffic. If we divide the network into layers of cells starting from the access node as shown in Fig. 3.3, the traffic for cells in each layer is the same order. Let's assume the traffic for each layer is $T_i$ where $i = 1, ..., \Theta(\sqrt{\frac{n}{\log n}})$. Then our bandwidth requirement for each layer is given by

$$\frac{W_0}{T_0} = \frac{W_1}{T_1} = ... = \frac{W_{\Theta(\sqrt{\frac{n}{\log n}})}}{T_{\Theta(\sqrt{\frac{n}{\log n}})}} = c(n). \tag{4.6}$$

Note that $W_0 = W_{\max}, T_o = T_{\max}$. This assumption basically means that more bandwidth is provided to a cell with higher traffic[1].

For the Physical Model, it is important to show that under the schedule given in Section 4.3.1, the required SINR threshold $\beta$ can be guaranteed. We can consider that all the interference comes from cells that are active at the same time. It is obvious to see that there are at most $8k$ interfering cells from the $k$th layer of the network. Moreover, the distance from an interfering cell is at least $k\sqrt{M}s_n - s_n$, where $M$ is the

[1] The bandwidth allocation in this paper is based on the common definition of throughput capacity that is utilized in literature. Under this assumption, the achievable througput capacity is based the fact that all the nodes in the network achieve the same rate. However, if one changes this definition of capacity and allows different nodes to have different throughput capacity, then the bandwidth allocation should accordingly changes in order to achieve the highest possible throughput.

number of non-interference groups and $s_n$ is the side length of each cell.

Thereafter, for each specific node $i$, we can calculate a lower bound on the achieved SINR as shown below.

$$\frac{\frac{P}{|X_i - X_{i(R)}|^\alpha}}{N + \sum_{k\in\mathcal{K},k\neq i}\frac{P}{|X_k - X_{i(R)}|^\alpha}} \overset{(a)}{\geq} \frac{\frac{P}{(2\sqrt{2}s_n)^\alpha}}{N + \sum_{k=1}^{\infty} 8k\frac{P}{(k\sqrt{M}s_n - s_n)^\alpha}}$$
$$= \frac{\frac{P}{(2\sqrt{2})^\alpha}}{Ns_n^\alpha + \frac{8P}{M^{\frac{\alpha}{2}}}\sum_{k=1}^{\infty}\frac{1}{(k-\frac{1}{\sqrt{M}})^\alpha}} \tag{4.7}$$

Figure 4.1 shows the relationship between traffic in a cell and allocated bandwidth as described in Eq. (4.6). Since each layer of cells has different bandwidth requirement, therefore only portion of the transmitted signal in a layer will interfere with adjacent cells. For example, when $T_k \geq T_i$, the interfering portion of bandwidth for the cells in layer $i$ from cells in layer $k$ is at most $T_i$. Similarly, when $T_k < T_i$, the interfering bandwidth for the cells in layer $i$ from cells in layer $k$ is at most $T_k$. So for each subchannel, the interference may come from part of every layer of the network that is active at the same time. Since in the inequality (a) in Eq. (4.7), we calculate the entire signal power while only portion of it may interfere with $i$, then this value is the lower bound.

The summation in the denominator converges to a constant value when $\alpha > 2$

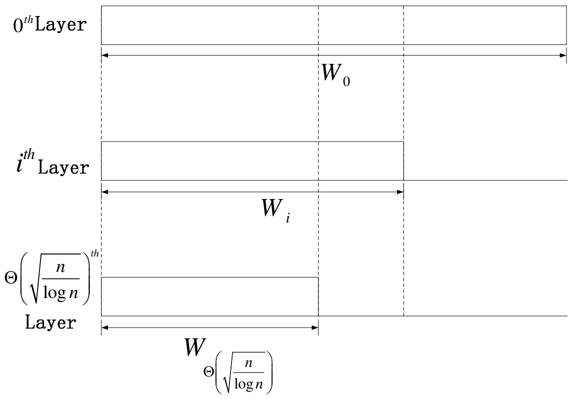


Figure 4.1: The distribution of the bandwidth in different cell layers of the network.

as described below.

$$\begin{aligned}
&\sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{M}})^{\alpha}} \\
&= \sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{M}})^{\alpha-1}} + \frac{1}{\sqrt{M}} \sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{M}})^{\alpha-1}} \\
&\leq \frac{1}{(k-\frac{1}{\sqrt{M}})^{\alpha-1}} + \int_{1-\frac{1}{\sqrt{M}}}^{\infty} \frac{1}{x^{\alpha-1}} \\
&+ \frac{1}{\sqrt{M}} \left( \frac{1}{(k-\frac{1}{\sqrt{M}})^{\alpha}} + \int_{1-\frac{1}{\sqrt{M}}}^{\infty} \frac{1}{x^{\alpha}} \right) \\
&= \frac{1}{(1-\frac{1}{\sqrt{M}})^{\alpha-1}} + \frac{\left(1-\frac{1}{\sqrt{M}}\right)^{-(\alpha-2)}}{\alpha-2} \\
&+ \frac{1}{\sqrt{M}} \frac{1}{(1-\frac{1}{\sqrt{M}})^{\alpha}} + \frac{1}{\sqrt{M}} \frac{\left(1-\frac{1}{\sqrt{M}}\right)^{-(\alpha-1)}}{\alpha-1} \\
&= \text{constant}
\end{aligned} \tag{4.8}$$

It is clear from these results that when $M$ is sufficiently large, then the SINR of an arbitrary subchannel can be made larger than the specific threshold $\beta$ to satisfy the *Physical Model* (4.1).

The average number of nodes in each cell is proportional to $\Theta(\log n)$, then the lower bound capacity is

$$\begin{aligned}
C_{\text{lower}} &= \frac{1}{MW_{\max}} \left( \sum_{l=1}^{\Theta(\sqrt{\frac{n}{\log n}})} \frac{8lW_l}{T_l} + \frac{W_0}{T_0} \right) \cdot \Theta(\log n), \\
&= \frac{1}{MW_{\max}} \left( \sum_{l=0}^{\Theta(\sqrt{\frac{n}{\log n}})} 8lc(n) + c(n) \right) \cdot \Theta(\log n), \\
&= \frac{1}{MW_{\max}} \cdot \Theta(\frac{n}{\log n} + \sqrt{\frac{n}{\log n}}) \cdot \Theta(\log n) \cdot c(n), \\
&= \Omega(\frac{c(n)n}{W_{\max}}) = \Omega(\frac{n}{T_{\max}}),
\end{aligned} \tag{4.9}$$

where $M$ is the TDMA parameter that is required to separate cells in order to satisfy the physical model.

Note that the capacity defined in this paper is the total capacity since the traffic for each node is different and per node capacity may not be meaningful.

#### 4.3.2.2 Case of $n - k = o\left(\sqrt{n \log n}\right)$

Under this condition, clearly most of the traffic is contributed by the access node and since each source is sending different packet to the access node, the achievable capacity is $\Omega(1)$ by allowing one source at the time to transmit its packet to the access node.

Combining the above results, we state the following theorem for the achievable lower bound.

**Theorem 4.2** *The achievable lower bound for a heterogeneous traffic with maximum number of traffic of $T_{max}$ in a cell can be given as follows.*

$$C_{lower} = \begin{cases} \Omega(\frac{n}{T_{max}}) & when\ n-k = \Omega\left(\sqrt{n\log n}\right) \\ \\ \Omega(1) & when\ n-k = o\left(\sqrt{n\log n}\right) \end{cases} \tag{4.10}$$

Finally, from the analysis above, we derive a tight bound for the capacity.

**Theorem 4.3** *In a random ad hoc network, under the heterogeneous traffic pattern with one node performing as the destination for $k$ source nodes and other nodes have unicast communications, the overall capacity is*

$$C = \Theta\left(\frac{n}{T_{max}}\right) \tag{4.11}$$

## 4.4 Discussion

Eq.(4.5) provides the value of $T_{\max}$ when $k$ is small value compared to $n$. But when $k$ is a large number, i.e., $n - k = o\left(\sqrt{n\log n}\right)$, then the dominant traffic in the network is the data gathering traffic and for computation of $T_{\max}$, one can ignore the contribution of unicast traffic. Under this assumption, then the data gathering traffic provides the maximum traffic for the access node, i.e., $T_{\max} = k$. Thus, Eq. (4.11) becomes

$$C = \begin{cases} \Theta\left(\sqrt{\dfrac{n}{\log n}}\right), & k = O(\sqrt{n \log n}) \\ \Theta\left(\dfrac{n}{k}\right), & k = \Omega(\sqrt{n \log n}) \end{cases} \tag{4.12}$$

Fig. 4.2 shows the throughput capacity of a wireless network obtained from (4.12) as a function of the number of sources for the access node $k$. Similarly as Chapter 3, when $k$ increases from 1 to $\Theta(\sqrt{n \log n})$, the capacity of the network is dominated by the unicast traffic and it is equal to the well known result computed by Gupta and Kumar for unicast communications as $\Theta(\sqrt{\frac{n}{\log n}})$. This region is called unicast region. Once the value of $k$ passes this threshold of $\Theta(\sqrt{n \log n})$, the capacity of the network is equal to $\Theta(\frac{n}{k})$ and it is affected by both the unicast and data gathering traffics. We call this capacity region as *Heterogeneous Traffic* region. This result implies that for the cells near the access node, we should assign more resources (bandwidth or time) to guarantee the data rate for each traffic. Finally when $k = \Theta(n)$, then the capacity is $\Theta(1)$ which is the same as broadcast transport capacity [10]. Since the number of sources is relatively large in this case, we call this capacity region as *All to One Traffic* region. We can see that almost all of the nodes have traffic for the access node, thus, for the extreme case that all the nodes have traffic to the access node, at each time, only one node can transmit.

The nodes with higher traffic consume more power for transmission of information. Our goal is to demonstrate the relationship between $k$ and maximum required power. From (4.7), it is easy to observe that the minimum transmit power $P$ for each

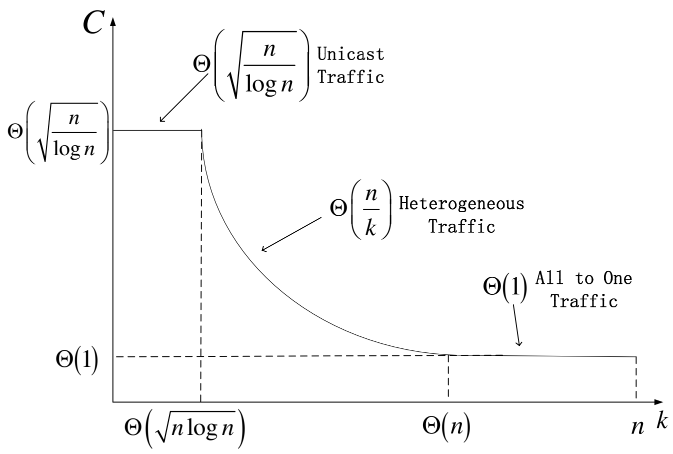


Figure 4.2: The capacity result

subchannel to guarantee the SINR $\geq \beta$ condition is

$$P_{\min} = \Theta(s_n^{\alpha}) = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}\right). \tag{4.13}$$

Thus, the maximum required power is

$$P_{\max} = P_{\min}W_{\max} = P_{\min}T_{\max}c(n). \tag{4.14}$$

Combining the above result with Eq. (4.5), we arrive at

$$P_{\max} = \begin{cases} \Theta\left(\frac{(\log n)^{\frac{\alpha}{2}+\frac{1}{2}}}{n^{\frac{\alpha}{2}-\frac{1}{2}}}\right)c(n), & k = O(\sqrt{n\log n}) \\ \Theta\left(\frac{(\log n)^{\frac{\alpha}{2}}}{n^{\frac{\alpha}{2}}}\right)kc(n), & \Omega(\sqrt{n\log n}) = k = O(n) \\ \Theta\left(\frac{(\log n)^{\frac{\alpha}{2}}}{n^{\frac{\alpha}{2}-1}}\right)c(n). & k = \Theta(n) \end{cases}, \tag{4.15}$$

Figure 4.3 shows the order of the maximum power as a function of $k$. It is clear that the node with the maximum required power is the access node since it carried more traffic than any other node in the network. In the unicast traffic region, the order of the maximum power for the access node is not growing because the unicast traffic is the dominant traffic. In the homogeneous traffic region, as $k$ increases, the traffic for the access node increases and accordingly, this node requires more transmit power. In the final region of all to one traffic, the traffic in the network is dominated by the data gathering scheme and the access node carries majority of the traffic in the network. The maximum transmit power is achieved in this region because the traffic for the access node has reached its order upper bound traffic. These results imply that if the traffic for the access node is restricted with $k = O(\sqrt{n \log n})$, then the optimal power consumption for the access node can be attained.

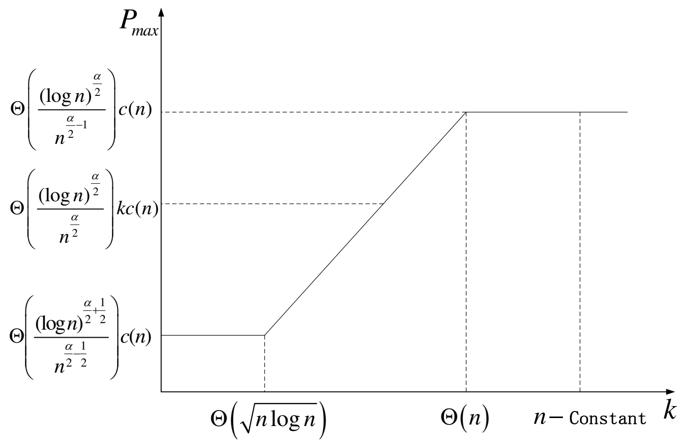


Figure 4.3: The Growth of Power as a function of $k$

## 4.5 Conclusion

This chapter presents a closed-form scaling law for the capacity of wireless ad hoc networks with heterogeneous traffic under physical model. More specifically, a combination of unicast communications and data gathering has been chosen for this paper. It is shown that the capacity of such heterogeneous network is $\Theta(\frac{n}{T_{\max}})$. Further, the capacity is equal to $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$ for $k = O(\sqrt{n \log n})$ and equal to $\Theta\left(\frac{n}{k}\right)$ for $k = \Omega(\sqrt{n \log n})$. The results confirms our intuition that the capacity of a heterogeneous network is dominated by the maximum traffic (congestion) in any area of the network.

# Chapter 5

# The Capacity of Wireless Ad-hoc Networks with Heterogeneous Traffic under Information Theoretical Model

In Chapter 3 and 4, we presented the capacity results of wireless network with a specific heterogeneous traffic type by using the protocol model and physical model. However, both of the protocol model and physical model are a simplified version of the information theoretical model. This means that it is too difficult to solve this problem by using information theoretical model so we have to seek some other simpler problem that we can solve. In this chapter, we try to solve the same network capacity problem as stated in Chapter 3 and 4 but using the original information theoretical approach without any constraint on the successful transmission condition.

This chapter is organized as follows. In Section 5.1, we will give the assumptions and wireless network model. Then we will give the first important result in this work in Section 5.2 which is the *Separation Theorem*. In Section 5.3, the upper bound of the capacity is calculated. The achievable aggregate throughput of the network will be computed in Section 5.4. Then we will discuss the result in Section 5.5. This work is published in [19].

## 5.1 Wireless Network Model

In this section, we provide the wireless network models. For different transmission schemes, we use the differently popular wireless network models which are the information theoretical model and the protocol model. We consider a network with nodes uniformly distributed in a dense network with constant area $A$. The bandwidth of the network is assumed to be a constant $W$. For the data-gathering traffic, a single node, called the access point or access node, is the destination for $k$ sources in the network. For the rest of the $s = n - k$ nodes in the network, source-destination pairs are selected randomly and uniformly. This network model is shown in Fig 3.1.

Notice that both $s$ and $k$ can be a function of $n$.

To get the actual aggregate throughput of the network, we use the information theoretical model to study this network which means that the actual information that can be transmitted in the network is calculated and we do not have any specific assumptions on the successful transmission condition.

For each source node $s_i$, where $i = 1, \cdots, n$, the data rate is denoted by $R_{s_i}(n)$.

Similarly, for each destination node $d_j$, where $j = 1, \cdots, s+1$, the rate is denoted by $R_{d_j}(n)$. The capacity of the network is defined as

$$C(n) = \max_{R_{s_i} \text{ and } R_{d_j}; i=1,\cdot,n; j=1,\cdot,s+1} \left( \min \left( \sum_{i=1}^{n} R_{s_i}(n), \sum_{j=1}^{s+1} R_{d_j}(n) \right) \right) \tag{5.1}$$

The total bandwidth in the network is $W$ hertz. Furthermore, the total power for transmission in the network is assumed to be $P$. The complex channel gain between nodes $i$ and $k$ at time $m$ is given by

$$H_{ik}[m] = \frac{\sqrt{G} \exp(j\theta_{ik}[m])}{d_{ik}^{\frac{\alpha}{2}}}, \tag{5.2}$$

where $d_{ik}$ is the distance between nodes $i$ and $k$, $\theta_{ik}[m]$ is the random phase at time $m$ which is uniformly distributed between $[0, 2\pi]$. For all pairs of $i$ and $k$, $\theta_{ik}[m]s$ are independent and identically distributed (i.i.d.) random variables. Note that $\theta_{ik}[m]$ and $d_{ik}$ are assumed to be independent. The parameters $G$ and $\alpha > 2$ are constants. The channel parameter is always known to the receiver during transmission and the phase is a fast fading[1]. The received signal by node $i$ at time $m$ is given by

$$Y_i = \sum_{k=1}^{n} H_{ik}[m] X_k[m] + Z_i[m], \tag{5.3}$$

[1]Our channel assumption is identical to the one used by Özgür et al. [1] and the reader can read the detailed justification for this channel model in [1].

where $X_k[m]$ is the transmitted signal by node $k$ at time $m$ and $Z_i[m]$ is white circularly symmetric Gaussian noise of variance $N_0$. The notations $K_i$ and $C_i$ for any integer $i$ represent constant values.

## 5.2 Separation Theorem

We first provide a *separation theorem* for heterogeneous traffic in wireless networks. Then, the achievable rate for this network model is presented.

**Theorem 5.1** Separation Theorem for Heterogeneous Traffic in Wireless Networks: *Consider a network with total available bandwidth $W$ in which $k_1$ traffic classes of equal priority are distributed uniformly in the network with each traffic class utilizing $\frac{W}{k_1}$ bandwidth. Separating the traffic classes and using a separate frequency and radio per node for each traffic class provides the optimum aggregate order capacity for the network.*

**proof 5.2** *Let $n$ be the total number of nodes in the network and for each traffic class $T_i, i = 1, \ldots, k_1$. We prove this theorem by induction.*

- *First, we assume that the types of traffic classes are $T_i, i = 1, \ldots, z$, where $z < k_1$. In this case, the optimum order of capacity can be achieved by using the separation theorem. Then, we will investigate capacity when the number of traffic classes is $z + 1$.*

  *Firstly, we consider two different networks each of which only contains one traffic class. One is the network containing traffic classes $T_i, i = 1, \ldots, z$ with bandwidth*

*$W$. By using the separation theorem, we can get the optimum order capacity is $\Theta(C_z)$. The other is the network containing traffic class $T_{z+1}$ with bandwidth $W$, by using the optimum transmission scheme, the maximum capacity we can get is $C_{z+1}$. Thus, from the order of the capacity point of view, since the optimum transmission schemes are used in both of the networks, when we combine the traffic classes in both networks into one network, which means we consider the network containing traffic classes $T_i, i = 1, \ldots, z+1$ with bandwidth $W$, the optimum order capacity we can get is at most $\Theta(C_z) + C_{z+1}$.*

*Secondly, we will use the separation theorem into the network containing the traffic classes $T_i, i = 1, \ldots, z+1$ with bandwidth $W$. When the number of traffic classes is $z$, the bandwidth for each traffic is $W_z = \frac{W}{z}$ according to the separation theorem. When the number of traffic classes is $z + 1$, the bandwidth for each traffic is $W_{z+1} = \frac{W}{z+1}$. We can find that $W_{z+1} = \frac{z}{z+1} W_z$. Because $z$ is a constant unrelated to $n$, the order of the bandwidth for each traffic is not changed. Now we separate the network into two parts. One is the network $N_z$ containing the traffic classes $T_i, i = 1, \ldots, z$ with bandwidth $zW_{z+1} = \frac{z^2}{z+1} W_z = \frac{z}{z+1} W$. The other is the network $N_{z+1}$ including the traffic classes $T_{z+1}$ with bandwidth $\frac{W}{z+1}$. By using the separation theorem, the optimum order capacity for network $N_z$ is $\Theta(C'_z)$ which is equal to $\Theta(C_z)$ since the order of the bandwidth is not changed. By using the optimum transmission scheme, the optimum capacity for the network $N_{z+1}$ is $C'_{z+1}$ which has the same order as $C_{z+1}$ still since the order of the bandwidth is not changed. Thus, in the network containing all the traffic with bandwidth $W$,* *the capacity by using separation theorem is* $\Theta(C'_z) + C'_{z+1} = \Theta(C_z) + C_{z+1}$ *which is the optimum order of the capacity for the network.*

- *Second, we prove the case that* $z = 2$*, which is the simplest case of the network with heterogeneous traffic. We assume that the two types of traffic are* $T_1$ *and* $T_2$*. Firstly, we consider the network only with traffic* $T_1$*. In this network, by operating the optimum transmission scheme, when the available bandwidth is* $W$*, we can get the capacity* $C_1$ *for this network. When the available bandwidth is* $\frac{W}{2}$*, the capacity of this network is* $C'_1$*. We can easily get* $\Theta(C_1) = \Theta(C'_1)$*. Then, we consider the network only with traffic* $T_2$*. Similarly, we can get that, by using the optimum transmission scheme, when the bandwidth is* $W$*, the capacity we can get is* $C_2$ *and when the bandwidth is* $\frac{W}{2}$*, the capacity we can get is* $C'_2$ *and* $\Theta(C_2) = \Theta(C'_2)$*. Since* $C_1$ *and* $C_2$ *are the capacity for these two different networks with different traffic, thus the optimum order of the capacity we can get in the network with both types of traffic is at most* $\Theta(C_1) + \Theta(C_2) = \Theta(C_1 + C_2)$*. If we use the separation scheme, the capacity we can get is* $\Theta(C'_1) + \Theta(C'_2) = \Theta(C_1) + \Theta(C_2) = \Theta(C_1 + C_2)$ *which is the optimum order capacity. Thus, we prove that when there are only two types of traffic in the network, the separation theorem can give us the optimum order of the capacity.*

*Thus, from above proof, we can get the* separation theorem *can give us the optimum order of the capacity for the network.*

This simple theorem provides a significant result for heterogeneous traffic networks. The theorem states that, by utilizing MC-MR systems, the nodes in the network that are not sources or destinations of a particular traffic class can be used as relays to improve the capacity of the network. We will show some examples of this intuitive result in this paper.

Another important implication of the above theorem is the fact that all analysis for homogeneous traffic can be used for heterogeneous traffic, as long as we allow some nodes in the network to participate as relays. Then the capacity of the network can be computed for that particular traffic pattern by changing the number of relays.

## 5.3 An Upper Bound on The Network Capacity

The information-theoretic upper bound of the aggregate throughput in wireless networks is derived. This upper bound is compared subsequently with the achievable lower bound by using MIMO cooperative scheme to demonstrate the effectiveness of the routing strategies utilized in this paper.

**Theorem 5.3** *In a wireless network with $k$ sources for the access node and $s$ source-destination pairs where $s + k = n$, the capacity is upper bounded by*

$$C(n) \leq K_1(s+1)\log n \tag{5.4}$$

*where $K_1$ is a constant.*

**proof 5.4** *It is clear from Eq. (6.12) that*

$$C(n) \leq \max_{R_{d_j};j=1,\cdot,s+1} \left( \sum_{j=1}^{s+1} R_{d_j}(n) \right). \tag{5.5}$$

*For any arbitrary destination $j$, the network is equivalent to a multiple-input single-output channel with upper bound given by ( [20], Eq. (5.31))*

$$R_{d_j}(n) \leq \log \left( 1 + \frac{P}{N_0} \sum_{k=1,k\neq j}^{k=n} \frac{G}{d_{kj}^{\alpha}} \right). \tag{5.6}$$

*Özgür et al. [1] showed that*

$$\Pr\left( d_{min} < \frac{1}{n^{1+\delta}} \right) \leq n \left( 1 - \left( 1 - \frac{\pi}{n^{2+2\delta}} \right)^{n-1} \right). \tag{5.7}$$

*This probability goes to zero as n tends to infinity, which means that the distance between any two nodes is at least $\frac{1}{n^{1+\delta}}$. Accordingly, Eq. (8.9) becomes*

$$\begin{aligned} C(n) &\leq (s+1) \log \left( 1 + \frac{PG}{N_0} n^{\alpha(1+\delta)+1} \right) \\ &= K_1(s+1) \log n \end{aligned} \tag{5.8}$$

From *Theorem* 5.3, we observe that the upper bound of the capacity scales as $\Theta((s+1)\log n)$.

## 5.4 Main Results by Using MIMO Cooperative Transmission Scheme

The following theorem establishes one of the main contributions of this paper, and the next two sections is dedicated to proving this theorem.

**Theorem 5.5** *Consider a network with one access node receiving information from $k$ sources and $s$ different nodes that select random destinations uniformly from all other nodes in the network. By using the MC-MR scheme, the achievable aggregate throughput* *is given by*

$$
R(n)_{Co} =
\begin{cases}
\Omega((s+1)\log n), & s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right), \\
\Omega\left(\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}} + \log n\right), & \Omega\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s \\
& = O\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right), \\
\Omega\left(\frac{s\log\log s}{\log s} + \log n\right), & \Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s \\
& = O\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right), \\
\Omega\left(\frac{n}{s} + \log n\right), & \Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right) = s \\
& = O\left(n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}}\right), \\
\Omega\left(s^{1-\varepsilon_1+\varepsilon_4} + \log n\right), & \Omega\left(n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}}\right) = s \\
& = O(n)
\end{cases}
\tag{5.9}
$$

*where $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, and $\varepsilon_4$ are positive small numbers between* $0$ *and* $1$ *and* $\varepsilon_4 < \varepsilon_1$.

Note that there are two terms corresponding to unicast and data gathering communications for each capacity region. For example, in the first capacity region of Eq. (5.9), the terms $s\log n$ and $\log n$ are related to unicast and data-gathering communication, respectively. It can be easily shown that the per-node throughput capacity of the unicast communication for our technique is always greater than that of [1], while the total unicast capacity is smaller than that of [1], because there are only

$s < n$ unicast pairs in our scheme. The reason for this capacity improvement is the use of relays in our scheme. We also note that in [1] all nodes were participating in unicast communication, while in [8], there are few unicast sessions and the rest of the nodes are relays. One of the results in this paper is the computation of the throughput capacity using MIMO cooperation when the number of relays in the network changes as a function of $n$. We also observe that, for the first capacity region, the throughput capacity is equal to the upper bound capacity that was derived in Section 5.3. This is the first paper to report per node throughput of $\log n$ for unicast communications.

In the following analysis of the achievable throughput for the network with heterogeneous traffic, we divide the bandwidth into $W_1$ and $W_2$ where $W_1 + W_2 = W$ and distribute bandwidth $W_1$ to the network with unicast traffic and $W_2$ to the data-gathering traffic. Moreover, we assume that the total power in the network is a positive constant $P$. we distribute $P_1$ to the network of unicast traffic and $P_2$ to the network with data-gathering traffic, where $P_1 + P_2 = P$.

### 5.4.1 Capacity Analysis for Unicast Traffic

Based on the *separation theorem*, it is sufficient to derive the throughput capacity for each traffic class independently without being concerned about the optimality of our result. Our main approach for the computation of unicast traffic capacity is based on the hierarchical MIMO cooperation approach introduced by Özgür et al. [1]. However, given that we take advantage of relays in this paper, we modify the *Three-Phase* scheme in [1] based on the number of relays available in the network in order to maximize the achievable capacity. The details of these schemes are described in the rest of this section.

#### 5.4.1.1 The Case of $s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$

First, we introduce a useful lemma from [7].

**Lemma 5.6** *Let $B(m,n)$ be the random variable that counts the maximum number of balls in any bin when we throw $m$ balls independently and uniformly at random into $n$ bins. Then*

$$B(m,n) = \begin{cases} \Theta\left(\dfrac{\log n}{\log \frac{n}{m}}\right), & \text{if } m < \dfrac{n}{\log n}, \\ \Theta\left(\dfrac{\log n}{\log \frac{n \log n}{m}}\right), & \text{if } \dfrac{n}{\textit{poly}\log n} \leq m \ll n \log n, \\ \Theta\left(\log n\right), & \text{if } m = c \cdot n \log n \\ & \text{for some constant } c, \\ \Theta\left(\dfrac{m}{n}\right), & \text{if } m \gg n \log n. \end{cases} \tag{5.10}$$

By dividing the network into $s^{1+\varepsilon_2}$ clusters and using *Lemma* 5.6, the following theorem can be proved.

**Theorem 5.7** *Consider a network with $n$ nodes and $s$ source nodes distributed uniformly in the network such that $s = n^{\alpha_1}$ and $0 \leq \alpha_1 < \frac{1}{1+\varepsilon_2+\varepsilon_3}$. If the network is divided into $s^{1+\varepsilon_2}$ clusters, there are at most $\Theta(1)$ source nodes and $\Theta\left(\frac{n}{s^{1+\varepsilon_2}}\right)$ nodes w.h.p. in each cluster. In any circle with radius $\frac{\sqrt{A}}{2s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}$ or $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$, where $0 < \beta_1 < 1$, there are $\Theta\left(\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}\right)$ or $\Theta\left(\frac{n}{n^{(1+\varepsilon_2)\beta_1}}\right)$ nodes, respectively w.h.p.*

The proof of this theorem is given in Appendix 8.1. To simplify the analysis, we assume that there are exactly $K_2 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}$ nodes in each circle with radius $\frac{\sqrt{A}}{2s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}$, $K_2 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}$ nodes in each circle with radius $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$, and $K_3$ source nodes in each cluster. We now introduce our *Three-Phase* communication scheme for each capacity region. Without loss of generality, the source nodes are considered at the center of each cluster to simplify the analysis. The total transmit power required to transmit all unicast traffic is $P_1$ Watts and for each phase is $P_1' = \frac{1}{3}P_1$.

**Phase 1. Distribution of packets from source to relays in the same cluster**: As in previous work [1], we divide the entire network into smaller cells or clusters of square shape. If the network area is divided into $s^{1+\varepsilon_2}$ clusters, then each cluster has an area of $\frac{A}{s^{1+\varepsilon_2}}$. In order to avoid interference, the cells are grouped into $K_6$ non-interfering groups using a TDMA scheme as shown in Fig. 5.1. We divide this region into two regions of $s = O\left(n^{\beta_1}\right)$ and $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$ where $\beta_1$ is an arbitrarily small positive constant number. Note that there are at most $K_3$ sources in each cluster. When $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, then we let $K_2 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}$ nodes in the circle of radius $\frac{\sqrt{A}}{2s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}$ help each source in the cluster to transmit information as shown in Fig. 5.2. For $s = O\left(n^{\beta_1}\right)$, it is easy to show that $K_2 \frac{n}{s^{1+\varepsilon_2}} > K_2 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}$. Therefore, only $K_2 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}$ relay nodes in the circle of radius $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$ help each source in the cluster to transmit information. These nodes operate as *relays* in the network. Each source node transmits $K_2 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}$ (or $K_2 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}$)blocks of information based on the size of $s$ to the relays in its cluster. Each block has a length of $L$. At the end of phase 1, each relay in the circle has received a different block of information. The next

Theorem describes the total aggregate throughput in the network.

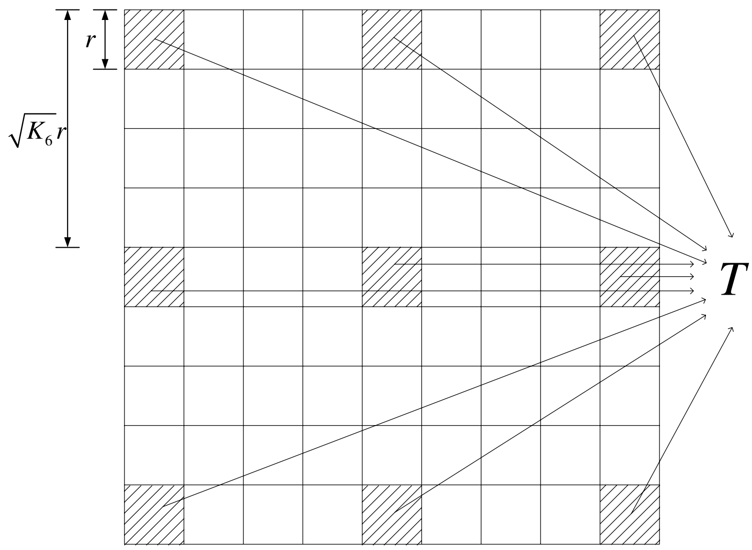


Figure 5.1: Grouping of interfering clusters in the TDMA scheme. $S_n$ is the length of each cluster. $K_6$ is the number of non-interfering group. $T$ is a non-interfering group.

**Theorem 5.8** *Consider a network organized into $s^{1+\varepsilon_2}$ clusters. Then by implementing the TDMA scheme described above when $s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, the aggregate throughput for the network is $\frac{s(K_4 \log n + K_5)}{K_6}$.*

The proof of this theorem is given in Appendix 8.2.

When $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, then the transmission time required to complete this phase is

$$t_{\text{Phase 1}} = \frac{sK_2L\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{\frac{s(K_4 \log n + K_5)}{K_6}} = \frac{K_6K_2L\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{K_4 \log n + K_5}. \tag{5.11}$$

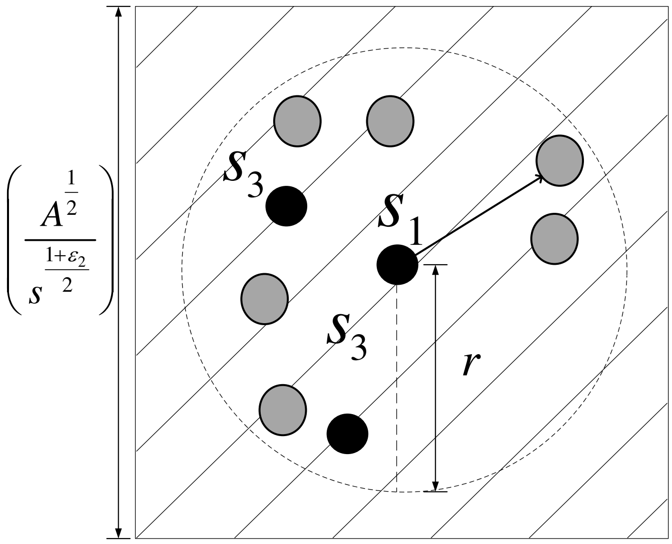


Figure 5.2: The broadcasting transmission in Phase 1. When $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, the radius $r$ is chosen as $\frac{\sqrt{A}}{2s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}$, and when $s = O\left(n^{\beta_1}\right)$, the radius $r$ is chosen as $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$.

When $s = O\left(n^{\beta_1}\right)$, then the transmission time required to complete this phase is

$$t_{\text{Phase 1}} = \frac{sK_2L\frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{\frac{s(K_4 \log n + K_5)}{K_6}} = \frac{K_6K_2L\frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{K_4 \log n + K_5}. \tag{5.12}$$

**Phase 2. MIMO Cooperation Transmission**: At the beginning of the second phase, all nodes in the cluster containing the source nodes decode the information into a finite number of bits. This information is mapped into $C_1\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}$ (or $C_1\frac{n}{n^{(1+\varepsilon_2)\beta_1}}$ when $s = O\left(n^{\beta_1}\right)$) symbols, where $C_1$ is a constant. Then the source nodes along with the relays form a distributed MIMO system to transmit their information to the destinations and the relays surrounding the destinations in that cluster (see Fig. 5.3). Given that there are $s$ sources in the network, there are $s$ MIMO transmissions to complete this phase.

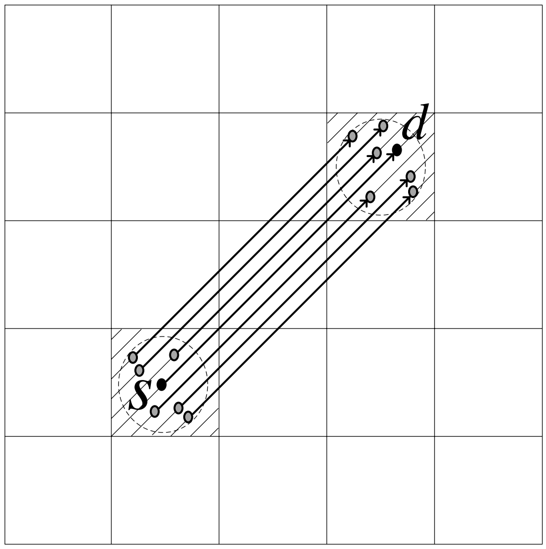


Figure 5.3: MIMO Cooperation Transmission. The black nodes represent sources or destinations. The gray nodes are the relays.

The aggregate throughput for this phase is given by the following lemma.

**Lemma 5.9** *The aggregate throughput for the MIMO cooperation transmission scheme is at least* $K_7 \frac{n}{s^{1+\epsilon_2+\epsilon_3}}$ *when* $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$ *and* $K_7 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}$ *when* $s = O\left(n^{\beta_1}\right)$ *for the MIMO quantized channel.*

This lemma is proved in [1]. It is easy to show that the total required time for Phase 2 is $t_{\text{Phase 2}} = \Theta(s) = \frac{C_1}{K_7} s$.

**Phase 3. Transmission from Relays to Destination**: Phase 3 is the reverse of phase 1 with relays in the destination cluster quantizing the observed information and transmitting them sequentially to the destination. The transmission procedure is shown in Fig. 5.4. Using *Lemma* 6.13 it can be proved [1] that $K_7 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}$

(or $K_7 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}$ for $s = O\left(n^{\beta_1}\right)$) throughput can be achieved. Note that the TDMA scheme for parallel transmissions in clusters is implemented for Phase 3.

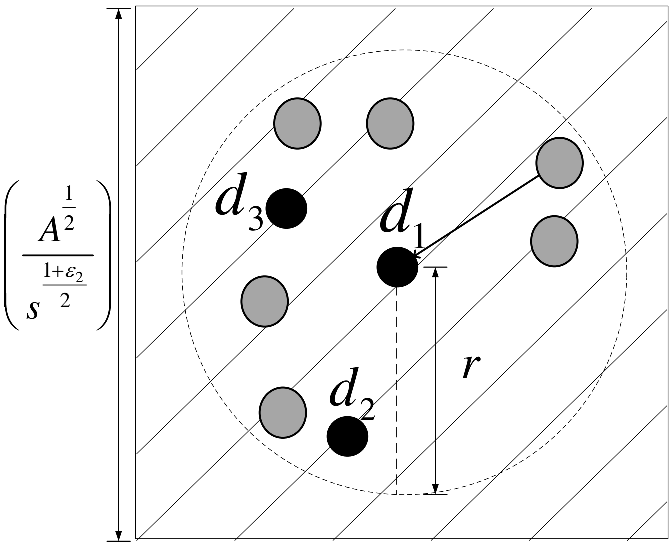


Figure 5.4: The one to one transmission in Phase 3. When $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, the radius $r$ is chosen as $\frac{\sqrt{A}}{2s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}$, and when $s = O\left(n^{\beta_1}\right)$, the radius $r$ is chosen as $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$.

From the above discussion, the time requirement for Phase 3, the total time and the aggregate throughput as the result of three phases can be given as follows.

When $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, then

$$t_{\text{Phase 3}} = \frac{sC_2C_3\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{\frac{s(K_5+K_4\log n)}{K_6}} = \frac{K_6C_2C_3\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{K_5+K_4\log n}, \tag{5.13}$$

$$\begin{aligned} t_{\text{total}} &= t_{\text{Phase 1}} + t_{\text{Phase 2}} + t_{\text{Phase 3}} \\ &= \frac{K_6 K_2 L \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{K_4 \log n + K_5} + \frac{C_1 s}{K_7} + \frac{K_6 C_2 C_3 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{K_5 + K_4 \log n}, \end{aligned} \tag{5.14}$$

and

$$\begin{aligned} R_1(n) &= \frac{K_2 L s \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{\frac{K_6 K_2 L \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{K_4 \log n + K_5} + \frac{C_1 s}{K_7} + \frac{K_6 C_2 C_3 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}}{K_5 + K_4 \log n}} \\ &\geq K_8 s \log n. \end{aligned} \tag{5.15}$$

The lower bound in (5.15) is correct when $s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$.

When $s = O\left(n^{\beta_1}\right)$, then

$$t_{\text{Phase 3}} = \frac{s C_2 C_3 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{\frac{s(K_5 + K_4 \log n)}{K_6}} = \frac{K_6 C_2 C_3 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{K_5 + K_4 \log n}, \tag{5.16}$$

$$\begin{aligned} t_{\text{total}} &= t_{\text{Phase 1}} + t_{\text{Phase 2}} + t_{\text{Phase 3}} \\ &= \frac{K_6 K_2 L \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{K_4 \log n + K_5} + \frac{C_1 s}{K_7} + \frac{K_6 C_2 C_3 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{K_5 + K_4 \log n}, \end{aligned} \tag{5.17}$$

and

$$\begin{aligned} R_1(n) &= \frac{K_2 L s \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{\frac{K_6 K_2 L \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{K_4 \log n + K_5} + \frac{C_1 s}{K_7} + \frac{K_6 C_2 C_3 \frac{n}{n^{(1+\varepsilon_2)\beta_1}}}{K_5 + K_4 \log n}} \\ &\geq K_8 s \log n. \end{aligned} \tag{5.18}$$

The lower bound in (5.18) is correct when $s = O\left(\frac{n^{1-(1+\varepsilon_2)\beta_1}}{\log n}\right)$.

For the rest of this section, we use the *Three-Phase* communication with slight modifications based on the value of $s$. Therefore, we only mention the differences between the cases and the *Three-Phase* scheme that we described above.

#### 5.4.1.2 The Case of $\Omega\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s = O\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right)$

**Phase 1**: The only differences in this region is the fact that we divide the network into $s$ clusters which results in $\Theta\left(\frac{\log s}{\log\log s}\right)$ sources for each cluster. Similar to *Theorem* 5.7, it can be proved that there are $\Theta\left(\frac{n}{s}\right)$ nodes in each cluster and all these nodes will be used as relays unlike previous section that we only used nodes inside a circle. The transmission scheme in this case is shown in Fig. 5.5 The following theorem can be proved for this phase.

**Theorem 5.10** *The link capacity between any two nodes in a cluster under* Phase One *and* Phase Three *of this capacity region is at least* $\Theta(1)$.

The proof of this theorem is omitted due to page limitations. The aggregate throughput in this phase is $\Theta(s) = K_{11}s$. Therefore, the time needed in this phase is given by

$$t'_{\text{Phase 1}} \leq \frac{sK_9\frac{\log s}{\log\log s}K_{10}L\left(\frac{n}{s}\right)}{K_{11}s} = \frac{K_9K_{10}Ln\frac{\log s}{\log\log s}}{K_{11}s}. \tag{5.19}$$

**Phase 2**: This phase is also identical to previous one except that the aggregate throughput is $K_{12}\left(\frac{n}{s}\right)$ for transmitting $C_4\frac{n}{s}$ symbols, since all nodes in the cluster are used for the cooperative MIMO transmission as shown in Fig. 5.6. Therefore, the

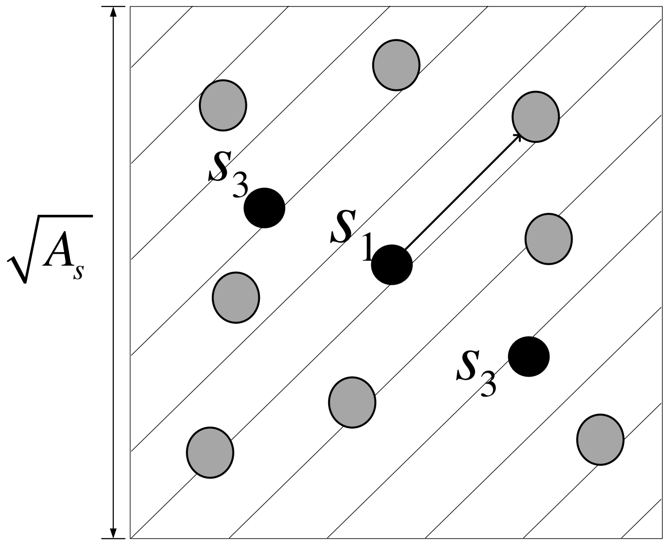


Figure 5.5: The one-to-one transmission in Phase 1, when $\Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s = O(n)$. $\sqrt{A_s}$ is the length of each cluster and its has different value for different region of $s$.

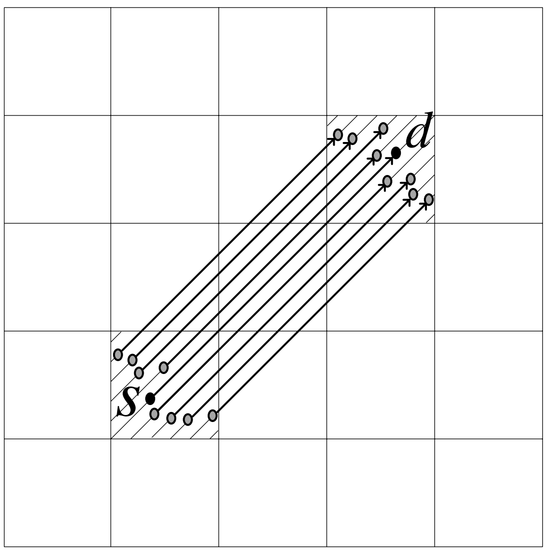


Figure 5.6: MIMO cooperation transmission in Phase 2, when $\Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s = O(n)$.

required time for this phase is

$$t'_{\text{Phase 2}} = \frac{sC_4\left(\frac{n}{s}\right)}{K_{12}\left(\frac{n}{s}\right)} = \frac{C_4 s}{K_{12}}. \tag{5.20}$$

**Phase 3**: This phase is similar to previous one except that the aggregate throughput is $K_{11}s$ since all nodes in the cluster are used for deliver the information. The transmission scheme is shown in Fig. 5.7.

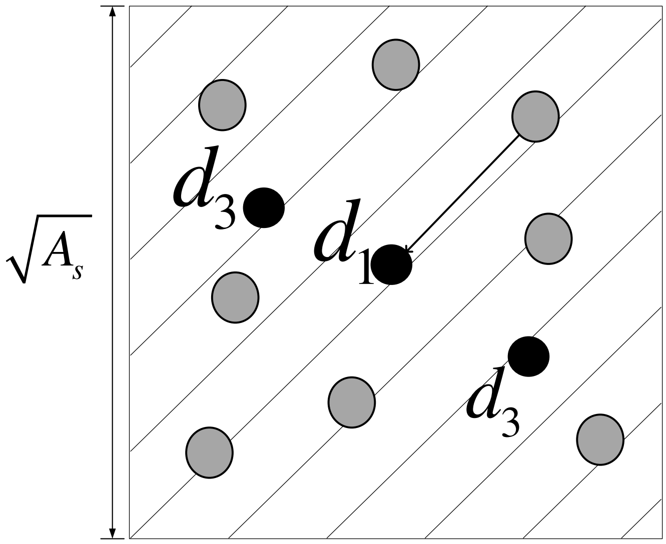


Figure 5.7: The one-to-one transmission in Phase 3, when $\Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s = O\left(n\right)$. $\sqrt{A_s}$ is the length of each cluster and its has different value for different region of $s$.

The total required time for this phase can be easily derived as

$$t'_{\text{Phase 3}} \leq \frac{sK_9\frac{\log s}{\log\log s}C_5C_3\left(\frac{n}{s}\right)}{K_{11}s} = \frac{K_9C_5C_3n\frac{\log s}{\log\log s}}{K_{11}s}. \tag{5.21}$$

Therefore, the aggregate throughput in this region of $s$ is

$$R_2(n) \geq \frac{sK_{10}L\left(\frac{n}{s}\right)}{\frac{K_9K_{10}Ln\frac{\log s}{\log\log s}}{K_{11}s} + \frac{C_4 s}{K_{12}} + \frac{K_9C_5C_3n\frac{\log s}{\log\log s}}{K_{11}s}}. \tag{5.22}$$

When $s = O\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right)$, then the first and third terms in the denominator are the dominant factors. Hence, the rate can be written as

$$\begin{aligned} R_2(n) &\geq \frac{sK_{10}L\left(\frac{n}{s}\right)}{2\left(\frac{K_9K_{10}L\frac{\log s}{\log\log s}}{K_{11}} + \frac{K_9C_5C_3\frac{\log s}{\log\log s}}{K_{11}}\right)\frac{n}{s}}, \\ &= K_{13}\frac{s\log\log s}{\log s}. \end{aligned} \tag{5.23}$$

The *Three-Phase* approach that was described in Section 5.4.1.1 is optimum for that region. However if we use this approach for the second capacity region, it will reduce the capacity from the peak of first capacity region. Now the question is that if we use Section 5.4.1.1 scheme in this region, at what point the throughput capacity for the two schemes are equal, i.e. $\Theta(R_1(n)) = \Theta(R_2(n))$? It turns out that when $\Omega\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\epsilon_2+\epsilon_3}}\right) = s$, then $R_1(n)$ can be approximated as $R_1(n) = \Theta\left(\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}\right)$. For the same capacity region, it is easy to show that $R_2(n) = \Theta\left(\frac{s\log\log n}{\log n}\right)$[2]. By making the two rates $R_1(n)$ and $R_2(n)$ equal, we arrive at $s = \Theta\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$. Therefore when $\Omega\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s = O\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, we use the transmission scheme

[2]we used the fact that in this region, $s = n^\gamma$ for some constant value of $\gamma$.

shown in Section 5.4.1.1, and for the rest of second capacity region, we utilize the second *Three-Phase* approach that we explained here. By doing this, the maximum throughput capacity is achieved in the second capacity region.

#### 5.4.1.3 The Case of $\Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right) = s = O(n)$

For this region, given that the number of source-destination pairs is large, the original Hierarchical MIMO cooperation scheme of [1] provides the highest throughput capacity. The main feature of the *Hierarchical Cooperation* scheme is the fact that each cluster is further divided into smaller clusters and the distributed MIMO system is utilized in a hierarchical fashion.

Let $A_s$ denote an area of a cluster. From *Theorem* 5.7, it can be shown that the number of sources in each cluster is $M_s = \Theta(A_s s) = K_{14}A_s s$ and the total number of nodes in each cluster is $N_s = \Theta\left(\frac{M_s n}{s}\right) = K_{15}\left(\frac{M_s n}{s}\right)$ as long as the following condition is satisfied.

$$A_s = \omega\left(\frac{A}{s}\right) \tag{5.24}$$

Note that the communication scheme for each hierarchy is very similar to that of the previous section. Due to page limitations, we only state the differences in each communication phase.

**Phase 1**: This phase is identical to Phase 1 in the previous section, except that there are $K_{14}A_s s$ source nodes and $K_{15}\left(\frac{M_s n}{s}\right)$ nodes in each cluster. The transmission scheme is shown in Fig. 5.5. Besides, each node transmits $K_{15}\left(\frac{M_s n}{s}\right)$ blocks of bits to relays and the link throughput is $K_{16}M_s^b$ where $b$ is a constant between zero and one related to the number of hierarchies. The time required for this phase is

$$t''_{\text{Phase 1}} = \frac{M_s \cdot K_6 K_{15} L\left(\frac{M_s n}{s}\right)}{K_{16} M_s^b}. \tag{5.25}$$

**Phase 2**: As shown in Fig. 5.6, this phase is also similar to that in the previous section, except that the total number of transmitted bits in one cluster for each source is $C_6\left(\frac{M_s n}{s}\right)$ and the aggregate throughput for the network is $K_{17}\left(\frac{M_s n}{s}\right)$ symbols. The required time to finish this phase is

$$t''_{\text{Phase 2}} = \frac{sC_6\left(\frac{M_s n}{s}\right)}{K_{17}\left(\frac{M_s n}{s}\right)} = \frac{C_6}{K_{17}} s. \tag{5.26}$$

**Phase 3**: This phase is similar to Phase 3 as shown in Fig. 5.7 in the previous section with link throughput in the network as $K_{16}M_s^b$. The required time to complete this phase is

$$t''_{\text{Phase 3}} = \frac{M_s \cdot K_6 C_7 C_3\left(\frac{M_s n}{s}\right)}{K_{16} M_s^b}. \tag{5.27}$$

Thus, the total required time is

$$t''_{\text{total}} = \frac{M_s \cdot K_6 K_{15} L\left(\frac{M_s n}{s}\right)}{K_{16} M_s^b} + \frac{C_6}{K_{17}} s + \frac{M_s \cdot K_6 C_7 C_3\left(\frac{M_s n}{s}\right)}{K_{16} M_s^b}. \tag{5.28}$$

Then, the aggregate throughput for the network is

$$R_3(n) = \frac{s \cdot K_{15} L\left(\frac{M_s n}{s}\right)}{t''_{\text{total}}} = \frac{K_{18} M_s n s}{K_{19} M_s^{2-b} n + K_{20} s^2}, \tag{5.29}$$

where $K_{18} = K_{15}L$, $K_{19} = \frac{K_6 K_{15} L}{K_{16}} + \frac{K_6 C_7 C_3}{K_{16}}$ and $K_{20} = \frac{C_6}{K_17}$.

By computing the derivative of $R_3(n)$ with respect to $M_s$ and equating it to zero, we have

$$M_s = \left(\frac{K_{20}}{K_{19}(1-b)}\right)^{\frac{1}{2-b}} \left(\frac{s^2}{n}\right)^{\frac{1}{2-b}} = K_{21}\left(\frac{s^2}{n}\right)^{\frac{1}{2-b}}, \tag{5.30}$$

where $K_{21} = \left(\frac{K_{20}}{K_{19}(1-b)}\right)^{\frac{1}{2-b}}$.

It can be shown from Eq. (6.21) that the number of sources in each cluster is at least a constant value when $s = \omega(\sqrt{n})$, which guarantees that the number of clusters is less than the number of sources and fulfills the condition in Eq. (6.24). Therefore, the aggregate throughput is given by

$$\begin{aligned}
R_3(n) &= \frac{K_{18}\left(K_{21}\left(\frac{s^2}{n}\right)^{\frac{1}{2-b}}\right) ns}{K_{19}\left(K_{21}\left(\frac{s^2}{n}\right)^{\frac{1}{2-b}}\right)^{2-b} n + K_{20}s^2}, \\
&= \frac{K_{18}K_{21}}{K_{19}K_{21}^{2-b} + K_{20}} n^{\frac{1-b}{2-b}} s^{\frac{b}{2-b}} \overset{(a)}{=} K_{22} s^{\frac{1}{2-b}} \left(\frac{n}{s}\right)^{\frac{1-b}{2-b}}, \\
&= K_{22} s^{\frac{1}{2-b} + \log_s\left(\frac{n}{s}\right)^{\frac{1-b}{2-b}}} = K_{22} s^{\left(\frac{1}{2-b} + \frac{1-b}{2-b}\log_s\left(\frac{n}{s}\right)\right)}, \\
&\overset{(b)}{=} K_{22} s^{1-\varepsilon_1+\varepsilon_4}.
\end{aligned} \tag{5.31}$$

Equality (a) in the above equation is derived by defining $K_{22} = \frac{K_{18}K_{21}}{K_{19}K_{21}^{2-b}+K_{20}}$ and equality (b) is obtained by letting $\varepsilon_1 = \frac{1-b}{2-b}$ and $\varepsilon_4 = \frac{1-b}{2-b}\log_s\left(\frac{n}{s}\right)$. It is easy to show that $\varepsilon_4 \leq \varepsilon_1$ in all cases. Note that the result from this scheme is similar to that of [1] when $s = n$. As in [1], if the capacity in the current hierarchy is $K_{22}s^b$, then the capacity in the next hierarchy is $K_{22}s^{\left(\frac{1}{2-b}+\frac{1-b}{2-b}\log_s\left(\frac{n}{s}\right)\right)}$, which can be shown to increase monotonically. Now we investigate the case when the maximum capacity is achieved or equivalently,

$$\frac{1}{2-b} + \frac{1-b}{2-b}\log_s\left(\frac{n}{s}\right) = 1. \tag{5.32}$$

This equality is satisfied when $s = \sqrt{n}$ or $b = 1$. However $s = \sqrt{n}$ is not acceptable, because it violates the condition in Eq. (6.24). In addition, $b$ is always smaller than one and, therefore, the capacity of phase three cannot reach its maximum of $\Theta(s)$.

Now the question is for what value of $s$ we have $\Theta(R_2(n)) = \Theta(R_3(n))$ in this capacity region. Following a similar procedure as in the previous section, it can be proved that $s = \Theta\left(n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}}\right)$. Similarly, this capacity region can be divided into two regions. When $\Omega\left(\left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right) = s = O\left(n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}}\right)$, then $R_2(n)$ provides a higher throughput capacity of $K_{13}'\frac{n}{s}$ and for $\left(n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}}\right) = s = O(n)$, $R_3(n)$ gives $K_{22}s^{1-\varepsilon_1+\varepsilon_4}$ throughput capacity.

### 5.4.2 Capacity Analysis for Data-Gathering Traffic

This section is dedicated to computation of achievable capacity for *Many-to-One Traffic*. We assume that a bandwidth of $W_2 = W - W_1$ is allocated to this traffic.

Our analysis is similar to the method used in [9], with the exception that nodes are uniformly distributed in a square plane in this paper as opposed to sphere in [9]. It can be proved that the upper bound is also $\Theta(\log n)$, which is similar to the lower bound.

We adopt a *Two-Phase* scheme that utilizes $\eta_4 P_2$ and $(1 - \eta_4)P_2$ Watts for power consumption in Phases 1 and 2, respectively.

**Phase 1. Broadcasting Transmission**: In the first phase, only one of the source nodes broadcast its information to the nodes of radius $r$ around it. The transmission is shown in Fig. 5.8.

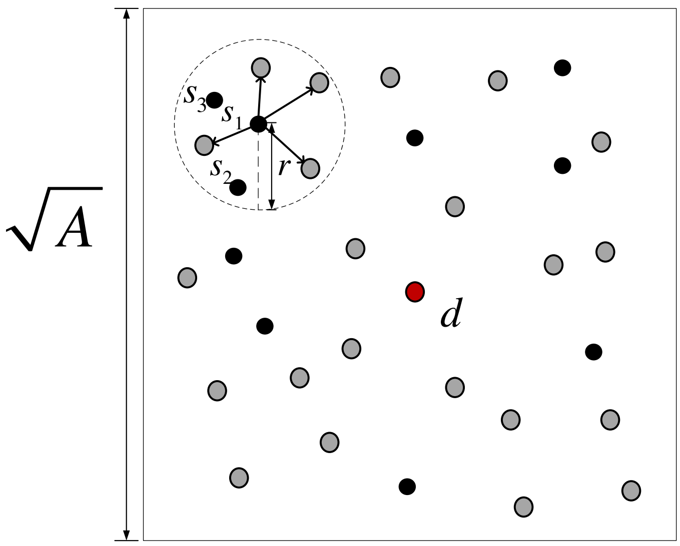


Figure 5.8: The broadcasting transmission in Phase 1. In this graph $s_1$, $s_2$ and $s_3$ are the source nodes and $d$ is the access node. $r = n^{-\varepsilon_5}$, where $\varepsilon_5$ is a constant between 0 and 1.

If $r$ is small enough, all the nodes in the circle with radius $r$ can decode the information. The aggregate throughput we can achieve is given by the following theorem.

**Theorem 5.11** *Let $r = n^{-\varepsilon_5}$, where $\varepsilon_5$ for $0 \le \varepsilon_5 \le 1$. Then with bandwidth $W_2$ and total transmit power $\eta_4 P_2$, an aggregate throughput of at least $K_{23} \log n + K_{24}$ can be achieved.*

This theorem is proved in Appendix 8.3.

**Phase 2. Cooperative Many-to-One Transmission**: As shown in Fig. 5.9, in the second phase, all the relays within a radius of $r$ transmit the data along with the source node, thus creating a distributed MISO system.

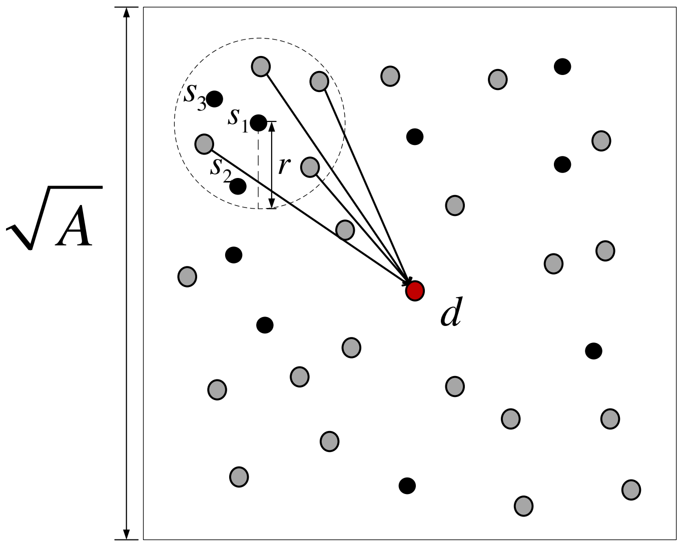


Figure 5.9: The cooperative *many to one* transmission in Phase 2. In this graph $s_1$, $s_2$ and $s_3$ are the source nodes and $d$ is the access node. $r = n^{-\varepsilon_5}$, where $\varepsilon_5$ is a constant between 0 and 1.

The aggregate throughput that can be achieved is given by the following theorem.

**Theorem 5.12** *The aggregate throughput of the cooperative* Many-to-One *transmission scheme is at least $K_{25} \log n + K_{26}$.*

This theorem is proved in Appendix 8.4. Therefore, the total throughput capacity for the data gathering scheme is $R_{\text{Many-to-One}}(n) = K_{27} \log n + K_{28}$ where $K_{27} = \min(K_{23}, K_{25})$ and $K_{28} = \min(K_{24}, K_{26})$.

The total throughput capacity in the network as a result of these two types of traffics is the summation of their individual rates, i.e., $R(n)_{\text{Co}} = R_1(n) + R_{\text{Many-to-One}}(n)$. The result is provided in *Theorem* 5.5.

## 5.5 Discussion

The first major contribution of this chapter is the separation theorem for heterogeneous traffic. This simple theorem states that when there are multiple classes of traffic in the network, a simple way to achieve the maximum order throughput capacity is to allow all nodes in the network to operate on a single traffic class for an assigned bandwidth. This result implies that multiple-radio multiple-channel systems are order-optimum for heterogeneous traffic. The main reason for this result is the fact that nodes that are not part of a specific traffic can be utilized as relays [8], which clearly improve the throughput capacity of the network.

The second major contribution of these two sections is the computation of the achievable throughput capacity when the number of relays and source-destination pairs are changing as a function of $n$. Gastpar and Vetterli [8] have solved this problem when there is only one source-destination pair in the network and the rest of the nodes are relays. We have shown different forwarding strategies when the ratio between relays and unicast sessions changes by utilizing an extended version of the three-phase approach introduced in [1]. Our results also corroborate previous results obtained in [1] when the number of source-destination pairs is $n$ and in [8] when there are $\Theta(1)$ source-destination pairs in the network.

Note that, in the last capacity region, the achievable capacity is $n^{1-\varepsilon_1+\varepsilon_4}$ instead of capacity of $n^{1-\varepsilon_1}$ as reported in [1]. The gain of $n^{\varepsilon_4}$ for $\varepsilon_4 = \frac{1-b}{2-b}\log_s\left(\frac{n}{s}\right)$ is achieved by employing relays to improve the throughput. This gain reduces as $s$ tends to $n$, because $\varepsilon_4 \to 0$.

The unicast capacity for $\Omega\left(S_2 = \left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right) = s = O\left(S_3 = \left(\frac{n\log n}{\log\log n}\right)^{\frac{1}{2}}\right)$ is $\Theta\left(\frac{s\log\log s}{\log s}\right)$. However, if the number of sources in each cluster is a constant value instead of a random variable, then it is easy to show that a capacity of $\Theta(s)$ can be achieved.

Fig. 5.10 plots the capacity region that was derived in (6.19). From this figure, we see that, when the number of unicast sessions is from 1 to $\Theta\left(S_1 = \left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$, the majority of nodes are part of *Many-to-One Traffic* and we call this region as *Many-to-One Traffic.* The achieved capacity in this region is the optimum value. When $\Omega\left(S_1\right) = s = O\left(S_4 = n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}}\right)$, then the number of nodes for both traffic patterns are comparable. Hence, we call it *Heterogeneous Traffic* region. It is not clear whether our achievable capacity region is optimum for this region. Finally when $s = \Omega\left(S_4\right)$, then majority of nodes are involved in unicast communication and we call this region *Unicast traffic.*

It is worthy of note that the capacity actually decreases in two regions as $s$

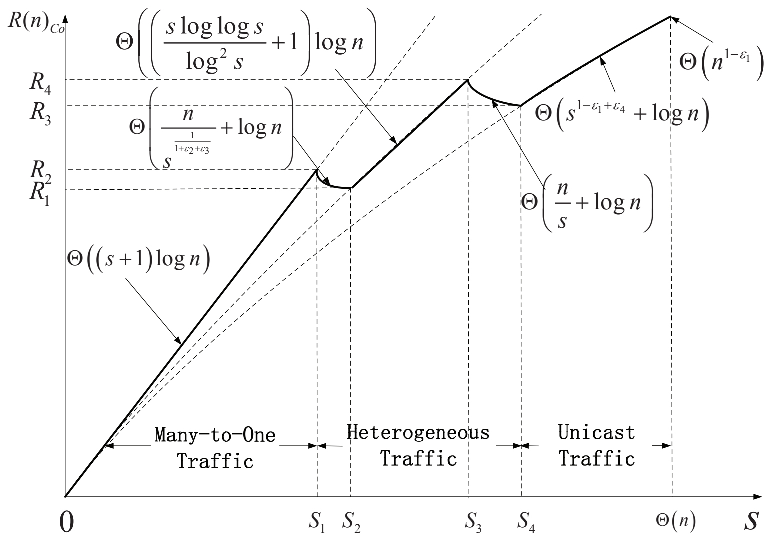


Figure 5.10: The achievable aggregate throughput.

increases. These two regions require more investigation to find better communication schemes, which is the subject of future study.

From next section, we will introduce the analysis of the capacity by using non-cooperative transmission protocol.

# Chapter 6

# The Capacity of Wireless Ad-hoc Networks with Heterogeneous Traffic under Protocol Model: Part 2

In this Chapter, we provide the analysis of the capacity of the network with heterogeneous traffic without using cooperative transmission schemes, which means that we use the simpler and practical multihop relay scheme for the information dissemination.

By using *Theorem* 5.1, we still treat the heterogeneous traffic in the network separately into two different networks with different traffics. One is the network with unicast relay traffic and containing $s$ source-destination pairs. The other is the network with data-gathering traffic and containing $n - s$ source nodes. Now the problem is simplified as calculating of the capacity of these two networks by using multihop relay scheme. Same as previous section, we distribute bandwidth $W_1$ to the network with unicast traffic and bandwidth $W_2$ to the network with data-gathering traffic, where $W_1 + W_2 = W$.

Thanks to the work of [10], we have known the scaling of the capacity of the network with data-gathering traffic, which is given by the lemma below.

**Lemma 6.1** *A uniformly deployed network using multi-hop transmission for data gathering communication can achieve per-node throughput* $R(n)_{DG} \geq \frac{W_2}{n} \frac{\pi r(n)^2 - \sqrt{\epsilon}}{4\pi r(n)^2 + 4\pi r(n)\delta + \pi\delta^2 + \sqrt{\epsilon}}$, *where* $\epsilon$ *and* $\delta$ *are positive constants.*

In the network with unicast traffic, the number of source-destination pairs, $s$, is a function of the total number of the nodes in the network, $n$. The transportation of information from sources to destinations occurs through multiple hops depending on the distance between each source-destination pairs. We define this network as *Multihop Relay Wireless Network (MRWN).* Moreover, since most wireless ad hoc networks such as military networks utilize multihop communications, this study is valuable in understanding the capacity behavior of these networks.

The rest of this chapter is organized as follows. Section 6.1 provides the main results of this section. We compute the upper bound in Section 6.2. The achievable throughput by using multihop transmission scheme is described in Section 6.3. Section 6.4 briefly discusses the implications of the results. We will conclude this chapter in Section 6.5. Part of this work is presented in [21].

For simplicity, the constants $K_i$ ($i$ is a positive constant number) we used in the proofs are reused from $K_1$, which means that the constants $K_i$ we used in this section are different from those of last section but with the same name.

## 6.1 Main Results without Cooperative Transmission Scheme

This section summarizes the main contributions of this paper. We first describe the upper bound capacity of the network with heterogeneous traffic using multihop transmission scheme.

- The Upper Bound:

  The upper bound of the capacity of the network with heterogeneous traffic using multihop transmission scheme is given by the following theorem.

  **Theorem 6.2** *In a wireless network with $k$ sources for the access node and $s$ source-destination pairs for the unicast traffic section, where $s+k=n$, the aggregate throughput of the network under protocol model is upper bounded by*

  $$R(n)_{NCo} = O\left(\min\left\{\sqrt{\frac{n}{\log n}}, s+1\right\}\right). \tag{6.1}$$

  From *Theorem* 6.2, to easy to analyze, we can get that the per-destination upper bound of the capacity is shown below

  $$R(n)_{\text{NCo per-destination}} = O\left(\min\left\{\frac{\sqrt{\frac{n}{\log n}}}{s+1}, 1\right\}\right). \tag{6.2}$$

We can see that if the number of sources for unicast communication is larger than a threshold, i.e., $s = \Omega\left(\sqrt{\frac{n}{\log n}}\right)$, then the per-destination upper bound capacity is dominated by this value. When $s$ increases in the network while the number of nodes with data-gathering traffic decreases as $n - s$, then the per-destination upper bound capacity decreases monotonically with $s$. The main reason is the fact that, from *Theorem* 5.1, we can view the nodes with data-gathering traffic as relays for the nodes with unicast communication, then as the number of nodes with data-gathering traffic decreases, there are not enough relays to facilitate the transportation of information between all source-destination pairs in the unicast communication session which contributes more capacity of the network, thus the lack of relays creates a bottleneck in the network. On the other hand, when $s$ is smaller than a threshold, i.e., $s = O\left(\sqrt{\frac{n}{\log n}}\right)$, then there are many relays for the nodes of unicast traffic to transport information for all source-destination pairs. Under this condition, it is clear that all source-destination pairs can transmit their information in parallel and the upper bound is 1. This result also indicates that when there is a large number of relays in the network, simple point-to-point communication does not utilize the full capability of the network. Under such conditions, it may be useful to develop cooperative techniques between nodes such that the network is able to fully take advantage of the relays in the network. Notice that when $s = \Theta(n)$, the upper bound becomes $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$ which is similar to the result given in [2]. Under this condition, there will be no relay for the unicast communication and there are $n$ simultaneous unicast sessions in the network.

- The lower Bound:

The achievable lower bound of the throughput is given by the following theorem.

**Theorem 6.3** *In a dense network with uniform node distribution, we assume there are $s$ source-destination pairs having unicast communication and $n-s$ nodes doing data-gathering traffic. Under protocol model, there exits a routing scheme which can lead to the following achievable throughput this network as*

$$R(n)_{NCo} = \begin{cases} \Omega\left(1\right), & s = \Theta\left(1\right) \\ \Omega\left(s \cdot \dfrac{\log\log s}{\log s} + 1\right), & \Omega\left(1\right) = s = O\left(S_1\right) \\ \Omega\left(\sqrt{\dfrac{n}{\log n}} + 1\right), & s(n) = \Omega\left(S_1\right) \end{cases} \tag{6.3}$$

*where* $S_1 = \Theta\left(\sqrt{\frac{n}{\log n}}\frac{\log n}{\log\log n}\right)$[1].

The result indicates that for two regions of $s = \Theta\left(1\right)$ and $s = \Omega\left(S_1\right)$, the achievable lower bound is the same as the upper bound. However, when $\Omega\left(1\right) = s = O\left(S_1\right)$, there exits a gap between the lower and upper bounds of the capacity. The reason behind this gap is in the randomness of the node distribution and the random selection of source-destination pairs.

The results of *Theorems* 6.2 and 6.3 are shown in Figure 6.1.

[1] Note that this particular representation of $S_1$ is intentional in order to easier find the common regions between relay traffic and source-destination traffic in each cell later on for the network with unicast traffic.

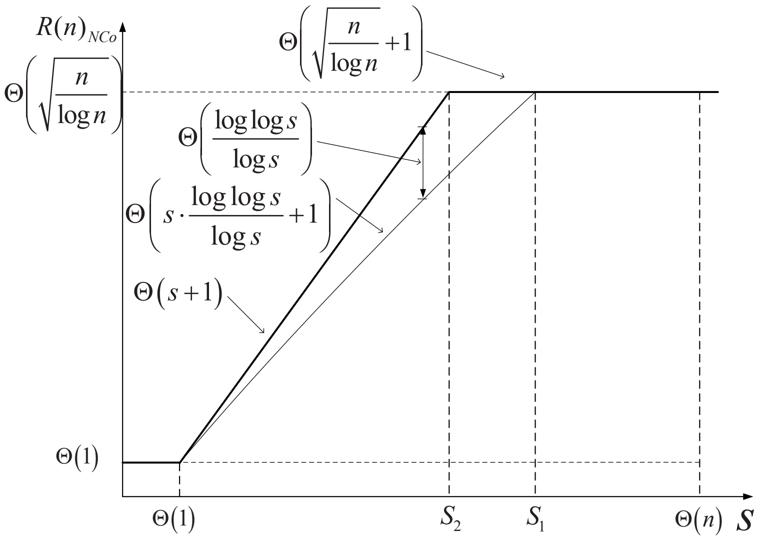


Figure 6.1: The upper and lower bound of the capacity in the network with heterogeneous traffic. The bold line is the upper bound of the capacity and the thin line is the achievable lower bound of throughput. In the regions of $(1, \Theta(1)]$ and $[\Theta(S_1), n)$, the upper and lower bounds are tight. In this figure, $S_1 = \Theta\left(\sqrt{\frac{n}{\log n}\frac{\log n}{\log\log n}}\right)$, $S_2 = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$.

## 6.2 Upper Bound

In this section, we prove the upper bound of the capacity for the network with heterogenous traffic by using multihop transmission scheme. For the wireless networks, we use the concept of *sparsity cut*, which is defined by Liu et al. [22], instead of min-cut, to take into account the differences between wired and wireless links.

**Definition 6.4 (Sparsity Cut:)** *A sparsity cut for a random network is defined as a cut induced by the line segment with the minimum length that separates the region into two equal area subregions (see Fig 6.2). The cut capacity is defined as the transmission bandwidth $W$ multiplied by the maximum possible number of simultaneous transmissions across the cut. This cut capacity is the information rate that the nodes from one side of the cut can deliver to the nodes at the other side. The cut length $l_\Gamma$ is defined as the length of the cut line segment in 2-D space. In another word, sparsity cut can be seen for random geometric graph (RGG) similar to min-cut concept in graph theory.*

From [2], we know that the disks centered at each receiver are disjoint and have radius of $\frac{\Delta r(n)}{2}$. By assuming the length of the sparsity cut as $l_\Gamma$, the following lemma provides the sparsity cut capacity which was originally proved in [22].

**Lemma 6.5** *The capacity of the cut $\Gamma$ for a 2D region has an upper bound of $\frac{K_1 l_\Gamma W}{r(n)}$, where $K_1 = \max\left\{\frac{16}{\pi\Delta^2}, \frac{\sqrt{3}}{\Delta}\right\}$, and $W$ is the link rate.*

Since the network area is assumed to be 1, we have $l_\Gamma = \Theta(1)$. To guarantee the connectivity [2], $r(n)$ is chosen as $K_2\sqrt{\frac{\log n}{n}}$. Thus, the upper bound of the cut capacity is $C_\Gamma = \frac{K_1 W}{K_2\sqrt{\frac{\log n}{n}}} = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$.

We use Lemma 6.5 to prove *Theorem* 6.2 as described below.

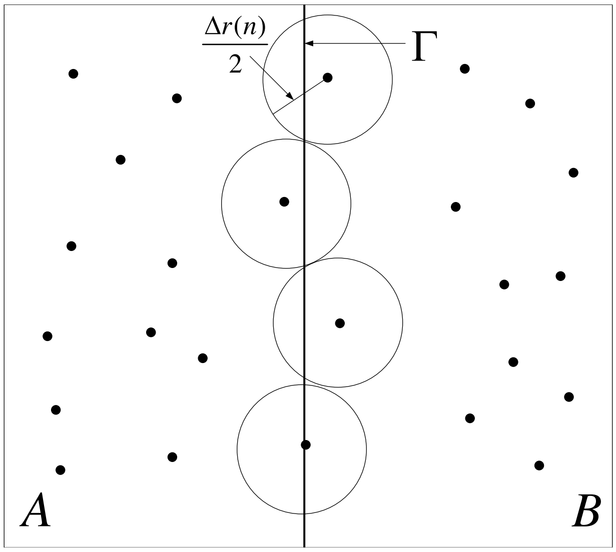


Figure 6.2: Γ is the sparsity cut which separates the network into A and B equal areas. The figure demonstrates four disjoint disks with radius $\frac{\Delta r(n)}{2}$ across the sparsity cut.

*Proof of Theorem 6.2:*

**proof 6.6** *Lemma 6.5 shows the upper bound of the aggregate capacity of the network is* $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$*. Since there are only* $s+1$ *destination nodes in the network, it is clear that this throughput can be divided between these* $s+1$ *destination nodes equally. Therefore as long as* $s+1$ *is larger than* $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$*, per-destination throughput capacity is upper bounded as* $\Theta\left(\frac{\sqrt{\frac{n}{\log n}}}{s+1}\right)$*. However, when the number of destinations is less than* $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$*, then there are plenty of capacity in the network such that each source can continuously transmit its data to destination. Under this condition, the per source throughput capacity is upper bounded as* $\Theta(1)$*. Thus, the per-destination capacity is* *upper bounded as*

$$R(n)_{NCo\ per\text{-}destination} = O\left(\min\left\{\frac{\sqrt{\frac{n}{\log n}}}{s+1}, 1\right\}\right). \tag{6.4}$$

*Therefore, the aggregate throughput in the network is given by*

$$R(n)_{NCo} = O\left(\min\left\{\sqrt{\frac{n}{\log n}}, s+1\right\}\right). \tag{6.5}$$

## 6.3 Lower Bound

By using *Theorem* 5.1, we have two different networks with different traffic. We will calculate the capacity for each network and sum them up to get the achievable lower bound of the throughput for the network with heterogeneous traffic.

From *Lemma* 6.1, we can get that the lower bound of the capacity for the network with data-gathering traffic is $R(n)_{\text{DG}} \geq \frac{W_1}{n}\frac{\pi r(n)^2-\sqrt{\epsilon}}{4\pi r(n)^2+4\pi r(n)\delta+\pi\delta^2+\sqrt{\epsilon}}$. As $n$ goes to infinity, we can have

$$R(n)_{\text{DG}} = \Omega(1). \tag{6.6}$$

Then, the proof for the achievable lower bound of the throughput capacity for the network with unicast traffic is presented in the rest of this section. Since we only use multihop transmission scheme in this relay network, we call this network as *Multihop Relay Wireless Network (MRWN)*. The achievable throughput of *MRWN* is given below.

**Theorem 6.7** *In a dense* MRWN *with uniform node distribution, we assume there are s source-destination pairs. There exits a routing scheme which can lead to the following per-source achievable throughput capacity for* MRWN *as*

$$R(n)_{MRWN} = \begin{cases} \Omega\left(1\right), & s = \Theta\left(1\right) \\ \Omega\left(\dfrac{\log\log s(n)}{\log s}\right), & \Omega\left(1\right) = s = O\left(S_1\right) \\ \Omega\left(\dfrac{\sqrt{\frac{n}{\log n}}}{s}\right), & s = \Omega\left(S_1\right) \end{cases} \tag{6.7}$$

*where* $S_1 = \Theta\left(\sqrt{\frac{n}{\log n}}\frac{\log n}{\log\log n}\right)$.

In the rest of this section, we first describe the access scheme in the *MRWN*, then we present the "straight line" routing scheme. In the end, the achievable lower bound of *Theorem* 6.3 is proven.

### 6.3.1 Access Scheme

We first divide the network into cells whose length is $s_n = K_3\sqrt{\frac{\log n}{n}}$ to guarantee the connectivity between cells in the the network. The cells are divided into $M$ groups and in each time slot, only one group of cells are activated as shown in Fig. 5.1. Notice that we use the letter $M$ instead of $K_6$ in Fig. 5.1. The cell separation is such that successful communication based on protocol model is guaranteed when the cells in one group are activated simultaneously. The value of $M$ is derived in the following lemma which was originally proposed in [16].

**Lemma 6.8** *There exits a positive integer $M = K_4(1+\Delta)^2$ such that if we divide the network into $M$ non interfering groups, then all the cells in a group can communicate every $M$ time slots.*

### 6.3.2 Routing Scheme

The routing scheme is "straight line" routing that was described originally in [16]. In this routing scheme, for each source, we randomly and uniformly pick a location in the network and choose the closest node to this location as the destination for the source. The routing trajectory is a straight line $L_i$ from the source node to this destination. Then the packets traverse from each source to the destination in a multihop fashion passing through all the cells that cross $L_i$.

### 6.3.3 Traffic In Each Cell

- preliminaries

  The achievable lower bound capacity is directly related to the number of lines passing through each cell. This achievable rate is proportional to the inverse of the number of lines passing through each cell.

  There are two types of traffics in a cell. One type of traffic is caused by relays in the cell and the other one is caused by the sources and destinations in the cell. Unlike the cases in [2, 23] where the traffic is dominated by the latter traffic, in *MRWN* this assumption is not correct. In order to compute these two traffic types in *MRWN*, we first present the Markov's and Chebyshev's inequalities [17] without any proof.

**Lemma 6.9** Markov Inequality:

*If $X$ is any random variable and $a > 0$, then*

$$\Pr\left(|X| \geq a\right) \leq \frac{\mathrm{E}\left(|X|\right)}{a} \tag{6.8}$$

**Lemma 6.10** Chebychev Inequality:

*Let $X$ be a random variable with mean and standard deviation of $\mu_x$ and $\sigma_x$ respectively. Then*

$$\Pr(|X - \mu_x| \geq \alpha_x) \leq \frac{\sigma_x^2}{\alpha_x^2} \tag{6.9}$$

*for any any $\alpha_x > 0$.*

- The Traffic Caused by Sources and Destinations

The traffic generated by the sources and the destinations in a cell is given by the following theorem.

**Theorem 6.11** *In* MRWN, *the maximum number of traffic $T_{SD}$ caused by the* *sources and destinations in a cell is*

$$T_{SD} = \begin{cases} \Theta\left(\frac{\log\left(\frac{n}{\log n}\right)}{\log\left(\frac{\frac{n}{\log n}}{s(n)}\right)}\frac{\log s}{\log\log s(n)}\right), & s = O\left(\frac{n}{\log^2 n}\right) \\ \Theta\left(\frac{\log\left(\frac{n}{\log n}\right)}{\log\left(\frac{\frac{n}{\log n}\log\left(\frac{n}{\log n}\right)}{s}\right)}\frac{\log s}{\log\log s}\right), & \Omega\left(\frac{n}{\log^2 n}\right) \\ & = s = o\left(n\right) \\ \Theta\left(\log\left(\frac{n}{\log n}\right)\frac{\log s}{\log\log s}\right), & s = \Theta\left(n\right) \end{cases} \tag{6.10}$$

.

**proof 6.12** *Let's denote the number of traffic caused by sources in each cell as* $T_S$ *and the number of traffic caused by destinations in each cell as* $T_D$*. Clearly,*

$$T_{SD} \leq \max\left\{T_S + T_D\right\} \leq \max\left\{T_S\right\} + \max\left\{T_D\right\} \tag{6.11}$$

*Therefore, we need to compute both* $\max\left\{T_S\right\}$ *and* $\max\left\{T_D\right\}$*.*

- *Maximum Traffic Caused by Sources in Each Cell:*

  *In each cell, since one source can only contribute to one flow, the maximum traffic caused by the sources in one cell is equal to the maximum number of sources in that cell. Since the side length of each cell is* $K_3\sqrt{\frac{\log n}{n}}$*, the total number of cells in the network is* $\frac{n}{K_3^2 \log n}$*. We can apply the classical bin-ball problem by considering sources as balls and the cells as bins. By using*

Lemma *5.6, w.h.p., the maximum number of sources in each cell is*

$$\max\{T_S\} = \begin{cases} \Theta\left(\dfrac{\log\left(\frac{n}{\log n}\right)}{\log\left(\frac{\frac{n}{\log n}}{s}\right)}\right), & s = O\left(\dfrac{n}{\log^2 n}\right) \\ \Theta\left(\dfrac{\log\left(\frac{n}{\log n}\right)}{\log\left(\frac{\frac{n}{\log n}\log\left(\frac{n}{\log n}\right)}{s}\right)}\right), & \Omega\left(\dfrac{n}{\log^2 n}\right) = s = o\left(n\right) \\ \Theta\left(\log\left(\dfrac{n}{\log n}\right)\right), & s = \Theta\left(n\right) \end{cases} \tag{6.12}$$

*In this derivation, the range of $s$ is computed by assuming $\log \frac{n}{\log n} \cong \log n$.*

– *Maximum Traffic Caused by Destinations in Each Cell:*

*Due to the randomness of the source-destination selection, there may exist several sources which have the same destination. Now, lets fit this problem into the bin-ball problem again. The balls and bins represent the sources and the destinations respectively. From* Lemma *5.6, it is clear that there are at most $\Theta\left(\frac{\log s}{\log\log s}\right)$ sources w.h.p. for each destination*[2]*. Note that the maximum number of destinations in each cell is the same as that of sources or equivalently* $\max\{T_S\}$ *given by (6.12). Thus* $\max\{T_D\}$ *is given by*

$$\max\{T_D\} = \max\{T_S\} \cdot \Theta\left(\frac{\log s}{\log\log s}\right). \tag{6.13}$$

[2]Note that we used the second line in the equation of *Lemma* 5.6 because the number of sources and destination is equal to $s$.

*Therefore, we arrive at*

$$\begin{aligned} T_{SD} &\leq \max\{T_S\} + \max\{T_D\}, \\ &= \max\{T_S\} + \max\{T_S\}\,\Theta\left(\frac{\log s}{\log\log s}\right), \\ &= (\max\{T_S\} + 1)\,\Theta\left(\frac{\log s}{\log\log s}\right). \end{aligned} \tag{6.14}$$

*By combining (6.12) and (6.14), the theorem follows.*

- Traffic Caused by Relays

In this section, we compute the traffic caused by relays. First, we introduce a lemma from [16].

**Lemma 6.13** *In a network with uniform distribution of nodes, there exists a positive constant* $K_5$ *such that for every line* $L_i$ *and cell* $C_j$*,*

$$\Pr\left(\textit{Line } L_i \textit{ intersects } C_j\right) \leq K_5\sqrt{\frac{\log n}{n}} \tag{6.15}$$

The maximum traffic caused by relays is given by the following theorem.

**Theorem 6.14** *In* MRWN, *the maximum relay traffic $T_R$ in a cell is given by*

$$\max\{T_R\} = \begin{cases} \Theta(1), & \Omega(1) = s = o\left(\sqrt{\dfrac{n}{\log n}}\right) \\ \Theta(f(n)), & s = \Theta\left(\sqrt{\dfrac{n}{\log n}}\right) \\ \Theta\left(s\sqrt{\dfrac{\log n}{n}}\right), & s = \omega\left(\sqrt{\dfrac{n}{\log n}}\right) \end{cases} \tag{6.16}$$

*where $f(n)$ can be any function of $n$ that fulfills the condition $\lim_{n\to\infty} f(n) = \infty$.*

**proof 6.15** *In this proof, we divide $s$ into three regions which are $s = \Theta(1)$, $\omega(1) = s = O\left(\sqrt{\frac{n}{\log n}}\right)$ and $s = \omega\left(\sqrt{\frac{n}{\log n}}\right)$.*

- *Case of $s = \Theta(1)$:*

  *In this region, $s$ is a positive constant. Clearly, the traffic caused by relays is at most equal to $s$. Thus, $\max\{T_R\} = \Theta(1)$.*

- *Case of $\omega(1) = s = O\left(\sqrt{\frac{n}{\log n}}\right)$:*

  *In this region, we will prove that when $\omega(1) = s = o\left(\sqrt{\frac{n}{\log n}}\right)$, then $\max\{T_R\} = \Theta(1)$. When $s = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$, then $\max\{T_R\} = \Theta(f(n))$, where $f(n)$ is defined above.*

  *We first introduce the following lemma.*

  ***Lemma 6.16*** *In* MRWN, *when the "straight line" routing scheme is used* *and $k$ is a positive constant, we have*

$$\begin{aligned}
&\Pr\left(k \textit{ lines intersecting any cell } C_j\right) \\
&\leq \binom{s}{k}\left(K_5\sqrt{\frac{\log n}{n}}\right)^k\left(1-K_5\sqrt{\frac{\log n}{n}}\right)^{s-k}
\end{aligned} \tag{6.17}$$

*This lemma is proved in the Appendix 8.5.*

*For $s=\omega(1)$ in conjunction with* Lemma *6.16 and large $n$, we arrive at*

$$\begin{aligned}
&\Pr\left(k \textit{ lines intersecting any cell } C_j\right) \\
&\leq \lim_{s\to\infty}\binom{s}{k}\left(K_5\sqrt{\frac{\log n}{n}}\right)^k\left(1-K_5\sqrt{\frac{\log n}{n}}\right)^{s-k}, \\
&= \lim_{s\to\infty}\binom{s}{k}\left(\frac{K_5 s\sqrt{\frac{\log n}{n}}}{s}\right)^k\left(1-\frac{K_5 s\sqrt{\frac{\log n}{n}}}{s}\right)^{s-k}, \\
&\leq \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{k!}\exp\left(-K_5 s\sqrt{\frac{\log n}{n}}\right).
\end{aligned} \tag{6.18}$$

*The last line of equation is derived by considering the fact that it is Poisson distribution. Let's define $K_6$ as the maximum number of lines passing through* *each cell, then one can obtain*

$$
\begin{aligned}
&\Pr\left(k \le K_6\right) \\
&= \sum_{k=0}^{K_6} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{k!} \exp\left(-K_5 s\sqrt{\frac{\log n}{n}}\right) \\
&\overset{(a)}{=} \frac{\sum_{k=0}^{K_6} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{k!}}{\sum_{i=0}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^i}{i!}} \\
&= \frac{\sum_{k=0}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{k!} - \sum_{k=K_6+1}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{k!}}{\sum_{i=0}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^i}{i!}} \\
&= 1 - \frac{\sum_{k=K_6+1}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{k!}}{\sum_{i=0}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^i}{i!}} \\
&\overset{(b)}{\geq} 1 - \frac{\sum_{k=K_6+1}^{\infty} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^k}{K_6+1}}{\exp\left(K_5 s\sqrt{\frac{\log n}{n}}\right)} \\
&= 1 - \frac{\frac{1}{K_6+1} \frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^{K_6+1}}{1-K_5 s\sqrt{\frac{\log n}{n}}}}{\exp\left(K_5 s\sqrt{\frac{\log n}{n}}\right)}
\end{aligned}
\tag{6.19}
$$

*(a) and (b) are due to the Maclaurin series that* $\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. *Now there are two cases with respect to* $s$.

- *When* $s = o\left(\sqrt{\frac{n}{\log n}}\right)$*:*

*Since $K_6$ is a constant value and as $n$ tends to infinity, from (6.19) it can be concluded that*

$$\Pr\left(k \le K_6\right) = \lim_{n\to\infty} 1 - \frac{\frac{1}{K_6+1}\frac{\left(K_5 s\sqrt{\frac{\log n}{n}}\right)^{K_6+1}}{1-K_5 s\sqrt{\frac{\log n}{n}}}}{\exp\left(K_5 s\sqrt{\frac{\log n}{n}}\right)}$$

$$= 1, \tag{6.20}$$

*which implies that* $\max\left\{T_R\right\} \le K_6$.

∗ *When $s = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$:*

*In this case, (6.19) cannot be used any more. However, Markov inequality in* Lemma *(6.9) implies that*

$$\begin{aligned}\Pr\left(T_R \ge K_6\right) &\le \frac{\mathrm{E}\left(T_R\right)}{K_6}, \\ &= \frac{\mathrm{E}\left(K_5 s\sqrt{\frac{\log n}{n}}\right)}{K_6}, \\ &= \frac{K_7}{K_6},\end{aligned} \tag{6.21}$$

*where $K_7$ is a positive constant. From (6.21), we can see that if $K_6 = f(n)$ where $f(n)$ is an arbitrary function of $n$ satisfying* $\lim_{n\to\infty} f(n) = \infty$*, then as $n$ goes to infinity, we can have*

$$\Pr\left(T_R \ge K_6\right) = 0. \tag{6.22}$$

*Thus* $\max\{T_R\} \leq f(n)$.

– *Case of* $s = \omega\left(\sqrt{\frac{n}{\log n}}\right)$*:*

*In this region of* $s$*, we will show that* $\max\{T_R\} = O\left(s\sqrt{\frac{\log n}{n}}\right)$.

*Let's assume* $\lim_{n\to\infty} \frac{\alpha}{sK_5\sqrt{\frac{\log n}{n}}} = \varepsilon$ *where* $\varepsilon$ *is a positive constant. By utilizing Chebychev inequality in* Lemma *6.10, it can be shown that*

$$\begin{aligned}
&\Pr\left(|T_R - sK_5\sqrt{\frac{\log n}{n}}| \geq \sqrt{\alpha s\sqrt{\frac{\log n}{n}}}\right) \\
&\leq \frac{sK_5\sqrt{\frac{\log n}{n}}\left(1 - K_5\sqrt{\frac{\log n}{n}}\right)}{\alpha sK_5\sqrt{\frac{\log n}{n}}} \\
&= \frac{1 - K_5\sqrt{\frac{\log n}{n}}}{\alpha} \qquad (6.23)
\end{aligned}$$

*which goes to zero as* $n$ *tends to infinity and we used the fact that the random variable has Poisson distribution. Thus* $\max\{T_R\} \leq sK_5\sqrt{\frac{\log n}{n}}$*. Notice that from this result we can find that when* $s = n$*,* $T_R = O\left(\sqrt{n\log n}\right)$ *which is the same as the result in [2]. Note that under this condition, the traffic is dominated by relays and source or destination traffic in each cell are simply negligible.*

*Thus, we finish the proof of* Theorem *6.14.*

### 6.3.4 Achievable Throughput of *MRWN*

The total traffic in any cell can be obtained by using the results shown above and that is summarized here.

$$
\begin{aligned}
T_{\text{total}} &\le \max\{T_{\text{SD}}\} + \max\{T_{\text{R}}\} \\
&= \begin{cases} O(1), & s(n) = \Theta(1) \\ O\left(\dfrac{\log\left(\frac{n}{\log n}\right)}{\log\left(\frac{\frac{n}{\log n}}{s}\right)} \dfrac{\log s}{\log\log s}\right), & \Omega(1) = s = O(S_1) \\ O\left(s\sqrt{\dfrac{\log n}{n}}\right), & s = \Omega(S_1) \end{cases}
\end{aligned} \tag{6.24}
$$

where $S_1 = \Theta\left(\sqrt{\frac{n}{\log n}} \frac{\log n}{\log\log n}\right)$. From *Lemma* 6.8, we can find that there exists a transmission scheme such that in every $M = K_4(1+\Delta)^2$ slots, each cell can get one slot to send packets at a rate $W$ bits/second. Thus, the rate for each cell is $\frac{W}{K_4(1+\Delta)^2}$. From Eq.(6.24), each cell can send packets at a rate equal to $T_{\text{total}}$ with probability one as $n$ goes to infinity. Therefore, the maximum achievable throughput $C_{\text{Lower}}$ for each source should satisfy

$$
R(n)_{\text{MRWN}} T_{\text{total}} = \frac{W}{K_4(1+\Delta)^2}. \tag{6.25}
$$

Hence

$$
R(n)_{\text{MRWN}} = \frac{\frac{W}{K_4(1+\Delta)^2}}{T_{\text{total}}} \ge \frac{\frac{W}{K_4(1+\Delta)^2}}{\max\{T_{\text{SD}}\} + \max\{T_{\text{R}}\}} \tag{6.26}
$$

*Theorem* 6.7 follows immediately. Note that in derivation of second line in

*Theorem* 6.7, we ignore the term $\frac{\log\left(\frac{n}{\log n}\right)}{\log\left(\frac{\frac{n}{\log n}}{s}\right)}$ because this term is asymptotically equal to $\Theta(1)$.

### 6.3.5 The Total Achievable Throughput

By using *Theorem* 6.7, *Lemma* 6.1 and Eq. 6.6, we can have that

$$R(n)_{\text{NCo}} = s \cdot R(n)_{\text{MRWN}} + R(n)_{DG}$$

which finish the proof of *Theorem* 6.3.

## 6.4 Discussion

In this subsection, we discuss some implication of the result obtained in this section.

### 6.4.1 The Comparison of the Results in Chapter 5 and Chapter 6

In this paper, we derived the upper bound of the capacity and achievable aggregate throughput of the network with a specific type of heterogeneous traffic including unicast communication and data-gathering traffic. For the derivation, we utilize two transmission models which are information theoretical model and protocol model. In the information theoretical model, we allow any transmission scheme and the actual maximum information that can be supported in the network is obtained. While the protocol model is a more practical more since only the non-cooperative routing scheme is allowed. Moreover, the point-to-point communication is constrained to a constant number. Thus, we can expect that the capacity result for the protocol model is much smaller than that of information theoretical model. Our analytical results confirmed this intuitive observation. In our result, the upper bound of the capacity for different models are shown in Eq. (5.4) and (6.1). Clearly, the upper bound of the capacity under information theoretical model is much higher than the upper bound of the capacity under protocol model. Similar, from the results of achievable aggregate throughput under both information theoretical model and protocol as shown in Eq.(5.9) and (6.3) respectively, we can see that the achievable lower bound under information theoretical model is much higher than that under protocol model. All of this is mainly due to the use of cooperative transmission scheme.

### 6.4.2 The Capacity and Gains from Multihop Relays for *MRWN*

From the upper bound of the capacity for the network with heterogeneous traffic as shown in (6.1), we can easily get the per-source capacity of *MRWN* as shown below.

$$R(n)_{\mathrm{MRWN}} = O\left(\min\left\{\frac{\sqrt{\frac{n}{\log n}}}{s}, 1\right\}\right) \tag{6.27}$$

From Section 6.3.1, 6.3.2 and 6.3.4, we have the lower bound of the achievable throughput per node shown as in (6.7). Thus, we have the capacity result of *MRWN* shown in Fig 6.3. From Fig 6.3, we can see that in most of the regions of $s$, the achievable lower bound of the throughput meets the upper bound of the capacity, however, in the region

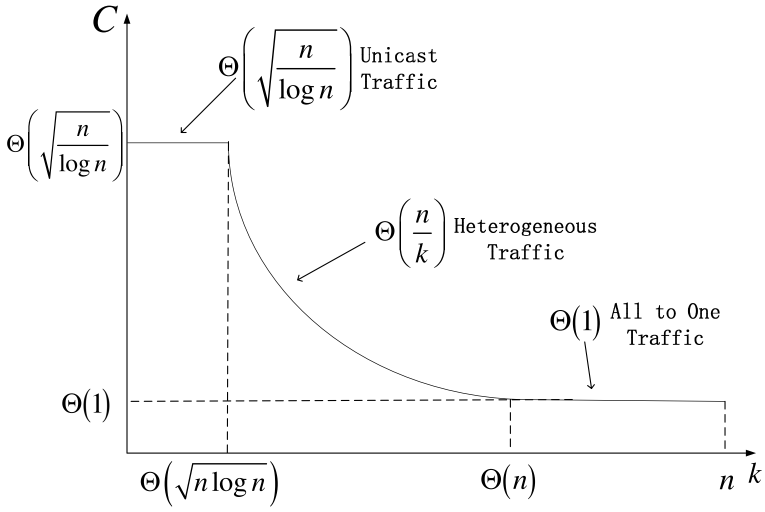


Figure 6.3: The upper and lower bound of the capacity in *MRWN* along with capacity of networks with no relays. The bold line is the upper bound of the capacity and the thin line is the lower bound of the capacity. In the regions of $(1, \Theta(1)]$ and $[\Theta(S_1), n)$, the upper and lower bounds are tight. In this figure, $S_1 = \Theta\left(\sqrt{\frac{n}{\log n}\frac{\log n}{\log\log n}}\right)$, $S_2 = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$.

of $[\Theta(1), S_1]$, there is a gap between the lower bound and upper bound. We will give the analysis of this phenomenon in Section 6.4.3. Moreover, form Fig. 6.3, we can see that achivable throughput of *MRWN* is much higher than the achievable throughput in the network without relays, which is shown in the dotted line of Fig. 6.3. Clearly, this gain from the relays in *MRWN*. We will give the analysis of the gain from the relays in the rest of this subsection.

From the definition of the network model, we know that the relays in *MRWN* only utilize the decode-and-forward operation to help the source to transmit information without any cooperation. Even with no cooperation, the network with relays provide order throughput gain compared to the case of no relays in the network. The throughput of this type of network with no relay, $s$ source-destination pairs and by only using plain multihop routing scheme is given in [2], which is $\Theta\left(\frac{1}{\sqrt{s\log s}}\right)$[3]. Therefore, the gains $G$ from the relays in the network is given by

$$G = \frac{R(n)_{\mathrm{MRWN}}}{\Theta\left(\frac{1}{\sqrt{s\log s}}\right)} = \begin{cases} \Omega\left(\sqrt{s\log s}\right), & s = \Theta(1) \\ \Omega\left(\log\log s\sqrt{\dfrac{s}{\log s}}\right), & \omega(1) = s = O(S_1) \\ \Omega\left(\sqrt{\dfrac{n}{s}\dfrac{\log s}{\log n}}\right), & s = \Omega(S_1) \end{cases} \tag{6.28}$$

The first interesting observation from (6.28) is the fact that by increasing $s$, the gain $G$ in some regions increases while in other regions decreases. More specifically, when $s = \Omega(S_1)$ then the gain $G$ decreases with the increase of $s$ and when $s = O(S_1)$, $G$

[3] Note that unlike [2] that considers $n$ source-destination pairs, we assume $s$ source-destination pairs in order to compare it with our technique

increases with the increase in $s$.

When $s = \Theta(S_1)$, the gain $G$ obtains the maximum value which means in order to get the optimal gain from relays, we need to make the traffic caused by the relays be comparable to the traffic caused by the sources and destinations in a cell. The reason of this interesting result is that in the case that $s = \Omega(S_1)$, as the increase of $s$, the number of relays becomes smaller and smaller, so the gain caused by relays decreases. While in the case that $s = O(S_1)$, $s$ is so small that not all of the relays can work in parallel, so as the increase of $s$, more and more relays help in the network for transportation of information, thus the gain obtained from relays increases with the increase of $s$. When all the relays helps sources in delivery of data and $s$ is not too large, the gain $G$ achieves its optimal value.

We know that in *MRWN*, the transmission scheme is plain multihop routing without any cooperative scheme. One important question is "why do we achieve such order gains in throughput capacity by using simple routing and relays?" This gain mainly comes from the increase in the number of concurrent transmissions by increasing the number of relays. When the total network area is constant and we we increase the number of relays, the distance between nodes decreases which can result in decrease in transmission range. By decreasing the transmission range, one can increase the number of concurrent transmissions in the network which is much larger than the increase in the number of hops due to reduction in transmission range. Next figure demonstrates the gain from relays in the networks compared to the case of no relay utilizing multihop point-to-point communications.

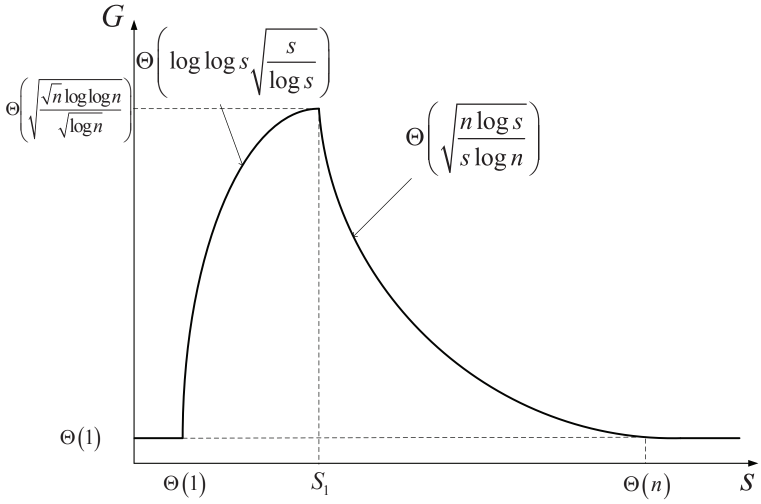


Figure 6.4: The gain in *MRWN* compared to that of networks with no relays. In both schemes, simple point-to-point communication protocol is utilized. In this figure, $S_1 = \Theta\left(\sqrt{\frac{n}{\log n}\frac{\log n}{\log\log n}}\right)$.

### 6.4.3 Gap Between Lower and Upper Bounds for MRWN

Another important observation is that, from Fig. 6.1, there is a gap between the lower and upper bound of the capacity for *MRWN* in the region of $s = [1, S_1]$. In the region of $s = [S_1, n]$, it is clear that the main restriction of the network is from the traffic caused by relays, while in the region of $s = [1, S_1]$, the main bottleneck of the network comes from the traffic caused by the sources and destinations in each cell. From the bin-ball problem, we observe that this gap comes from the combination of the maximum number of destinations in each cell and the maximum number of sources for each destination. The upper bound of the traffic in each cell is caused by these two values. These maximum numbers are much larger than the mean number of destinations in each cell or mean number of sources for each destination. This large difference is caused by the randomness of the network. However, if we let each cell have the same number of sources and destinations, and the traffic becomes the permutation traffic which means that each destination only has one source, then this gap will disappear.

This observation gives us a hint that randomness can lead to the loss of the capacity in the real network. In cells with dominant traffic dictate the throughput capacity for that network. In order to achieve higher capacity, we should do more averaging and remove the randomness.

## 6.5 Conclusion

In this chapter, we give an analysis of the capacity for the wireless network with heterogeneous traffic. First, we proposed a *separation theorem* which tells us that by distributing different radio to different traffic type in the network, the optimum capacity can be obtained from the order point of view.

Second, we analyzed the capacity of the wireless network with a specific traffic type which includes a unicast communication session and a data-gathering traffic. Under the *separation theorem*, we used a cooperative three-phase scheme and two-phase scheme for the network with unicast communication traffic and with data-gathering traffic respectively, and obtained a achievable throughput for the network with this heterogeneous traffic type. In some regions of our result, the achievable lower bound of the throughput does not meet the upper bound of the capacity, however, it is the best result so far as we know. Moreover, it is clearly that the network with unicast traffic is large relay network which is not quite well-studied. In the literature, we only know the results that only a constant number of source-destination pairs or all the nodes are sources or destinations, in this paper, we give a results that when the number of source-destination pairs is a function of the number of the total nodes in the network.

Third, under the same heterogeneous traffic assumption, we proposed the scaling laws of the capacity when a more practical protocol model is assumed and a more practical pure routing scheme is utilized. The results indicate that by using the simpler routing scheme, both of the upper bound of the capacity and the achievable throughput in the network is decreased significantly. However, we believe that the delay requirements for the pure routing scheme is much less than that of cooperative transmission scheme. The delay analysis by using both scheme is our topic for future study. Moreover, for the network with unicast communication, since only multihop pure routing scheme is allowed, we call this network *MRWN*. By our analysis, we can find that the network can still get big gains from the relays by only using pure routing scheme and the the largest gain can be obtained in some value of source-destination pairs in the network.

# Chapter 7

# Conclusion and Future Work

## 7.1 Conclusion

In this thesis, the capacity of wireless ad-hoc networks with heterogeneous traffic has been studied. Specifically, we studied one particular type of heterogeneous traffic which is that there are two types of traffic in the network. One is uniform unicast. The other is data-gathering traffic. However, thanks to the *Separation Theorem* introduced in this thesis, we can extend our method to any constant number of traffic in the network, which is the reality. Under different network configurations, the capacity of the network with heterogeneous traffic are different.

In this thesis, first, we studied the scaling law of the capacity of the network by protocol model. In this model, we assume the bandwidth of each node is proportional to the traffic of the corresponding cell or cluster. Under protocol model, the capacity is scaled according to the number of nodes performing the data-gathering traffic.

Second, we studied the similar network model as the first one but under physical model. In this part, besides the assumption that the bandwidth of each node is proportional to the traffic of the corresponding cell or nodes, we also assume the power of each node for the unit bandwidth is a constant number. Thus, we get a same capacity result as the first part of the thesis.

Third, we released the assumption of the bandwidth and study the same network under information theoretical model. In this chapter, we introduce an important theorem which is the *Separation Theorem* stating that as long as we distribute certain bandwidth to each type of traffic in the network, then we can get the optimum scaling of the capacity in the network if the number of traffic type is a constant. By using this theorem, and the so-called *Three Phase* schemes which are modified Hierarchical Cooperative MIMO approach first introduced in [1], the aggregate throughput of the network is obtained and is the optimum results so far.

Fourth, due to the complexity of the *Three Phase* schemes, we studied the same problem by under protocol model again and released the bandwidth assumption. Under *Separation Theorem*, we have a significant gain comparing to the result with the bandwidth assumption. This result indicates that the assumption of the bandwidth can waste the bandwidth significantly.

## 7.2 Future Work

The scaling laws of the ad-hoc networks with heterogeneous traffic have been studied in this work. However, the scaling laws are only asymptotic results which means that the number of nodes have to go to infinity or a very large number to make the result reliable. However, in practice, it is not the case. The more valuable result is the real capacity of the ad-hoc networks not just the scaling laws. Thus, in the future, we will investigate and study the actual capacity of the wireless ad-hoc networks with or without heterogeneous properties.

# Chapter 8

# Appendix

## 8.1 Proof of Theorem 5.7

For the number of source nodes in each cluster, we can consider the problem as bins and balls problem. Here the number of balls is $s$ and the number of bins is $s^{1+\varepsilon_2}$. By using Lemma 5.6, the maximum number of source nodes in each cluster is given by

$$\begin{aligned} B(s, s^{1+\varepsilon_2}) &= \Theta\left(\frac{\log s^{1+\varepsilon_2}}{\log \frac{s^{1+\varepsilon_2}}{s}}\right), \\ &= \Theta\left(\frac{(1+\varepsilon_2)\log s}{\varepsilon_2 \log s}\right) = \Theta(1). \end{aligned} \tag{8.1}$$

For the number of the nodes in each cluster, we can use Chebychev's inequality given in *Lemma* 6.10. Assume $\alpha_x = \sqrt{\alpha \times \frac{n}{s^{1+\varepsilon_2}}}$ where $\alpha$ is defined as a sequence such that $\lim_{\frac{n}{s^{1+\varepsilon_2}} \to \infty} \frac{\alpha}{\frac{n}{s^{1+\varepsilon_2}}} = \gamma_1$ for any positive value of $\gamma_1$. Define the random variable $V_n$ as the number of nodes in each cluster. Because the nodes uniformly distributed in the network, then the mean and variance of $V_n$ is given by $\mu_x = \frac{n}{s^{1+\varepsilon_2}}$ and $\sigma_x^2 = \frac{n}{s^{1+\varepsilon_2}}\left(1 - \frac{1}{s^{1+\varepsilon_2}}\right)$ respectively. By using Chebychev's inequality, we arrive at

$$\begin{aligned}\Pr\left(|V_n - \frac{n}{s^{1+\varepsilon_2}}| \geq \sqrt{\alpha \times \frac{n}{s^{1+\varepsilon_2}}}\right) &\leq \frac{\frac{n}{s^{1+\varepsilon_2}}\left(1 - \frac{1}{s^{1+\varepsilon_2}}\right)}{\alpha \times \frac{n}{s^{1+\varepsilon_2}}} \\ &\leq \frac{1}{\alpha}\left(1 - \frac{1}{s^{1+\varepsilon_2}}\right)\end{aligned} \tag{8.2}$$

The second term on the right hand side of (8.2) goes to zero as $\frac{n}{s^{1+\varepsilon_2}} \rightarrow \infty$. Thus with probability close to one $|V_n - \frac{n}{s^{1+\varepsilon_2}}| \leq \sqrt{\alpha \times \frac{n}{s^{1+\varepsilon_2}}}$ or equivalently, $V_n = \Theta\left(\frac{n}{s^{1+\varepsilon_2}}\right)$. Similarly, it can be proved that in a circle with radius of $\frac{\sqrt{A}}{2s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}$ or $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$, the number of nodes is $\Theta\left(\frac{n}{s^{1+\varepsilon_2+\varepsilon_3}}\right)$ or $\Theta\left(\frac{n}{n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}\right)$ respectively.

## 8.2 Proof of Theorem 5.8

To prove *Theorem* 5.8, we consider two cases. First, we calculate the aggregate throughput when $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$. Then, we consider the case when $s = O\left(n^{\beta_1}\right)$.

### 8.2.1 When $\Omega\left(n^{\beta_1}\right) = s = O\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}}\right)$

The capacity between a source node $k$ and the relay $i$ is

$$C_{ki}^{\text{Case 1}} = W_1 \log\left(1 + \frac{\frac{P_1'}{s}|H_{ki}|^2}{W_1 N_0 + \sum_{j\in T, j\neq k} \frac{P_1'}{s}|H_{ji}|^2}\right)$$

$$= W_1 \log\left(1 + \frac{\frac{P_1'}{s}\left(\frac{\sqrt{G}}{d_{ki}^{\frac{\alpha}{2}}}\right)^2}{W_1 N_0 + \sum_{j\in T, j\neq k} \frac{P_1'}{s}\left(\frac{\sqrt{G}}{d_{ji}^{\frac{\alpha}{2}}}\right)^2}\right)$$

$$\geq W_1 \log$$

$$\left(1 + \frac{\frac{P_1'}{s}\left(\frac{\sqrt{G}}{\left(\frac{\sqrt{A}}{s^{\frac{1+\varepsilon_2+\varepsilon_3}{2}}}\right)^{\frac{\alpha}{2}}}\right)^2}{W_1 N_0 + \sum_{l=1}^{\infty} 8l \frac{P_1'}{s}\left(\frac{\sqrt{G}}{\left((l\sqrt{K_6}-1)\frac{\sqrt{A}}{s^{\frac{(1+\varepsilon_2)}{2}}}\right)^{\frac{\alpha}{2}}}\right)^2}\right)$$

$$= W_1 \log$$

$$\left(1 + \frac{\frac{P_1'G}{A^{\frac{\alpha}{2}}} s^{\frac{(1+\varepsilon_2+\varepsilon_3)\alpha}{2}-1}}{W_1 N_0 + \left(\frac{P_1'G}{A^{\frac{\alpha}{2}}}\sum_{l=1}^{\infty}\frac{8l}{(l\sqrt{K_6}-1)^{\alpha}}\right) s^{\frac{(1+\varepsilon_2)\alpha}{2}-1}}\right)$$

$$> W_1 \log\left(\frac{\frac{P_1'G}{A^{\frac{\alpha}{2}}} s^{\frac{(1+\varepsilon_2+\varepsilon_3)\alpha}{2}-1}}{2\left(\frac{P_1'G}{A^{\frac{\alpha}{2}}}\sum_{l=1}^{\infty}\frac{8l}{(l\sqrt{K_6}-1)^{\alpha}}\right) s^{\frac{(1+\varepsilon_2)\alpha}{2}-1}}\right)$$

$$
\begin{aligned}
&= W_1 \log\left(\frac{1}{2\sum_{l=1}^{\infty}\frac{8l}{(l\sqrt{K_6}-1)^{\alpha}}}\right) + W_1 \log\left(s^{\frac{\varepsilon_3\alpha}{2}}\right) \\
&\overset{(a)}{\geq} K_5 + W_1 \log\left(\left(n^{\beta_1}\right)^{\frac{\varepsilon_3\alpha}{2}}\right) \\
&= K_5 + M_1 \log n,
\end{aligned} \tag{8.3}
$$

where inequality $(a)$ is derived because when $\alpha > 2$, then $\sum_{l=1}^{\infty}\frac{8l}{(l\sqrt{K_6}-1)^{\alpha}}$ converges to a constant value and $M_1$ is a positive constant value.

### 8.2.2 When $s = O\left(n^{\beta_1}\right)$

Under the condition it is easy to show that $\frac{\sqrt{A}}{s^{\frac{1+\varepsilon_2}{2}}} > \frac{\sqrt{A}}{n^{\frac{(1+\varepsilon_2)\beta_2}{2}}} > \frac{\sqrt{A}}{n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$ for appropriate value of $\beta_2$ such that $\beta_2 < \frac{[(1+\varepsilon_2)\alpha-2]}{(1+\varepsilon_2)\alpha}\beta_1$.

If the source node transmits information within a circle of radius $\frac{\sqrt{A}}{2n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}$, then the achievable rate between nodes $k$ and $i$ is given below.

$$
\begin{aligned}
&C_{ki}^{\text{Case 2}} \\
&= W_1 \log\left(1+\frac{\frac{P_1'}{s}\left(\frac{\sqrt{G}}{d_{ki}^{\frac{\alpha}{2}}}\right)^2}{W_1N_0+\sum_{j\in T, j\neq k}\frac{P_1'}{s}\left(\frac{\sqrt{G}}{d_{ji}^{\frac{\alpha}{2}}}\right)^2}\right) \\
&\geq W_1 \log \\
&\left(1+\frac{\frac{P_1'}{s}\left(\frac{\sqrt{G}}{\left(\frac{\sqrt{A}}{n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}\right)^{\frac{\alpha}{2}}}\right)^2}{W_1N_0+\sum_{l=1}^{\infty}8l\frac{P_1'}{s}\left(\frac{\sqrt{G}}{\left((l\sqrt{K_6}-1)\frac{\sqrt{A}}{s^{\frac{(1+\varepsilon_2)}{2}}}\right)^{\frac{\alpha}{2}}}\right)^2}\right) \\
&\overset{(a)}{\geq} W_1 \log \\
&\left(1+\frac{\frac{P_1'}{n^{\beta_1}}\left(\frac{\sqrt{G}}{\left(\frac{\sqrt{A}}{n^{\frac{(1+\varepsilon_2)\beta_1}{2}}}\right)^{\frac{\alpha}{2}}}\right)^2}{W_1N_0+\sum_{l=1}^{\infty}8l\frac{P_1'}{1}\left(\frac{\sqrt{G}}{\left((l\sqrt{K_6}-1)\frac{\sqrt{A}}{n^{\frac{(1+\varepsilon_2)\beta_2}{2}}}\right)^{\frac{\alpha}{2}}}\right)^2}\right) \\
&= W_1 \log\left(1+\frac{\frac{P_1'G}{A^{\frac{\alpha}{2}}}n^{\frac{(1+\varepsilon_2)\alpha\beta_1}{2}-\beta_1}}{W_1N_0+\left(\frac{P_1'G}{A^{\frac{\alpha}{2}}}\sum_{l=1}^{\infty}\frac{8l}{(l\sqrt{K_6}-1)^{\alpha}}\right)n^{\frac{(1+\varepsilon_2)\alpha\beta_2}{2}}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&> W_1 \log \left( \frac{\frac{P_1'G}{A^{\frac{\alpha}{2}}} n^{\frac{(1+\varepsilon_2)\alpha\beta_1}{2} - \beta_1}}{2\left(\frac{P_1'G}{A^{\frac{\alpha}{2}}} \sum_{l=1}^{\infty} \frac{8l}{(l\sqrt{K_6}-1)^\alpha}\right) n^{\frac{(1+\varepsilon_2)\alpha\beta_2}{2}}} \right) \\
&= W_1 \log \left( \frac{1}{2\sum_{l=1}^{\infty} \frac{8l}{(2l-1)^\alpha}} \right) + W_1 \log \left( n^{\frac{(1+\varepsilon_2)\alpha(\beta_1-\beta_2)}{2} - \beta_1} \right) \\
&= K_5 + M_2 \log n \qquad (8.4)
\end{aligned}
$$

(a) is derived by replacing distance for interference with smaller distance and replacing $s$ in numerator by its maximum value and replace it with 1 in the denominator. Note that $M_2 = W_1 \left( \frac{(1+\varepsilon_2)\alpha(\beta_1-\beta_2)}{2} - \beta_1 \right)$ is a positive value given the condition above for $\beta_2$.

Thus, the achievable rate between nodes $k$ and $i$ is given by

$$
\begin{aligned}
R_{ki}(n) &= \min\left(C_{ki}^{\text{Case 1}}, C_{ki}^{\text{Case 2}}\right), \\
&= \min\left(M_1 \log n + K_5, M_2 \log n + K_5\right), \\
&= \min(M_1, M_2) \log n + K_5, \\
&= K_4 \log n + K_5 \qquad (8.5)
\end{aligned}
$$

where $K_4 = \min(M_1, M_2)$.

Given the TDMA parameter $K_6$, there are on average $\frac{s}{K_6}$ nodes sending their information. Thus, the aggregate throughput is given by

$$
R_{\text{Phase1}}(n) \geq \frac{s(K_4 \log n + K_5)}{K_6}. \qquad (8.6)
$$

## 8.3 Proof of Theorem 5.11

Under the *Many-to-One* transmission model, the capacity between source node $k$ and the relay node $i$ in the circle with radius $r = n^{-\varepsilon_5}$ is given by

$$\begin{aligned} C_{ki} &= W_2 \log\left(1 + \frac{\eta_4 P_2 |H_{ki}|^2}{W_2 N_0}\right) \\ &\geq W_2 \log\left(1 + \frac{\eta_4 P_2 \left(\frac{\sqrt{G}}{r^{\frac{\alpha}{2}}}\right)^2}{W_2 N_0}\right) \\ &= W_2 \log\left(1 + \frac{\eta_4 P_2 G}{W_2 N_0} n^{\varepsilon_5 \alpha}\right) \geq W_2 \log\left(\frac{\eta_4 P_2 G}{W_2 N_0} n^{\varepsilon_5 \alpha}\right) \\ &= W_2 \log\left(\frac{\eta_4 P_2 G}{W_2 N_0}\right) + W_2 \varepsilon_5 \alpha \log(n) = K_{24} + K_{23} \log n \end{aligned} \tag{8.7}$$

where $K_{23} = W_2 \varepsilon_5 \alpha$ and $K_{24} = W_2 \log\left(\frac{\eta_4 P_2 G}{W_2 N_0}\right)$.

## 8.4 Proof of Theorem 5.12

Similar to *Theorem* 5.7, we can prove that the number of nodes in the circle with radius of $n^{-\varepsilon_5}$ is $\Theta\left(n^{1-2\varepsilon_5}\right) = M_4 n^{1-2\varepsilon_5}$. Then the capacity is computed as

$$\begin{aligned} C_{\text{Many-to-One}} &= W_2 \log\left(1 + \frac{(1-\eta_4) P_2 \sum_{j=1}^{M_4 n^{1-2\varepsilon_5}} \left(\frac{\sqrt{G}}{d_j^{\frac{\alpha}{2}}}\right)^2}{W_2 N_0}\right) \\ &\geq W_2 \log\left(\frac{(1-\eta_4) P_2 G M_4}{W_2 N_0 A^{\frac{\alpha}{2}}} n^{1-2\varepsilon_5}\right) \\ &= K_{25} \log n + K_{26} \end{aligned} \tag{8.8}$$

where $K_{25} = 1 - 2\varepsilon_5$ and $K_{26} = W_2 \log\left(\frac{(1-\eta_4)P_2 M_4}{W_2 N_0 A^{\frac{\alpha}{2}}}\right)$.

## 8.5 The Proof of *Lemma* 6.16

We assume that $\Pr(\text{Line } L_i \text{ intersects } C_j) = p$. Since the events that different line $L_i$ intersects $C_j$ are independent, then

$$\begin{aligned} &\Pr(\text{There are k lines intersecting any cell } C_j) \\ &= \binom{s}{k} (p)^k (1-p)^{s-k} \end{aligned} \tag{8.9}$$

By doing the derivative of (8.9), one arrives at the maximum value of (8.9) given by $p = \frac{k}{s} = p^*$. when $p < p^*$, then this probability increases with increase in $p$. Let's assume $K_5\sqrt{\frac{\log n}{n}} = p^\dagger$. Since $s = O\left(\sqrt{\frac{n}{\log n}}\right)$, it is clear that $p^* \geq kK_6\sqrt{\frac{\log n}{n}}$, where $K_6$ is a positive constant. By allowing $k$ large enough, we obtain $p^\dagger \leq p^*$. By using *Lemma* 6.13, it is clear that $p \leq p^\dagger$. Due to monotonic increase of $\Pr(\text{There are k lines intersecting any cell } C_j)$ with the increase in $p$, the result follows.

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