Transmit Diversity for Arrays in Correlated Rayleigh Fading

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Abstract

Transmit diversity is usually presented for the case of independently faded channels. In this paper we derive the structure of a linear transmitter which can be optimized for a Rayleigh fading environment in which the fading may be correlated. The transmitter achieves the best mix of array gain – obtained by beamforming, and diversity gain – obtained by using multiple transmit beamformers and space-time coding. We use a multi input single output (MISO) transmitter and receiver structure to present a detailed performance analysis, that shows the array gain vs. diversity gain trade-off as the fading correlation changes. This analysis is validated by simulation results.

Keywords

Array Processing, Communication, Fading, Estimation and Detection.

I. Introduction

In this paper we study the optimal use of an antenna array at a cellular base station (BS), while the mobile station (MS) has a single antenna, also called a multi input single output (MISO) system. We are interested in the use of a combination of beamformers and transmit diversity. Transmit diversity systems are typically optimal in the rich scattering environments existing in indoor or pico-cellular environments, which means that the fading between the antenna elements are independent. Thus diversity gain is maximized when the variance of the receiver SNR is minimized. On the other hand, antenna gain is maximized by doing beamforming, typically in line of sight, or low scattering environments where the fading is correlated. Maximizing antenna gain means that we are maximizing the average SNR at the receiver. We are interested in the performance of these technologies as the fading correlation changes from perfectly correlated to completely independent.

Some of the recent work in using arrays in the downlink addressed optimal solutions for particular environments. Transmit diversity systems which are optimal in independently faded environments have been proposed by [1], [2], [3],[4], [5] and others. In [1] the authors introduced the concept of space time block codes (STBC) which includes the Alamouti codes as a special case. The same information is transmitted on all the antenna elements, but is multiplexed in a different way on each antenna. In the Space Time Spreading (STS) system proposed by [2] the information is code multiplexed on different antennas for a CDMA system. Both systems introduce diversity at the cost of losing array gain.

In this work we consider Frequency Division Duplexing (FDD) systems, where the up and downlink channels are fading independently. Therefore the BS is unable to estimate the downlink channel, which is needed for designing the downlink beamformer. To overcome this problem, some authors such as [6] proposed a feedback system where this channel information is transmitted back to the base station. This feedback is not desirable in practice because it uses up some of the uplink capacity. More recently [7] proposed a method that combines beamforming and STS for a CDMA2000 system. They used 2 arrays, each with 2 elements, and did beamforming with each of the 2 element arrays. Others, like [8] also proposed combining space-time codes and beamformers for a MIMO system. They considered a scenario of a number of distinct point sources, and showed through simulations how the Frame Error Rate improves as they use different sized space-time codes. Lately, some performance analyses of MIMO and MISO systems was published by [9],[10],[11] and [12]. We will elaborate on this work in the discussion section, since these results agree with the results in this paper.

In this paper we describe a linear open-loop transmit diversity system which does not require feedback. This system combines STBC's with multiple beamformers, and the performance can be optimized for a given fading environment. Since the STBC system has pure diversity gain and a beamformer has pure array gain, we strive to find a system with an optimum diversity/antenna gain ratio. Indeed our system changes from a pure STBC transmit diversity system if the channels are independently faded, to some hybrid system for partially correlated fading channels, to a single beamformer for perfectly correlated fading channels. Based on the available knowledge at the transmitter, we consider the following scenarios:

1. The channel statistics are known at the transmitter.
2. The channel statistics are unknown at the transmitter.
3. The Channel is known at the Transmitter. This represents the ideal case where Maximum Ratio Combining (MRC) is possible and will have the best possible performance.

Scenarios (1) and (2) are useful in demonstrating the antenna gain vs. diversity gain trade off which happens here. Scenario (3) is possible either in a time division duplexing (TDD) system, or by using excessive feedback from the MS and is therefore unrealistic from an FDD implementation perspective, but is interesting as a reference. In all the scenarios we assume that the channel is known perfectly at the receiver, however we will present simulation results where the channel is estimated. In the absence of feedback from the mobile, the downlink channel covariance matrix is not available at the base station (BS). Our basic assumption here is that the covariance matrix estimated from uplink measurements can be substituted for the downlink covariance matrix. This assumption is supported by the fact that the covariance matrix is a function of the mobile location and the location of the scatterers, which are the same for the uplink and the downlink. We present closed form expressions for the Signal to Noise Ratio (SNR), and the bit error rate for each scenario considered. Finally we show the results of some numerical simulations to demonstrate the improved performance of this system, and to validate the analytical results.

Notation: We use lower case boldface letters to denote vectors and upper case boldface letters to denote matrices. In addition $()^H$ means the conjugate transpose, $()^T$ means transpose, $\otimes$ means the Kronecker product, $\Re(\mathbf{A})$ means the real part of $\mathbf{A}$, $\Im(\mathbf{A})$ means the imaginary part of $\mathbf{A}$, E[] means the expected value, and $\mathbf{I}$ means the identity matrix.

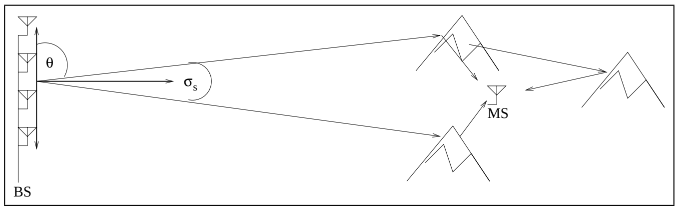

Fig. 1. The Communication System and array coordinates

II. Problem Statement

Consider an antenna array at a BS used to transmit on an FDD system to a MS with a single antenna as depicted in Fig.1. In this section we derive the structure of the transmit diversity system which adapts to the different types of fading environments corresponding to different angular spreads of the signal.

A. The Transmitter Structure

Consider a general linear transmitter $\mathcal{W}$, transmitting a block of complex data $\mathbf{x}$, separated into real and imaginary parts $\mathbf{x}_r$ and $\mathbf{x}_i$ each of length N on M antennas over K symbol periods. Note that $\mathcal{W}$ performs linear *temporal and spatial* processing of the data, and the transmitter matrix $\mathcal{W}$ is thus $2MK \times 2N$. The elements of $\mathbf{x}$ are assumed to have zero mean and unit variance. The transmitted sequence can be represented in real and imaginary form as

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_{1r} \\ \mathbf{y}_{1i} \\ \vdots \\ \mathbf{y}_{Kr} \\ \mathbf{y}_{Ki} \end{bmatrix} = \sqrt{\frac{S}{N}} \begin{bmatrix} \mathbf{W}_{r1} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{i1} \\ \vdots & \vdots \\ \mathbf{W}_{rK} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{iK} \end{bmatrix} \begin{bmatrix} \mathbf{x}_r \\ \mathbf{x}_i \end{bmatrix} = \mathcal{W}\mathbf{x}, \tag{1}$$

where both $\mathbf{W}_{rk}$ and $\mathbf{W}_{ik}$ are $M \times N$ and the transmitted power is normalized to s, the signal power, assuming that the squared norm of $\mathcal{W}\mathbf{x}$ is N, for example $s_{QPSK} = s_{BPSK}/2$. This block of transmitted symbols ($\mathcal{W}\mathbf{x}$) is then received by a receiver with a single receive antenna over a complex channel $\mathcal{A}_c$,

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_r \\ \mathbf{r}_i \end{bmatrix} = \begin{bmatrix} \Re\{\mathcal{A}_c\mathcal{J}\} \\ \Im\{\mathcal{A}_c\mathcal{J}\} \end{bmatrix} \mathbf{y} + \begin{bmatrix} \mathbf{n}_r \\ \mathbf{n}_i \end{bmatrix} = \mathcal{A}\mathcal{W}\mathbf{x} + \mathbf{n} \tag{2}$$

where $\mathcal{A}$ is the real representation of the complex fading channel, and

$$\mathcal{J} = \mathbf{I}_K \otimes ([1\ j] \otimes \mathbf{I}_M) \tag{3}$$

is a matrix that converts from a real to a complex representation, and $\mathbf{n}$ is a $2K \times 1$ vector of i.i.d. real white Gaussian noise with elements having unit variance.

B. The Receiver Structure

In this analysis we will impose certain constraints on the receiver, namely,

- it must be linear,
- it must be able to reconstruct the transmitted data perfectly in the noiseless case, thus have zero inter symbol interference and
- it may not color the noise.

This receiver $\mathcal{V}$, will reconstruct the transmitted data $\mathbf{x}$,

$$\hat{\mathbf{x}} = \mathcal{V}\mathbf{r} = \mathcal{V}\mathcal{A}\mathcal{W}\mathbf{x} + \mathcal{V}\mathbf{n}. \tag{4}$$

Based on the receiver constraints listed we can state that

$$\mathcal{V}\mathcal{A}\mathcal{W} = \sqrt{\gamma}\mathbf{I}_{2N}, \tag{5}$$

which means that $\mathcal{V}$ must be the scaled left pseudo inverse of $\mathcal{A}\mathcal{W}$, namely

$$\mathcal{V} = \sqrt{\gamma}(\mathcal{W}^{\mathcal{T}}\mathcal{A}^{\mathcal{T}}\mathcal{A}\mathcal{W})^{-1}\mathcal{W}^{\mathcal{T}}\mathcal{A}^{\mathcal{T}}, \tag{6}$$

and to ensure that the detected noise is i.i.d. with unit variance, $\mathcal{V}$ has to be orthonormal which implies that

$$\mathcal{W}^{\mathcal{T}}\mathcal{A}^{\mathcal{T}}\mathcal{A}\mathcal{W} = \gamma\mathbf{I}_{2N}. \tag{7}$$

This results in

$$\hat{\mathbf{x}} = \sqrt{\gamma}\mathbf{x} + \hat{\mathbf{n}}, \tag{8}$$

where γ represents the received signal to noise ratio, and $\hat{\mathbf{n}}$ the corresponding noise.

C. Transceiver Structure for Flat-Fading Channels

Let us now place some restrictions on the channel matrix $\mathcal{A}$. If we are dealing with a quasi static flat fading channel (i.e. one which is constant during the transmission of the block of data $\mathbf{x}$), then $\mathcal{A}_c = \mathbf{I}_K \otimes \mathbf{a}^{\mathbf{H}}$ where $\mathbf{a}$ is the $M \times 1$ channel vector from the M transmit antennas to the receive antenna. Thus

$$\mathcal{A}_c\mathcal{J} = \mathbf{I}_K \otimes \left[\mathbf{a}^{\mathbf{H}} \; j\mathbf{a}^{\mathbf{H}}\right], \tag{9}$$

$$\Re\{\mathcal{A}_c\mathcal{J}\} = \Re\begin{bmatrix} [\mathbf{a}^{\mathbf{H}} \; j\mathbf{a}^{\mathbf{H}}] & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & [\mathbf{a}^{\mathbf{H}} \; j\mathbf{a}^{\mathbf{H}}] \end{bmatrix} = \begin{bmatrix} [\mathbf{a}_r^T \; \mathbf{a}_i^T] & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & [\mathbf{a}_r^T \; \mathbf{a}_i^T] \end{bmatrix}, \tag{10}$$

$$\Im\{\mathcal{A}_c\mathcal{J}\} = \Im\begin{bmatrix} [\mathbf{a}^{\mathbf{H}} \; j\mathbf{a}^{\mathbf{H}}] & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & [\mathbf{a}^{\mathbf{H}} \; j\mathbf{a}^{\mathbf{H}}] \end{bmatrix} = \begin{bmatrix} [-\mathbf{a}_i^T \; \mathbf{a}_r^T] & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & [-\mathbf{a}_i^T \; \mathbf{a}_r^T] \end{bmatrix}. \tag{11}$$

Under these channel conditions we can write

$$\mathcal{W}^{\mathcal{T}}\mathcal{A}^{\mathcal{T}} = \begin{bmatrix} \mathbf{W}_{1r}^T\mathbf{a}_r & \ldots & \mathbf{W}_{Kr}^T\mathbf{a}_r & -\mathbf{W}_{1r}^T\mathbf{a}_i & \ldots & -\mathbf{W}_{Kr}^T\mathbf{a}_i \\ \mathbf{W}_{1i}^T\mathbf{a}_i & \ldots & \mathbf{W}_{Ki}^T\mathbf{a}_i & \mathbf{W}_{1i}^T\mathbf{a}_r & \ldots & \mathbf{W}_{Ki}^T\mathbf{a}_r \end{bmatrix} \tag{12}$$

$$\mathcal{W}^{\mathcal{T}}\mathcal{A}^{\mathcal{T}}\mathcal{A}\mathcal{W} = \sum_{k=1}^{K} \begin{bmatrix} \mathbf{W}_{kr}^T(\mathbf{a}_r\mathbf{a}_r^T + \mathbf{a}_i\mathbf{a}_i^T)\mathbf{W}_{kr} & \mathbf{W}_{kr}^T(\mathbf{a}_r\mathbf{a}_i^T - \mathbf{a}_i\mathbf{a}_r^T)\mathbf{W}_{ki} \\ \mathbf{W}_{ki}^T(\mathbf{a}_i\mathbf{a}_r^T - \mathbf{a}_r\mathbf{a}_i^T)\mathbf{W}_{kr} & \mathbf{W}_{ki}^T(\mathbf{a}_r\mathbf{a}_r^T + \mathbf{a}_i\mathbf{a}_i^T)\mathbf{W}_{ki} \end{bmatrix}. \tag{13}$$

To ensure that $\mathcal{W}^{\mathcal{T}}\mathcal{A}^{\mathcal{T}}\mathcal{A}\mathcal{W}$ is proportional to identity, the range space of $\mathbf{W}_{kr}$ has to lie in the null space of $\mathbf{W}_{mr}, \forall\, m \neq k$, and similarly for $\mathbf{W}_{ki}$. Furthermore, $\mathbf{W}_{mr}$ must be orthogonal to $\mathbf{W}_{mi}$, thus

$$\mathbf{a}_r^T\mathbf{W}_{kr}\mathbf{W}_{mr}^T\mathbf{a}_r = \mathbf{a}_r^T\mathbf{W}_{ki}\mathbf{W}_{mi}^T\mathbf{a}_r = \begin{Bmatrix} 0 & k \neq m \\ \mathbf{a}_r^T\mathbf{a}_r & k = m \end{Bmatrix} \tag{14}$$

$$\sum_{k=1}^{K} \mathbf{a}_r^T\mathbf{W}_{ki}\mathbf{W}_{kr}^T\mathbf{a}_r = \sum_{k=1}^{K} \mathbf{a}_r^T\mathbf{W}_{kr}\mathbf{W}_{ki}^T\mathbf{a}_r = 0, \tag{15}$$

for any real vector $\mathbf{a}_r$. In other words, $\{\mathbf{W}_{kr}, \mathbf{W}_{ki}\}$ are a set of rotation matrices which will take any given real vector and rotate it into an orthogonal set of vectors. Such square matrices can be constructed for $M = 2, 4, 8$ for real constellations of $\mathbf{x}$, for $M = 2$ for complex constellations of $\mathbf{x}$, but can be constructed for more M's for both real and complex constellations, if the matrices don't need to be square as was shown by [1]. This means that the receiver can be simplified to

$$\mathcal{V} = (\mathbf{a}^{\mathbf{H}}\mathbf{a})^{-\frac{1}{2}} \left[\bar{\mathbf{W}}_1^T \mathbf{a}_c, ..., \bar{\mathbf{W}}_{2K}^T \mathbf{a}_c\right], \tag{16}$$

where

$$\bar{\mathbf{W}}_k = \begin{cases} \begin{bmatrix} \mathbf{W}_{kr} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{ki} \end{bmatrix} & \forall\ k \leq K \\ \\ \begin{bmatrix} \mathbf{0} & -\mathbf{W}_{k-K,r} \\ \mathbf{W}_{k-K,i} & \mathbf{0} \end{bmatrix} & \forall\ K < k \leq 2K \end{cases} \tag{17}$$

$$\mathbf{a}_c = \begin{bmatrix} \mathbf{a}_r \\ \mathbf{a}_i \end{bmatrix}. \tag{18}$$

Note that $\mathbf{a}_c^T \mathbf{a}_c = \mathbf{a}^{\mathbf{H}}\mathbf{a}$. We will only consider the stricter case of $N = M$, that is square matrices, in this paper. Note that $N > M$ for the generalized Space Time codes as presented by [1]. For simplicity we will assume $K = N$, so that there is no spreading of the bandwidth.

When we generalize the structure of the channel to $\mathbf{B}^{\mathbf{H}}\mathbf{a}$, for some arbitrary matrix $\mathbf{B}$, (the purpose will be explained later), then we need to replace all instances of $\mathbf{a}_r$ with $\Re(\mathbf{B}^{\mathbf{H}}\mathbf{a})$, and all instances of $\mathbf{a}_i$, with $\Im(\mathbf{B}^{\mathbf{H}}\mathbf{a})$, in the previous equations. Note that the introduction of $\mathbf{B}$ does not change anything as far as the $\bar{\mathbf{W}}_k$ orthogonal matrices are concerned, however the sum of the norms squared of the columns of $\mathbf{B}$ must equal L in order to preserve the total transmitted power. The introduction of $\mathbf{B}$ makes it possible to let $\mathbf{a}$ have different dimensions than that of $\mathbf{W}_{kr}$ and $\mathbf{W}_{ki}$. For example, $\mathbf{W}_{kr}$ and $\mathbf{W}_{ki}$ could be chosen to be $L \times L$ matrices, for any $1 \leq L \leq N$ as long as $\mathbf{B}$ is $M \times L$, and $N = M$. The reason for choosing the dimension L to be different from N (or M) will become clear later. Recall that $\mathbf{B}\mathcal{J}\mathcal{W}\mathbf{x}$ is the vector of transmitted signals at the M

antennas. We can interpret the matrix $\mathbf{B}$ as a bank of L complex beamformers, whose L inputs are the elements of $\mathcal{JW}\mathbf{x}$. In other words, the transmitter structure is as shown in Fig.2. We can thus rewrite the receiver as

$$\mathcal{V} = (\mathbf{a^H BB^H a})^{-\frac{1}{2}} \left[\bar{\mathbf{W}}_1^T \mathbf{a}_c, ..., \bar{\mathbf{W}}_{2L}^T \mathbf{a}_c\right], \tag{19}$$

and then

$$\mathcal{VAW} = \sqrt{\frac{s}{L}} \frac{\sum_{k=1}^{2L} \bar{\mathbf{W}}_k^T \mathbf{a}_c \mathbf{a}_c^T \bar{\mathbf{W}}_k}{\sqrt{\mathbf{a^H BB^H a}}} \tag{20}$$

$$\gamma = \frac{s}{L} \frac{(\mathbf{a^H BB^H a})^2}{\mathbf{a^H BB^H a}} = \frac{s}{L} \mathbf{a^H BB^H a}. \tag{21}$$

The receiver in (16) is basically the inverse operation of the transmitter, it rotates the received signal (using the STBC's $\bar{\mathbf{W}}_l^T$) into orthogonal directions to remove interference from the different transmitted signals. Also, the receiver only needs to estimate $\mathbf{B^H a}$ which only has L complex unknowns compared to estimating $\mathbf{a}$, which has M complex unknowns.

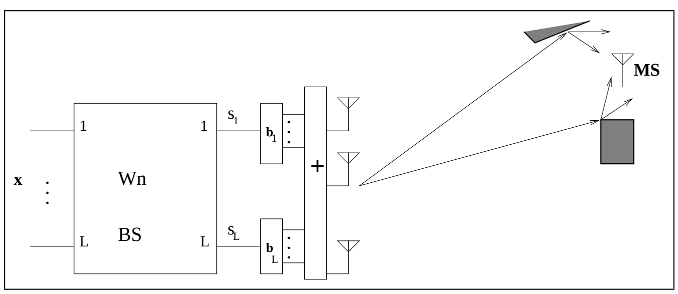

Fig. 2. The structure of the BS transmitter

D. Beamformer Design

Here, we attempt to design the bank of beamformers to maximize the average SNR at the receiver, which is equivalent to maximizing the effective *array gain*. Let us assume that the matrix $\mathbf{B}$ is an $M \times L$ matrix with orthonormal columns (this assumption is not strictly necessary but is convenient for the following interpretation). Then (21) can be written as

$$\gamma = \frac{s}{L} ||P_{\mathbf{B}} \mathbf{a}||^2 \tag{22}$$

where $P_{\mathbf{B}} = \mathbf{B}\mathbf{B}^H$ denotes the projection operator into the subspace spanned by the columns of $\mathbf{B}$. Examination of the equation above shows that the optimization of the average SNR involves a trade-off between increasing the dimension of the subspace $\mathbf{B}$ to maximize the numerator (projecting $\mathbf{a}$ on a bigger subspace), and decreasing the subspace dimension in order to minimize the denominator (i.e. maximize $1/L$). To make this more precise, consider the eigendecomposition of the covariance matrix $\mathbf{R}$

$$\mathbf{R} = E[\mathbf{a}\mathbf{a}^H] = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^H \tag{23}$$

where $\mathbf{\Gamma}$ is the matrix whose columns are the eigenvectors of $\mathbf{R}$ and $\mathbf{\Lambda}$ is the diagonal matrix consisting of λ_m, the (real) eigenvalues arranged in descending order. Let us further denote by $\mathbf{\Gamma}_L$ a matrix consisting of the first L columns of $\mathbf{\Gamma}$. In other words, this matrix consists of the eigenvectors corresponding to the L largest eigenvalues of $\mathbf{R}$. Selecting

$$\mathbf{B} = \mathbf{\Gamma}_L \tag{24}$$

will give the largest average value of $||P_{\mathbf{B}}\mathbf{a}||^2$ over all other possible choices of $\mathbf{B}$. It is straightforward to show that

$$E[||P_{\mathbf{\Gamma}_L}\mathbf{a}||^2] = \sum_{m=1}^{L} \lambda_m. \tag{25}$$

In the next section we do a performance analysis which captures both the array gain and the diversity advantage.

III. BER Performance Analysis

The BER performance analysis is done for BPSK modulation, due to the fact that it is a real constellation, and therefore it gives us the greatest flexibility in showing the array gain vs. diversity gain trade-off. This is because STBC's of sizes 2,4 and 8 can be used. Let us consider the 3 basic cases mentioned before.

A. Unknown covariance at the BS

When $\mathbf{R}$ is unknown at the BS, the transmitter does not do any beamforming, ($\mathbf{B} = \mathbf{I}$) and then the SNR in (21) is given by

$$\gamma = \frac{s}{M}\mathbf{a}^{\mathbf{H}}\mathbf{a} = \sum_{m=1}^{M} \frac{s}{M}\lambda_m \chi_2^2, \tag{26}$$

where χ_2^2 denotes a chi-square random variable with 2 degrees of freedom. The probability density function (PDF) of the SNR in (26) is a weighted sum of magnitudes squared of complex Gaussians which has been described by [13] as

$$p(\gamma) = \sum_{m=1}^{M} \alpha_m p_m(\gamma), \tag{27}$$

where

$$p_m(\gamma) = \frac{M}{s\lambda_m} e^{-\frac{M\gamma}{s\lambda_m}}, \tag{28}$$

$$\alpha_m = \prod_{k=1}^{M-1} \frac{\lambda_m}{(\lambda_m - \lambda_k)} \ \forall \ \lambda_m \neq \lambda_k. \tag{29}$$

In order to find the probability of error (P_e) of a BPSK system operating in this fading environment, we would have to evaluate

$$P_e = \int_0^\infty Q(\sqrt{2\gamma}) p(\gamma) d\gamma, \tag{30}$$

where $Q()$ is the Marcum's Q-function [14]. This derivation was done in ([14], Chapter 14), for the case of $M = 1$. A simple extension of this derivation leads to

$$P_e = \sum_{m=1}^{M} \alpha_m \frac{1}{2}\left(1 - \sqrt{\frac{\frac{s}{M}\lambda_m}{1 + \frac{s}{M}\lambda_m}}\right) = P(M, \frac{s}{M}, \lambda_1 ... \lambda_M). \tag{31}$$

where P is used as a notational device to denote that P_e is a function of the dimension M, the SNR s, and the eigenvalues.

B. Known covariance at the BS

When the subspace is known at BS, then the SNR in (21) is given by

$$\gamma = \frac{s}{L} \mathbf{a^H B B^H a} = \sum_{m=1}^{L} \frac{s}{L} \lambda_m \chi_2^2. \tag{32}$$

Therefore the average SNR and BER are

$$\overline{\text{SNR}} = E[\gamma] = \frac{s}{L} \sum_{m=1}^{L} \lambda_m \tag{33}$$

$$P_e = P(L, \frac{s}{L}, \lambda_1 ... \lambda_L). \tag{34}$$

Note that if weighted (w_m) eigenbeamformers are used instead such that $\sum_m w_m = L$, therefore transmitting proportionally more power in the directions where the eigenvalues are strong, the performance will be

$$\overline{\text{SNR}} = E[\gamma] = \frac{s}{L}\sum_{m=1}^{L}\lambda_m w_m \tag{35}$$

$$P_e = P(L, \frac{s}{L}, \lambda_1 w_1 ... \lambda_L w_L). \tag{36}$$

C. Known channel at the BS (MRC)

Let us consider the special case where the channel is known at the BS. This represents the Maximum Ratio Combining (MRC) case which will have the best performance possible. In this case we will choose $L = 1$,

$$\mathbf{B} = \frac{1}{\sqrt{\mathbf{a^H a}}}\mathbf{a}, \tag{37}$$

and consequently

$$\gamma = s\mathbf{a^H a} = s\sum_{m=1}^{M}\lambda_m \chi_2^2. \tag{38}$$

Therefore the average SNR and BER are

$$\overline{\text{SNR}} = E[\gamma] = sM \tag{39}$$

$$P_e = P(M, s, \lambda_1 ... \lambda_M). \tag{40}$$

D. Performance Summary

The Probability of error equation in (31) takes the general form of

$$P_e = P(D, \frac{\overline{\text{SNR}}}{\sum_{m=1}^{D}\lambda_m}, \lambda_1 ... \lambda_D), \tag{41}$$

where D represents the diversity order of the system and $\overline{\text{SNR}}$ – the average SNR. In Table I we summarize these quantities for the different scenarios considered in this paper. Note that when $L = M$ we have $\sum_{m=1}^{M}\lambda_m = M$. For a well designed beamformer with $L < M$ we will usually have $\sum_{m=1}^{L}\lambda_m \approx M$.

	Beamformer	Known at BS	Known at MS	SNR $= \gamma$	$\overline{\text{SNR}}$	D
1	$\mathbf{B} = \mathbf{\Gamma}_L$	subspace	channel	$\frac{s}{L}\mathbf{a^H B B^H a}$	$\frac{s}{L}\sum_{m=1}^{L}\lambda_m$	L
2	$\mathbf{B} = \mathbf{I}$	–	channel	$\frac{s}{M}\mathbf{a^H B B^H a}$	s	M
3	$\mathbf{B} = \frac{\mathbf{a}}{\sqrt{\mathbf{a^H a}}}$	channel(MRC)	–	$s\mathbf{a^H B B^H a}$	sM	M

TABLE I

Summary of Performance Analysis

IV. Discussion

The performance analysis makes it clear that maximizing the average SNR will depend on the distribution of the eigenvalues of $\mathbf{R}$. When there is only one non-zero eigen value the fading is perfectly correlated. The other eigen values grows as the correlation decreases until all the eigen values are equal when the fading is independent. Therefore $E[\gamma]$ is independent of the choice of L. In this case it is desirable to let $L = M$ in order to maximize the diversity advantage which is *not* reflected in the average SNR value. Note that $E[\gamma] = s$, which means that we have no array gain (i.e. unity gain). When there is no knowledge of the channel subspace at the transmitter, we have to assume that $\mathbf{a}$ exists in the entire M dimensional subspace and we will also choose $\mathbf{B} = \mathbf{I}$, which means that $L = M$. In the case of small angular spread $\mathbf{R}$ will essentially be a rank one matrix in which case $\lambda_1 = M$ while all the other eigenvalues are very small or zero. In this case $L = 1$ will maximize the average SNR. Note that $E[\gamma] = sM$, which means that we have maximum array gain (i.e. gain of M). When the angular spread has some intermediate value the choice of L which will maximize the average SNR will depend on the actual eigenvalue distribution and it must be determined on a case by case basis. In this case the effective array gain will be somewhere between unity and M. It should be noted that in general the value of L which will minimize BER may be different from the value of L which maximizes the average SNR because of the effect of diversity. As L is increased, diversity gain is increased. Thus, it may be advantageous to increase L beyond the value that maximizes the average SNR. In other words, we trade off a decrease in array gain (reflected in the average SNR) for an increase in diversity gain.

Some papers appeared quite recently which address a similar topic, namely the performance of MIMO and MISO systems in correlated fading. We show in this paper that the SNR for both a transmit diversity method and a beamformer is a function of the sum of all the eigenvalues, thus the received power. However, in [15] the authors showed that the capacity of a MIMO spatial multiplexer is proportional to the *minimum eigenvalue only.* In fact, the authors in [16] proposed an adaptive method that switches between MIMO spatial multiplexing and MIMO transmit diversity to maximize throughput based on the channel statistics. Interestingly, these authors used 4x4 non-orthogonal complex STBC's with a rate of 1, in order to do transmit diversity using a 4 element array. They found that the non-orthogonality severely restricts the performance when the SNR is high. In [10] they showed that the MIMO capacity scales linearly with number of antennas, even in correlated fading.

The paper by [9] (specifically the section on STBC) addresses the same MISO problem, and they arrive at the same result, although using a different approach. Their goal was to maximize throughput, whereas our goal is to study the array-gain vs. diversity gain trade-off when the correlation changes. Close inspection will show that their 2-D eigen-beamformer is similar (apart from the power loading, which we refer to in (36)) to the transmitter in Fig.2, for the case where $L = 2$. We believe that our analysis clearly demonstrates the array-gain vs. diversity-gain trade-off that is at play as the fading correlation changes. This is because we present the BER in a closed form and consider arbitrary sized STBC's restricted to only BPSK, while they consider performance bounds using 2x2 Alamouti STBC's for many different modulation schemes.

Other authors, like [11],[17] and [12], have studied the impact of fading correlation on the performance of STBC's. A very interesting interpretation is presented by [17], where they derive the relationship between the pole location of the characteristic function and the resulting PDF. They show that the pair wise error probability can be bounded based on the position of the poles. This bound is tighter than the traditional Chernoff bound.

In this paper we fix the rate by setting $N = K$ and try to minimize the BER. We show that a tradeoff exists between attaining array gain at the cost of decreased diversity gain. Alternatively, in [18] the authors try to maximize the throughput (rate) by switching between spatial multiplexing and transmit diversity for a MIMO system in an uncorrelated fading environment. Here, they show that a similar tradeoff exists where attaining multiplexing gain comes at the cost of decreased diversity gain.

V. Numerical Results

In the simulation examples we considered an $M = 8$ element uniform linear array (with half-wavelength spacing) BS transmit antenna. The MS is placed at broadside relative to the array. The angular spread of the signal in two experiments were set to $\sigma_s = 10°$ and $\sigma_s = 1°$ respectively, and the corresponding number of beamformers were selected to be $L = 2$ and $L = 1$, respectively. We thought that 1 and 10 degree scatterings are quite realistic for tower mounted BS antennas. In a BPSK scheme the imaginary components are all zero. Thus we used the space time codes as presented by [1], specifically when $L = 2$ then

$$\mathbf{W}_{1r} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \mathbf{W}_{2r} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{W}_{1i} = \mathbf{0}, \mathbf{W}_{2i} = \mathbf{0}, \tag{42}$$

and the receiver is

$$\mathcal{V} = (\mathbf{a^H B B^H a})^{-\frac{1}{2}} \begin{bmatrix} \mathbf{W}_{1r}^T \Re\{\mathbf{B^H a}\} & \mathbf{W}_{2r}^T \Re\{\mathbf{B^H a}\} & -\mathbf{W}_{1r}^T \Im\{\mathbf{B^H a}\} & -\mathbf{W}_{2r}^T \Im\{\mathbf{B^H a}\} \end{bmatrix}. \tag{43}$$

The receiver and codes for when $L = 4$ and $L = 8$ are not shown to save space.

A. Parameter Estimation

Practical implementation of the transmit diversity scheme described here requires knowledge of the the channel $\mathbf{B^H a}$ at the receiver. To estimate the channel we rewrite the complex received signal as

$$\mathbf{r}_c = \mathbf{a^H B S}_t + \mathbf{n}_c, \tag{44}$$

where the known transmitted signal is

$$\mathbf{S}_t = \sqrt{\frac{S}{L}} \begin{bmatrix} \bar{\mathbf{W}}_1 \mathbf{x} & \cdots & \bar{\mathbf{W}}_{2L} \mathbf{x} \end{bmatrix}, \tag{45}$$

and then estimate the channel by using a least squares method

$$\widehat{\mathbf{a^H B}} = \mathbf{r}_c \mathbf{S}_t^T (\mathbf{S}_t \mathbf{S}_t^T)^{-1}. \tag{46}$$

We will simulate a case where the channel is estimated just as a reference. In these simulations we used a training sequence of length $N_t = 8$ to do the channel estimation, followed by a data burst of $N_d = 160$ symbols.

B. Finding the optimum L

In this section we compare the effect that the number of beamformers has on the performance. The scenario considered here is where the transmitted signal has an angular spread of 1°, and the eigenvalues of $\mathbf{R}$ are

$$\lambda = [7.9886, 0.0114, 0, 0, 0, 0, 0, 0].$$

If the angular scattering region is increased to $\sigma_s = 10°$, then the eigenvalues are

$$\lambda = [6.9966, 0.9792, 0.0241, 0.0002, 0, 0, 0, 0].$$

In Figure 3 we plot the BER performance as the number of beamformers are increased at the BS from 1 to 8 for the 2 cases of signal spread. For the case of $\sigma_s = 1°$ (the top graph), the single beamformer is optimal which justifies our choice of $L = 1$ as used in Figure 6. For the case of $\sigma_s = 10°$ (the bottom graph), a single beamformer performs best in the low SNR case, but as the SNR increases the multi-beamformer systems outperform it due to their diversity gain. The BS with 8 beamformers performs more poorly because the increased diversity gain does not compensate for the complete loss of array gain. This bottom graph justifies our choice of $L = 2$ in Figure 5.

In Figure 4 the performance is shown as the number of beamformers is increased for different angular spreads ranging from $\sigma_s = 5°$ to $\sigma_s = 120°$. Once again we can see that each curve has a unique optimal choice of L. Note also that the optimal performance improves as the angular spread increases, because of the increased diversity.

C. Monte Carlo Simulations

In Figure 5 the Probability of Error curves derived in the Performance Analysis is superimposed on the MC simulation results, and they corresponds very closely. The case where $\mathbf{R}$ is known at the BS, (here we choose $L = 2$) performs nearly 6dB better than the case where $\mathbf{R}$ is unknown. This is confirmed by comparing the $\overline{\text{SNR}}$s in rows 1 and 2 in table I. Figure 6 shows the results of MC simulations when the angular spread is 1°, and here we choose $L = 1$.

VI. Conclusions

Transmit diversity is usually presented for the case of independently faded channels. In this paper we derive the structure of a transmit diversity system which can be optimized for a Rayleigh fading environment in which the fading may be correlated. The system attempts to achieve the best mix of array gain – obtained by beamforming, and diversity gain – obtained by using multiple transmit beamformers and space-time coding.

After deriving the transmitter and receiver structures we presented a performance analysis in the context of a digital communication system which uses this system. We derive closed form expressions for the bit-error-rate curves for a BPSK system for a Rayleigh fading environment characterized by the covariance matrix of the downlink channel response vector. The analysis and selected numerical results reveal the interplay of array gain and diversity gain. When the angular spread of the signal is very wide, the fading is uncorrelated between the different channels, and the system provides full diversity advantage but no array gain. On the other extreme, when the angular spread is very small the fading is highly correlated, and the system provides full array gain but no diversity gain. In intermediate situations the system provides the best possible combination of array gain and diversity gain.

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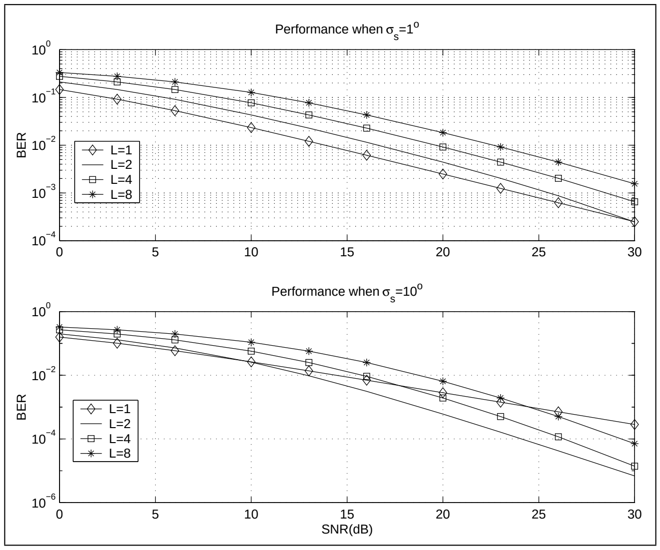

Fig. 3. The Performance of the transmit diversity system in a 1 and 10 degree scattering environment as L is increased from 1 to 8. Optimizing L is equivalent to choosing the L corresponding to the lowest curve for that SNR operating range.

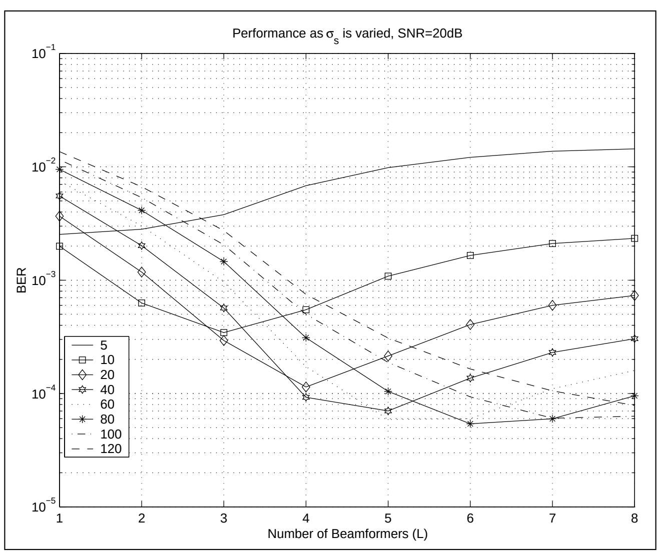

Fig. 4. The Performance of the transmit diversity system as σ_s is varied from $5-120°$. The SNR is fixed at 20dB. This shows that there is maximum performance gain if L is either 2 or 3 when $\sigma_s = 10°$.

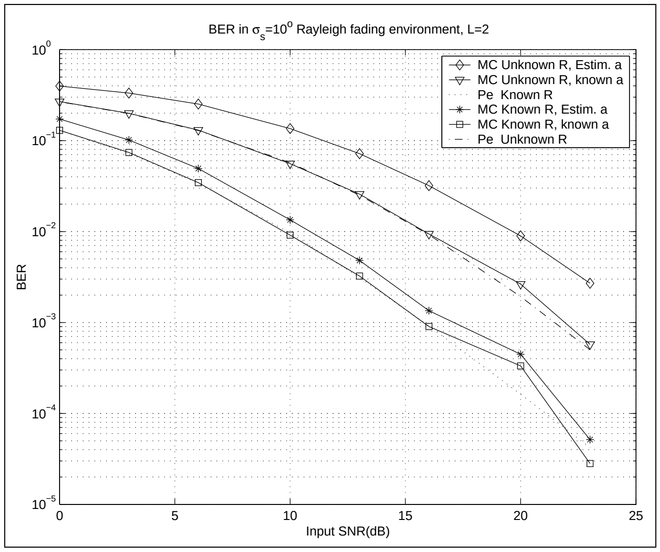

Fig. 5. The Predicted BER curves superimposed on the Monte Carlo Simulation results, for the 10° scattering environment with $L = 2$. The predicted curves were calculated using (31).

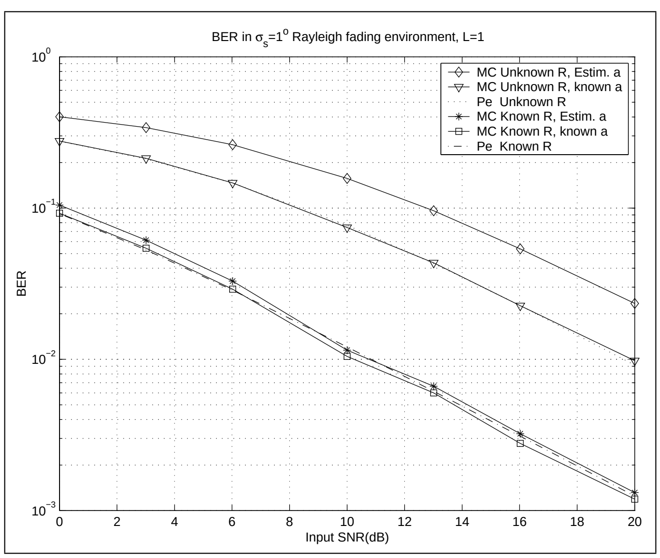

Fig. 6. The BER Performance plots of the Monte Carlo simulations in a 1° scattering environment with $L = 1$.