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On the Hough transform of a polygon

Peyman Milanfar

SRI International, 333 Ravenswood Ave., MS 409-44, Menlo Park, CA 94025, USA

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Abstract

It was recently shown that the Hough transform of a convex polygon uniquely determines it. In this note we generalize this result and lend further insight into why a non-convex polygon is, in general, not uniquely determined by its Hough transform.

Keywords: Hough transform; Polygon

It was recently shown in (Rosenfeld and Weiss, 1995) that a convex polygon is uniquely determined by its Hough transform. By invoking inherently geometric arguments, the authors also pointed out why the same is *not* true for non-convex polygons. In this note, using a more algebraic argument, we generalize this result to show that the *vertices* of a class of non-convex polygons are uniquely determined by their Hough transform. This argument will also further illuminate why an arbitrary non-convex polygon is, in general, not uniquely determined by its Hough transform.

To begin, consider a convex polygon P with vertices $v_i = [x_i, y_i]^T$, $i = 1, \dots, N$ (with $v_{N+1} = v_1$) ordered in, say, the counterclockwise direction. The Hough transform of this polygon is given by a set of N spikes in the Hough domain where each side s_i (connecting v_i to v_{i+1}) of the polygon is mapped to a unique point (θ_i, t_i). Here $0 \leqslant \theta_i \leqslant 2\pi$ is the direction of the normal to the side s_i, and t_i is the normal distance from the origin to s_i. See Fig. 1. Furthermore, the height of any spike is proportional to the length of the corresponding side.

First, we note that each *vertex* of P belongs to exactly two sides. That is to say, v_i is on both s_{i-1} and s_i. Therefore, given the Hough transform peaks for these two sides, (θ_{i-1}, t_{i-1}) and (θ_i, t_i), we can write

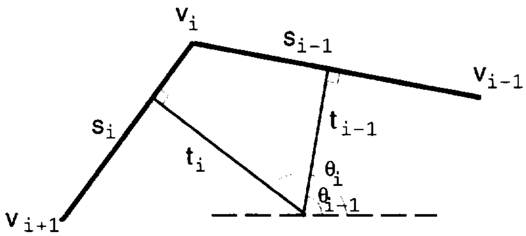

Fig. 1. Two adjacent sides and their Hough counterparts.

$$\begin{bmatrix} t_{i-1} \\ t_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_{i-1}) & \sin(\theta_{i-1}) \\ \cos(\theta_i) & \sin(\theta_i) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix}. \tag{1}$$

Assuming that P is non-degenerate (no two sides are collinear), we have that

$$\det \begin{bmatrix} \cos(\theta_{i-1}) & \sin(\theta_{i-1}) \\ \cos(\theta_i) & \sin(\theta_i) \end{bmatrix} = \sin(\theta_i - \theta_{i-1}) \neq 0. \tag{2}$$

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Hence (1) can be solved for the vertex v_i. In fact, given $0 \leqslant \theta_1 < \theta_2 \cdots < \theta_N \leqslant 2\pi$, we can solve (1) for every vertex and hence reconstruct the convex polygon.

Generalization to non-convex polygons. The reconstruction of vertices of a certain class of simply-connected *non-convex* polygons is also possible in the same fashion as described above. Namely, if no two sides of the polygon are collinear, then the correspondence between the sides of the polygon and the number of peaks in the transform domain will be one-to-one and the vertices will be recoverable as in (1). The difference here is that once the vertices are known, the interior of the polygon is not necessarily uniquely determined. For there is, in general, more than one way to "connect the dots". See Fig. 2. In this case, it may be possible to use the height of each spike to reconstruct the unique polygon that gave rise to the given Hough transform. Interestingly, we have come across quite similar non-uniqueness issues while studying the reconstruction of polygons from moments in (Milanfar et al., 1995).

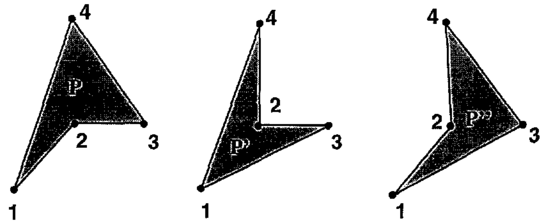

Fig. 2. Multiple ways of connecting the same set of vertices.

As Rosenfeld and Weiss correctly point out, the Hough transform of any "key-like" polygon (i.e., one with collinear non-adjacent sides) will *not* determine it uniquely. In our algebraic framework, this is because the mapping from the sides of the polygon to peaks in the Hough domain will not be one-to-one. Hence, for such polygons, (1) will be under-determined for some set of vertices and no unique solution will exist.

References

Milanfar, P., W.C. Karl and A.S. Willsky (1995). Reconstructing polygons from moments with connections to array processing. *IEEE Trans. Signal Process.* 43 (2), 432–443.

Rosenfeld, A. and I. Weiss (1995). A convex polygon is determined by its Hough transform. *Pattern Recognition Lett.* 16, 305–306.