Errata for "The Power of Temporal Proofs"

September 8, 1989

The proof of Theorem 6.2 in "The Power of Temporal Proofs" is seriously flawed. The claim was that all valid temporal formulas are arithmetical in the proof system T_2. The claim does not appear to be true. Indeed, Andréka, Németi, and Sain, who uncovered the problem, recently sent me a paper with a counterexample, "On the Strength of Temporal Proofs."

The main role of the claim was as a stepping stone towards Theorem 7.2. This claim, in turn, says that T_1 is complete for arithmetical formulas while T_2 is complete for all formulas. With the failure of Theorem 6.2, the result proved in 7.2 is weaker. It holds simply that both T_1 and T_2 are complete for arithmetical formulas. More precisely, Theorem 7.2 should read:

> For every formula u, $\vdash_O P(u) \Rightarrow \vdash_{T_1} u$ if u is arithmetical in $\vdash_{T_1}$.
> For every formula u, $\vdash_P P(u) \Rightarrow \vdash_{T_2} u$ if u is arithmetical in $\vdash_{T_2}$.

This final result may well be enough in practice—and, in particular, for the purposes of program verification. On the other hand, it would be pleasant to have a proof system complete for all formulas, rather than only for arithmetical ones.

Incidentally, the paper by Andréka, Németi, and Sain addresses some of the open questions in "The Power of Temporal Proofs," and I recommend it to the interested readers. It was presented at the 1989 International Symposium on Mathematical Foundations of Computer Science. I would also like to thank its authors for discovering this problem.